UPPSC-AE 2020

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Mechanical Engineering

Mechanics of Solids

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY Publications

Mechanics of Solids

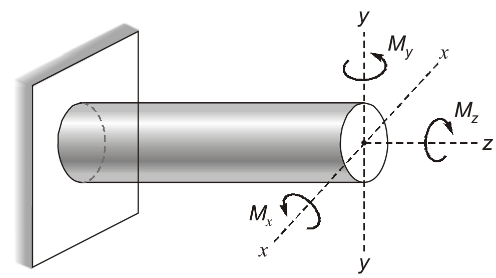
Contents

CHAPTER 5

Torsion of Shafts and Springs

5.1 Introduction

- Consider a circular prismatic bar subjected to M_x, M_y and M_z as shown in figure.
- In above case moments, M_x and M_y causes rotations about transverse axis. Hence, M_x and M_y are bending moments but M_z causes rotations about longitudinal axis or polar axis. Hence, M_z is torsional moment or twisting moment.



5.2 Difference between Bending Moment and Twisting Moment

Bending Moment (BM)

1. In bending, moment or couple acts about transverse axis i.e., x-x or y-y
2. In bending, the plane of cross-section rotates about NA
3. Due to bending, normal stresses are produced which vary from zero at NA to maximum at top and bottom surface i.e., at extreme fibres.

Twisting Moment (TM)

1. In twisting, couple acts about longitudinal axis or polar axis. i.e., z-z
2. In twisting, plane of cross-section rotates about polar axis. Hence, radii rotate about polar axis.
3. Due to twisting, only shear stresses are produced which acts in two mutually perpendicular planes and vary from zero at polar axis to maximum at the surface in circumferential directions.

5.3 Assumptions in the Theory of Pure Torsion

- The material of the shaft is uniform throughout.
- Torque is constant along the length.
- Shaft is of uniform circular section throughout, which may be hollow or solid.
- Cross-sections of the shaft, which are plane before twist remain plane after twist.
- All radii which are straight before twist remain straight after twist.

NOTE: When torque is applied on non-circular sections, the shear stress distribution is non-uniform, also wrapping may occur. Hence non-circular sections are torsionally weak.

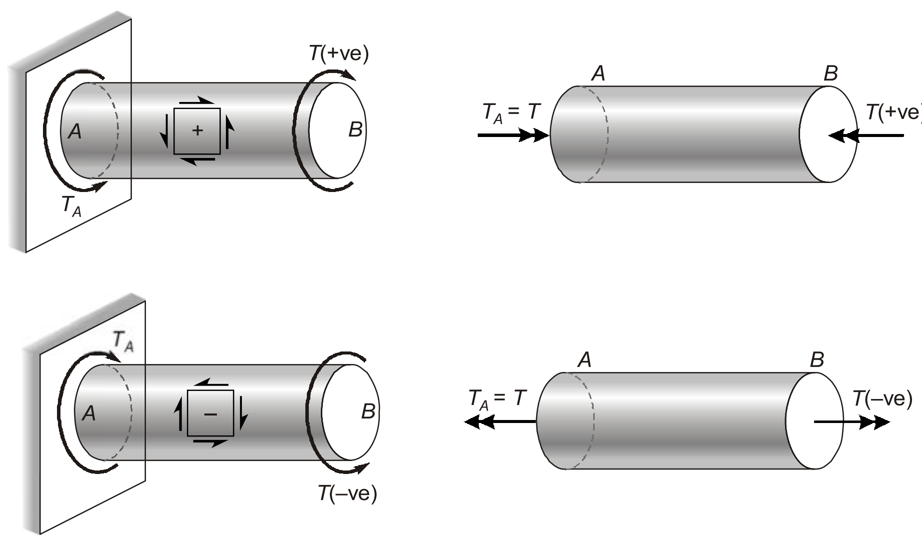
Example - 5.1 Which one of the following assumptions in the theory of pur torsion is false?

(a) The twist is uniform along the length
(b) The shaft is of uniform circular section throughout
(c) Cross-section plane before torsion remain plane after torsion
(d) All radii get twisted due to torsion

Solution: (a)

A member is said to be pure torsion if it is subjected to twisting couple in such a way that the magnitude of twisting moments remains constant throughout the length of the member. However, angle of twist is not uniform along the length, but varies linearly with length and is maximum at the free end.

5.4 Sign Convention of Torque



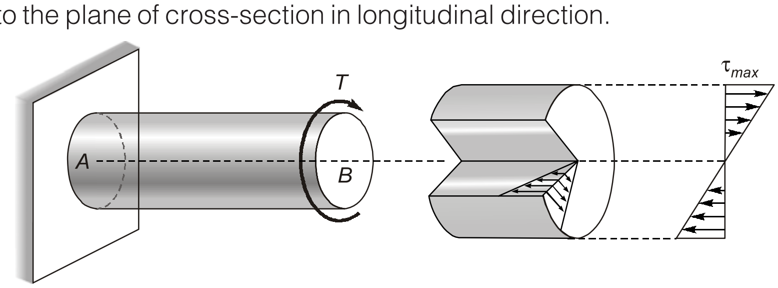
The sign convention can be explained with the help of right hand thumb rule. If torque is applied in the direction of right hand finger, the movement of nut due to torque will be in the direction of right hand thumb. If right hand thumb pointed towards cross-section, then torque is treated positive at that section. A positive torque will produce positive shear stress element on the surface and vice-versa.

5.5 Effects of Torsion

Due to torque, shear stresses are developed in two mutually perpendicular planes.

(i) In the plane of cross-section in circumferential direction.

(ii) Normal to the plane of cross-section in longitudinal direction.



The shear stress produced by torque vary from zero at centre of cross-section to maximum at surface in circumferential directions. Shear stresses in direction normal to the cross-section plane are always complimentary in nature and have equal magnitude.

Shear stress at any point at a distance r from the centre is given by torsional formula

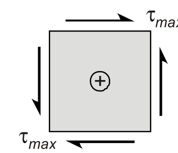
$$\boxed{\frac{T}{I_p} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

where, I_P = Polar moment of inertia; G = Modulus of rigidity; θ = Angle of twist in radian;
L = Length of shaft; R = Radius of shaft

Maximum shear stresses will occur when

$$r = r_{max} = R$$

$$\therefore \quad \tau_{max} = \frac{T}{I_P} \times r_{max} = \frac{T}{I_P} \times R \text{ (at surface of shaft)}$$



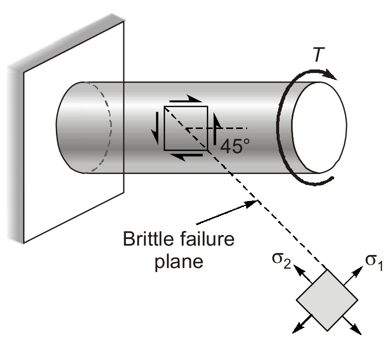
Hence, at surface of shaft any element as shown in figure will be subjected to τ_{max}.

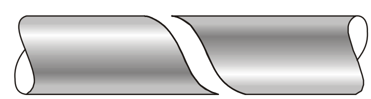
We know, in pure shear, principal stresses are

$$\sigma_1 = +\tau_{max} \text{(tensile)} \quad \text{and} \quad \sigma_2 = -\tau_{max} \text{(compressive)}$$

and maximum shear stress $$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\tau_{max} - (-\tau_{max})}{2} = \tau_{max}$$

Since brittle materials have minimum tensile strength, hence, brittle materials fail in tension.

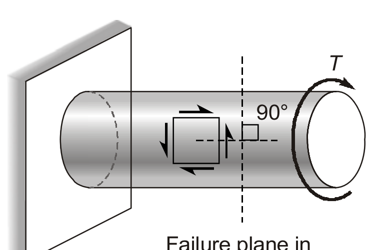


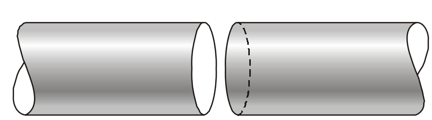


Torsional failure in brittle material

In torsion, maximum principal stresses are developed at 45° from longitudinal axis. Hence, brittle material failure plane is at an angle 45° with the longitudinal axis.

Since, ductile materials are weak in shear. Hence, ductile materials fail due to principal shear stress. In torsion test maximum shear stress is in the direction perpendicular to longitudinal axis. Hence, ductile failure plane in torsion will be perpendicular to longitudinal axis.





Torsional failure in ductile material

- Polar modulus = $\dfrac{\text{Polar moment of Inertia}}{\text{Maximum radius}}$

$$Z_p = \frac{I_P}{R}$$

Example - 5.2 A solid shaft transmitting torque T, the allowable shear stress is. The diameter is

(a) $\sqrt[3]{\dfrac{16T}{\pi\tau}}$ (b) $\sqrt[3]{\dfrac{32T}{\pi\tau}}$

(c) $\sqrt[3]{\dfrac{4T}{\pi\tau}}$ (d) $\sqrt[3]{\dfrac{64T}{\pi\tau}}$

Solution: (a)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$\Rightarrow \quad \tau = \frac{TR}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^4} \quad \Rightarrow \quad d = \left(\frac{16T}{\pi\tau}\right)^{1/3}$$

Example - 5.3 The ratio of the polar moment of inertia to the radius of the shaft is known as

(a) Shaft stiffness (b) Flexural rigidity
(c) Torsional rigidity (d) Torsional section modulus

Solution: (d)

- The greatest twisting moment which a given shaft section can resist = Maximum permissible shear stress × polar modulus

$$\Rightarrow \quad T = \tau_s Z_P$$

- For solid shaft, $T = \tau_s \dfrac{\pi d^3}{16}$

for hollow shaft, $T = \tau_s \dfrac{\pi}{16 d_o}\left(d_o^4 - d_i^4\right)$; where, d_o = outer diameter, d_i = inner diameter

5.5.1 Strength of shaft

The ability of shaft to resist the action of twisting moment is known as **strength of shaft**.

$$T = \tau \times Z_p$$

where, Z_p = Polar section modulus of shaft; τ = Permissible shear stress in shaft

For a shaft to be strongest its polar section modulus should be maximum. Generally, hollow circular shafts are more stronger than solid circular shafts.

5.5.2 Torsional Rigidity

Torsional rigidity is the product of modulus of rigidity G and polar moment of inertia I_P.

$$\text{Torsional} = GI_P$$

Its unit is Nm^2.

5.5.3 Torsional Stiffness

It is defined as "the torque required to produce unit angle of twist in the direction of twist.

Torsional stiffness, $K = \dfrac{GI_P}{L} = \dfrac{T}{\theta}$

It is also defined as "the torsional rigidity per unit length of shaft. It unit is N-m/radian.

Torsional Flexibility: It is the angle of twist produced by unit torque.

$$\text{Torsional flexibility} = \frac{L}{GI_P}$$

Its unit is radian/N-m.

Example - 5.4 If two shafts of the same length, one of which is hollow, transmit equal torques and have equal maximum stress, then they should have equal

(a) Polar moment of inertia
(b) Polar modulus of section
(c) Diameter
(d) Angle of twist

Solution: (b)

For Solid shaft, $\tau = \frac{16T}{\pi d^3} = \frac{T}{Z_P}$

For Hollow shaft, $\tau = \frac{16T}{\pi d^3\left(1-k^4\right)} = \frac{T}{Z_{P_1}}$

For same τ and T, $Z_p = Z_{p1}$

5.6 Power Transmitted by a Shaft

$$P = \frac{2\pi NT}{60} \text{ kW } = \text{Torque} \times \text{angle turned per second}$$

where, P = Power transmitted (kW); N = rotation per minute (rpm); T = mean torque (kN-m)

Example - 5.5 A solid shaft transmits 44 kW power at 700 rps. Calculate the torque produced (in Nm)

(a) 60
(b) 100
(c) 600
(d) 10

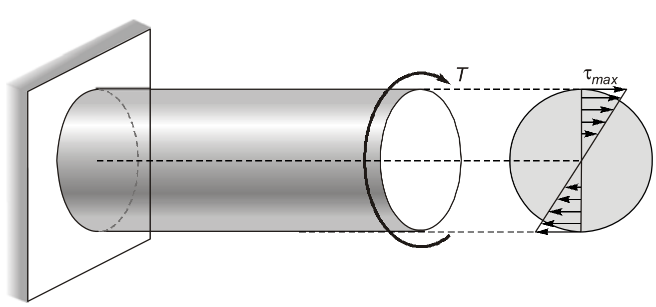
Solution: (d)

$$P = T\omega = T2\pi N$$

$$T = \frac{P}{2\pi N} = \frac{44000}{2\pi \times 700} = 10 \text{ Nm}$$

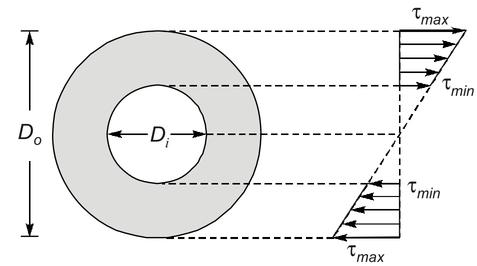
5.7 Shear Stress Distribution in Circular Section

(i) Solid circular shaft



$$I_P = \frac{\pi}{32}D^4 \text{ and } Z_P = \frac{\pi}{16}D^3$$

(ii) Hollow circular shaft

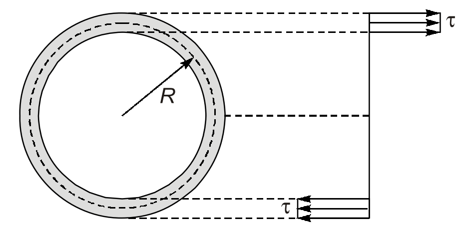


$$D_i = \text{internal diameter}$$
$$D_o = \text{outer diameter}$$
$$I_P = \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$$

and
$$Z_P = \frac{\pi\left(D_o^4 - D_i^4\right)}{16D_o}$$

(iii) Thin circular tube with mean radius R

Let t is thickness of tube



$$I_P = AR^2$$

where
$$A = (2\pi R)\times t$$

∴
$$I_P = (2\pi Rt)R^2 = 2\pi R^3 t$$

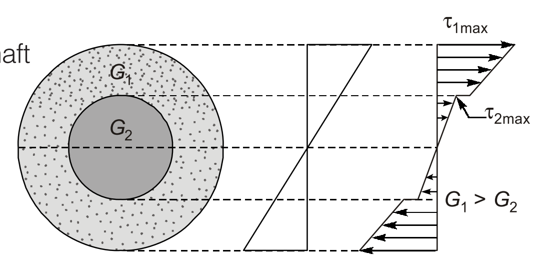
Now,
$$Z_p = \frac{I_P}{R_{max}} = \frac{2\pi R^3 t}{R} = 2\pi R^2 t$$

The shear stress distribution is assumed uniform across the thickness and is given by

$$\tau = \frac{T}{Z_P}$$

(iv) Composite circular shaft: Consider a composite circular shaft made from two materials whose modulus of rigidity are G_1 and G_2

In composite shaft, total torque T is shared by both shaft



$$T_1 + T_2 = T$$

∴
$$\tau_{1,\,max} = \frac{T_1}{Z_{P1}}$$

and
$$\tau_{2,\,max} = \frac{T_2}{Z_{P2}}$$

5.8 Design of Shaft

Shaft is designed on the basis of following two criteria,

(i) Strength criteria

$$\tau \le \tau_P$$

where, τ_P is permissible shear stress, τ is given as,

$$\tau = \frac{T}{I_P}R \Rightarrow \frac{T}{Z_P}$$

(ii) Stiffness criteria

$$\theta \le \theta_P$$

Where, θ_P is permissible angle of twist, θ is given by,

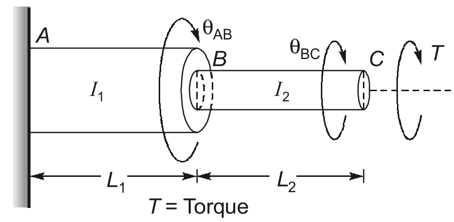
$$\frac{T}{I_P} = \frac{G\theta}{L}$$

∴
$$\theta = \frac{TL}{GI_P}$$

The diameter of shaft will be greater value that is calculated by strength criteria or stiffness criteria.

5.9 Shafts in Series and Shafts in Parallel

5.9.1 Shafts in Series



- Torque T will be same for both the shafts.
- The twists θ_1 and θ_2 will be different for both the shafts.

$$\frac{T}{I_{p_1}} = \frac{G_1\theta_1}{l_1}$$

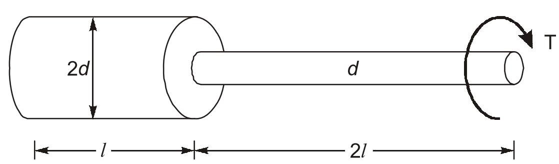
$$\frac{T}{I_{p_2}} = \frac{G_2\theta_2}{l_2}$$

Where, T = Torque; G_1, G_2 = Modulus of rigidity for shafts 1 and 2; l_1, l_2 = length of shaft 1 and 2

$$\frac{\theta_1}{\theta_2} = \frac{I_{p_2}}{I_{p_1}} \cdot \frac{l_1}{l_2} \cdot \frac{G_2}{G_1} = \left(\frac{d_2}{d_1}\right)^4 \cdot \frac{l_1}{l_2} \cdot \frac{G_2}{G_1}$$

Where, θ_1, θ_2 = angle of twist; I_{p_1}, I_{p_2} = polar moments of inertia

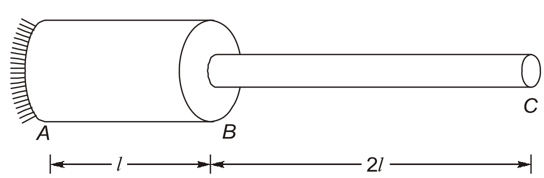
Example - 5.6 Calculate the total angle of twist for a stepped shaft which is subjected to the torque (T) as shown in the figure below:



(a) $\frac{Tl}{\pi G d^4}$ (b) $\frac{66Tl}{\pi G d^4}$

(c) $\frac{Tl}{66\pi G d^4}$ (d) $\frac{36Tl}{\pi G d^4}$

Solution: (b)



For AB,

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{Tl}{GJ} = \frac{Tl \times 32}{G\pi(2d)^4} = \frac{2Tl}{G\pi d^4}$$

For BC,

$$\theta = \frac{T \times 2l \times 32}{G\pi d^4} = \frac{64Tl}{G\pi d^4}$$

Total angle of twist = $\theta_{AB} + \theta_{BC}$

$$= \frac{2Tl}{G\pi d^4} + \frac{64Tl}{G\pi d^4}$$

$$\theta_{AC} = \frac{66Tl}{G\pi d^4}$$

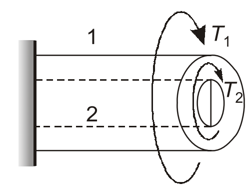
Example - 5.7 Which of the following conditions is correct for the shafts connected in series to each other?

(a) $\theta = \theta_1 - \theta_2$
(b) $T = T_1 = T_2$ and $\theta = \theta_1 + \theta_2$
(c) $\theta_1 = \theta_2$
(d) $\theta_1 = \theta_2$ and $T = T_1 + T_2$ both

Solution: (b)

For the shafts connected in series torques in all the shafts are equal and angle of tist is added.

5.9.2 Shafts in Parallel



- In this case applied torque T is distributed to two shafts.
- $T = T_1 + T_2$
- The angle of twist will be same for each shaft, $\theta_1 = \theta_2 = \theta$
- $T = T_1 + T_2 = \theta\left[\frac{I_1G_1}{l_1} + \frac{I_2G_2}{l_2}\right] \Rightarrow \theta = \frac{T}{\left[\frac{I_1G_1}{l_1} + \frac{I_2G_2}{l_2}\right]}$

5.10 Comparison between Solid and Hollow Shafts

- Let hollow shaft and solid shafts have same material and length.
  D_o = external diameter of hollow shaft; $D_i = nD_o$ = Internal diameter of hollow shaft;
  D = Diameter of the solid shaft

Case 1. When the hollow and solid shafts have the same torsional strength.

- In this case polar modulus section of two shafts would be equal.
- $\frac{\text{Weight of hollow shaft}}{\text{Weight of solid shaft}} = \frac{1-n^2}{\left(1-n^4\right)^{2/3}}$
- ***USE:*** % Saving in weight can be calculated for same torsional strength.

Case 2. When the hollow and solid shafts are of equal weights.

- In this case torsional strength is compared.

$$\frac{T_{hollow}}{T_{solid}} = \frac{1+n^2}{\sqrt{1-n^2}}$$

- ***USE:*** ratio of strength for same weight can be calculated.

Case 3. When the diameter of solid shaft is equal to the external diameter of the hollow shaft.

- $\frac{T_{hollow}}{T_{solid}} = 1 - n^4$

DO YOU KNOW?

- For a given diameter (i.e., when radial space is constrained) solid circular shafts are preferred than Hollow circular shaft due to their higher power transmission capacity. $[(Z_p)_S > (z_p)_H]$.
- For a given cross-section area, Hollow circular shafts are preferred for power transmission then solid circular shaft due to their higher power transmission capacity i.e., $[(Z_p)_H > (Z_p)H]$

5.11 Thin Circular Tube Subjected to Torsion

- Let D = external diameter of circular tube
  t = thickness of tube
  also, $t << D$
- Torsional resistance, $T = \frac{\tau_s \pi D^2 t}{2}$ where τ_s is allowable shear stress
- Twist of the tube in a length l,

$$\theta = \frac{Tl}{GI_P} = \frac{4Tl}{\pi D^3 t \cdot G}$$

5.12 Shear and Torsional Resilience

5.12.1 Shear Resilience

Let τ_d = shear stress intensity at faces of a square block

- Strain energy stored per unit volume,

$u = \frac{\tau_d^2}{2G}$ (uniform throughout the section) where G = rigidity modulus

5.12.2 Torsional Resilience

- In this case shear stress due to torsion varies uniformly from zero at the axis to the maximum value τ_s at the surface.
- Strain energy stored, per unit volume

(a) For solid shaft, $u = \frac{\tau_s^2}{4G}$

(b) For hollow shaft, $u = \frac{\tau_s^2}{4G}\left[\frac{D^2 + d^2}{D^2}\right]$

Where, D = diameter of solid shaft; d = internal diameter of hollow shaft

5.13 Springs

- Spring is a mechanical element, in which the material is arranged in such a way that it can undergo a considerable change, without getting permanently distorted.
- A spring is used to absorb energy due to resilience which may be restored as potential energy.
- The quality of a spring is judged from the energy it can absorb. The spring which is capable of absorbing the greatest amount of energy for the given stress is the best one.

5.13.1 Stiffness of a Spring

- The load required to produce a unit deflection in a spring is called stiffness of spring.

5.13.2 Types of Springs

There are two types depending upon the type of springs:

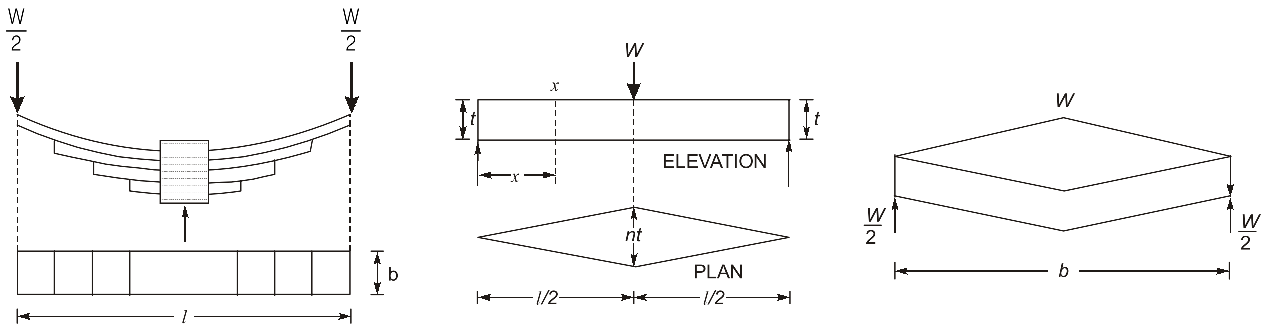
(i) Bending springs (leaf spring)

(ii) Torsion spring (helical spring)

5.13.2.1 Leaf Springs

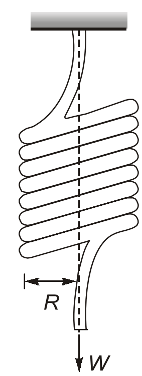
Central deflection δ is given by

$$\delta = \frac{3}{8}\frac{Wl^3}{Enbt^3}$$



Where, l = Span of spring; t = Thickness of plates; b = Width of plates;
n = Number of plates; W = Load acting on the spring; E = Young's modulus

5.13.2.2 Closed Coiled Helical Springs Subjected to Axial Loading



- Deflection in the spring due to load W,

$$\delta = \frac{64WR^3n}{Cd^4}$$

- Stiffness of the spring, $k = \frac{W}{\delta} = \frac{Cd^4}{64R^3n}$

where, W = axial load; n = number of turns of spring;
d = diameter of the wire of the spring; R = mean radius of the coil;
G = modulus of rigidity for the spring material.

- Energy stored, $U = \frac{1}{2}W\delta = \frac{32W^2R^3n}{Gd^4}$

Example - 5.8 If a helical spring is halved in length, its spring stiffness

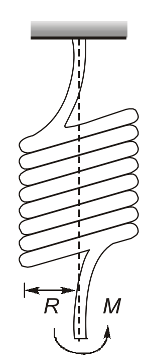
(a) Remain same (b) Halves
(c) Doubles (d) Amples

Solution: (c)

Stiffness of the spring, $k = \frac{w}{8} = \frac{Gd^4}{8D^3N}$ $\left(\therefore k \propto \frac{1}{N}\right)$

$\therefore$ When N is halved then k will doubled.

5.13.2.3 Closely Coiled Helical Spring Subjected to Axial Twist



- Total angle of bend = ϕ

$$\phi = \frac{Ml}{EI} = 2\pi(n' - n)$$

where $l = 2\pi Rn = 2\pi R' n'$

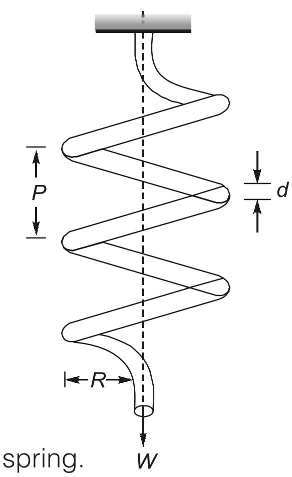
Also $\frac{d\phi}{dl} = \frac{M}{EI}$ = constant

Thus, the change in curvature or angle of bend per unit length, is constant, throughout the spring.

- Energy stored $= \frac{1}{2}M.\phi = \frac{1}{2}.\frac{M^2 l}{EI}$

  where, R = near radius of the spring coil; n = number of turns or coils;
  M = Moment or axial twist applied on the spring; R' = decreased mean radius due to twist
  n' = increased no. of turns due to twist; I = moment of inertia of spring rod section;
  E = Young's modulus

5.13.2.4 Open Coiled Helical Springs Subjected to Axial Load



- In this case load W will cause both twisting and bending of coils.
- Deflection of the spring as a result of axial load,

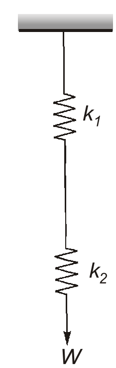
$$\delta = \frac{64WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

  Where, d = Diameter of spring wire; R = Mean radius of spring coil;
  p = pitch of the spring coils; n = number of turns or coils;
  C = modulus of rigidity for spring material;
  W = axial load on the spring; α = angle of helix;
  E = Young's modulus

- If we put $\alpha = o$ $\quad \delta = \frac{64WR^3 n}{Cd^4}$ = deflection of closed coiled spring.

5.13.3 Springs in Series and Parallel

5.13.3.1 Springs in Series



- Total extension = Σ (individual extensions)

$$\delta = \delta_1 + \delta_2$$

- Load applied (W) will be same on both the springs.

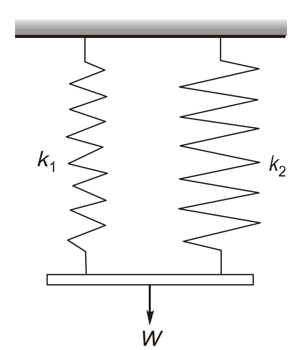
$$W_1 = W_2 = W$$

- The equivalent stiffness is given by

$$\frac{1}{K_{eq.}} = \frac{1}{K_1} + \frac{1}{K_2}$$

  where K_1 and K_2 are individual stiffness of the springs.

5.13.3.2 Springs in Parallel



- Both the springs have same extension.

$$\delta = \delta_1 = \delta_2$$

- Load applied (W) is shared by the springs

$$W = W_1 + W_2$$

- The equivalent stiffness is given by,

$$K_{eq.} = K_1 + K_2$$

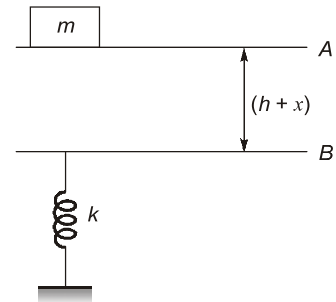
Example - 5.9 A body weighing 1000 kg falls 8 cm and strikes a 500 kg/cm spring. The deformation of spring will be _______ cm.

(a) 8 (b) 4
(c) 16 (d) 2

Solution: (a)

When body rolls on spring.

Applying energy conservation



$$(PE)_A + (kE)_A = (PE)_B + (kE)_B$$

$$mg(h + x) + 0 = \frac{1}{2}kx^2 + 0$$

$$\Rightarrow \quad 1000 \times 10 + (8 + x) = \frac{1}{2} \times 500 \times 10 \times x^2$$

$$\Rightarrow \quad 32 + 4x = x^2$$

$$\Rightarrow \quad x^2 - 4x - 32 = 0 \quad \Rightarrow \quad x = 8, -4 \quad = 8 \text{ cm}$$

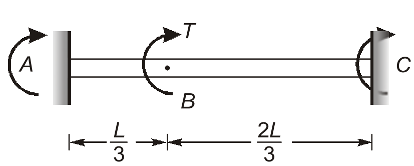
Example - 5.10 If springs are subjected to twisting moment, then spring wire is subjected to

(a) Bending stress
(b) Compressive stress
(c) Tensile stress
(d) Shear stress

Solution: (a)

Student's Assignment

Q.1 A circular shaft of length 'L' is held at two end without rotation. A twisting moment 'T' is applied at a distance $\frac{'L'}{3}$ from left support as shown in the given figure. The twisting moment in the portion AB will be:



(a) T
(b) T/3
(c) T/2
(d) 2T/3

Q.2 In an analysis of a closely coiled helical spring under axial load, which of the following is/are negligible?

I. Torsion
II. Bending
III. Axial force in the wire

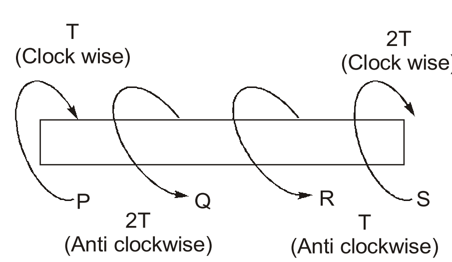
Select the correct answer using the codes given below.

Codes:

(a) I alone
(b) II and III
(c) I and II
(d) I and III

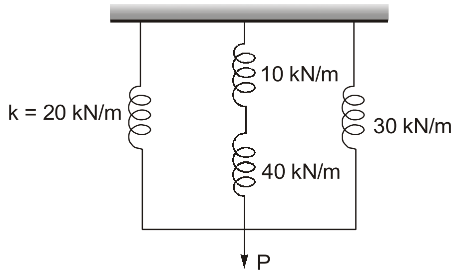
Q.3 A shaft PQRS is subjected to torques at P, Q, R, S as shown in the given figure.

The maximum torque for the shaft section occurs between



(a) P and Q
(b) P and R
(c) Q and R
(d) R and S

Q.4 For the system of springs shown in the figure, the equivalent spring stiffness is



(a) 58 kN/m (b) 62 kN/m
(c) 78 kN/m (d) 90 kN/m

Q.5 The diameter of shaft B is twice that of shaft A. Both shafts have the same length and are of the same material. If both are subjected to the same torque, then the ratio of the angle of twist of shaft A to that of shaft B will be

(a) 2 (b) 4
(c) 8 (d) 16

Q.6 A circular shaft of length 'L', a uniform cross-sectional area 'A' and modulus of rigidity 'G' is subjected to a twisting moment that produces maximum shear stress 'τ' in the shaft. Strain energy in the shaft is given by the expression

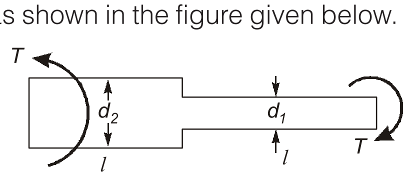
$\frac{\tau^2 AL}{kG}$ where k is equal to

(a) 2 (b) 4
(c) 8 (d) 16

Q.7 A close-coiled helical spring has 100 mm mean diameter and is made of 20 turns of 10 mm diameter steel wire. The spring carries an axial load of 100 N. Modulus of rigidity is 84 GPa. The shearing stress developed in the spring in N/mm^2 is

(a) $120/\pi$ (b) $160/\pi$
(c) $100/\pi$ (d) $80/\pi$

Q.8 Two shafts of same material and same length are connected in series and subjected to a torque T as shown in the figure given below.



If $\frac{d_2}{d_1} = 2$, what is the value of the ratio of $\frac{\tau_1}{\tau_2}$?

(where τ_1 and τ_2 are extreme shear stresses in either of the two segments)

(a) 4 (b) 2
(c) 8 (d) 16

Q.9 The stiffness of spring A and B are K and 2K respectively. If each spring is subjected to load 'p' for a while and the load is removed suddenly, the ratio of period of resulting simple harmonic of A to that of B is

(a) 2 (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

Q.10 Consider the following relation for the torsional stiffness (k_T)

1. $k_T = \frac{T}{\theta}$ 2. $k_T = \frac{GJ}{L}$ 3. $k_T = \frac{G\theta}{L}$

Which of the following option is correct?

(a) 1, 2 and 3 (b) only 1 and 3
(c) only 1 and 2 (d) only 2 and 3

Q.11 Two shafts are solid and other hollow are made of the same materials are having same length and weight. The hollow shaft as compared to solid shaft is

(a) have same strength
(b) more strong
(c) none of the option
(d) less strong

Q.12 The diameter of a shaft is increased from 30 mm to 60 mm all other conditions remaining unchanged. How many times is its torque carrying capacity increased?

(a) 2 times (b) 4 times
(c) 8 times (d) 16 times

Q.13 For a power transmission, shaft transmitting power P at N rpm, diameter is proportional to

(a) $\left(\frac{P}{N}\right)^{1/3}$ (b) $\left(\frac{P}{N}\right)^{1/2}$
(c) $\left(\frac{P}{N}\right)^{2/3}$ (d) $\left(\frac{P}{N}\right)$

Q.14 For the two shafts connected in parallel
(a) torque in each shaft is the same
(b) shear stress in shaft is the same
(c) angle of twist of each shaft is the same
(d) torsional stiffness of each shaft is the same

Q.15 A solid shaft is designed to transmit 100 kW while rotating at N rpm. If the diameter of the shaft is doubled and allowed to operate at 2 N rpm, the power that can be transmitted by the latter shaft is
(a) 200 kW (b) 400 kW
(c) 800 kW (d) 1600 kW

Q.16 The diameter of shaft A is twice the diameter of shaft B and both are made of the same material. Assuming both the shafts to rotate at the same speed, the maximum power transmitted by B is
(a) the same as that of A
(b) half of A
(c) 1/8th of A
(d) 1/4th of A

Q.17 The compliance of the spring is the
(a) Reciprocal of the spring constant
(b) Deflection of the spring under compressive load
(c) Force required to produce a unit elongation of the spring
(d) Square of the stiffness of the spring

Q.18 A spring with 25 active coils cannot be accommodated within a given space. Hence 5 coils of the spring are cut. What is the stiffness of the new spring?
(a) Same as the original spring
(b) 1.25 times the original spring
(c) 0.8 times the original spring
(d) 0.5 times the original spring

Q.19 For a helical spring, spring index is
(a) The ratio of the mean coil diameter to the pitch of helix
(b) The ratio of the wire diameter to the pitch of helix
(c) The difference between the mean coil diameter and the wire diameter
(d) The ratio of mean coil diameter to wire diameter

Q.20 The hollow shaft will transmit greater _______ then the solid shaft of the same weight.
(a) Bending moment
(b) Shear stress
(c) Torque
(d) Sectional Modulus

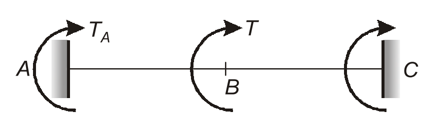
Q.21 Calculate the power transmitted in the shaft rotating at 150 rpm, and torque as 9000Nm.
(a) 140 kW (b) 150 kW
(c) 160 kW (d) 175 kW
[UPPSC]

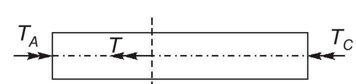
ANSWER KEY — STUDENT'S ASSIGNMENT

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (d)
6. (b)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (d)
16. (c)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (a)				

HINTS & SOLUTIONS — STUDENT'S ASSIGNMENT

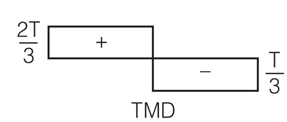
1. (d)





$T_A = T + T_C$

$$\frac{T_A L}{3GI_p} + \frac{(T_A - T)2L}{3GI_p} = 0$$



$$\Rightarrow T_A + 2T_A - 2T = 0$$

$$\Rightarrow \quad 3_A = 2T$$

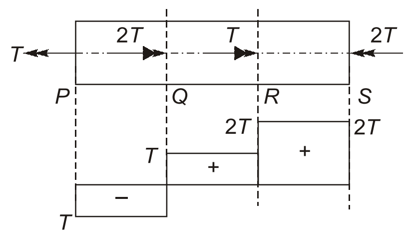
$$\Rightarrow \quad T_A = \frac{2T}{3}$$

$$\text{and } T_C = T_A - T = \frac{2T}{3} - T = -\frac{T}{3}$$

2. (b)

Axial load causes torsion in the wire in closely coiled helical spring. For closely coiled helical spring subjected to torque, in which bending and axial force in the wire is neglected.

3. (d)



TMD

4. (a)

The equivalent stiffness of 10 kN/m and 40 kN/m springs in series is

$$= \frac{1}{\frac{1}{10}+\frac{1}{40}} = 8 \text{ kN/m}$$

Thus equivalent stiffness of the system

= 20 + 8 + 30 = 58 kN/m

5. (d)

$$\frac{\theta_A}{\theta_B} = \frac{(TL/GJ)_A}{(TL/GJ)_B}$$

Given, $d_B = 2d_A$; $l_A = l_B$; $T_A = T_B$ and $G_A = G_B$

Then, $$\frac{\theta_A}{\theta_B} = \frac{J_B}{J_A} = \left(\frac{d_B}{d_A}\right)^4 = 16$$

6. (b)

$$\text{Strain energy} = \frac{1}{2}T\theta$$

Torsion formula

$$\frac{G}{L/\theta} = \frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow \quad T = \frac{\tau \cdot \pi D^3}{16} = \left(\left(\frac{\tau A}{4}\right)D\right)$$

and $$\theta = \frac{\tau L}{GR}$$

$$\therefore \quad \text{Strain energy} = \frac{1}{2}\left(\frac{\tau AD}{4}\right)\left(\frac{\tau L}{GR}\right) = \frac{\tau^2 AL}{4G}$$

$$k = 4$$

7. (d)

Closed coiled helical spring subjected to axial load (W) means that every section is subjected to torsion of WR where R is the radius of spring.

From torsion formula

$$\frac{WR}{J} = \frac{\tau_{max\,1}}{\left(\frac{d}{2}\right)} \qquad d \rightarrow \text{diameter of wire}$$

$$J = \frac{\pi d^4}{32}$$

$$\tau_{max\,1} = \frac{16\,WR}{\pi d^3}$$

$$R = \frac{100}{2} = 50 \text{ mm}$$

$$d = 10 \text{ mm}$$

$$W = 100 \text{ N}$$

shear stress developed due to torque

$$\tau_{max\,1} = \frac{16 \times 100 \times 50}{\pi \times (10)^3} = \frac{80}{\pi}$$

8. (c)

Given: $d_2 = 2d_1$; $T_1 = T_2 = T$

$$\frac{\tau_1}{\tau_2} = \frac{(T/Z_P)_1}{(T/Z_P)_2} = \frac{Z_{P_2}}{Z_{P_1}} = \left(\frac{d_2}{d_1}\right)^3 = 8$$

9. (b)

Time Period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\therefore \quad \frac{T_A}{T_B} = \sqrt{\frac{k_B}{k_A}} = \sqrt{2}$$

10. (c)

Torsional stiffness, $K_T = \frac{GJ}{l} = \frac{T}{\theta}$

11. (b)

Strength of the shaft is its torsional moment of resistance (T_R)

$$T_R = Z_p \tau_{Per}$$

$$T_R \propto Z_p$$

(for given material τ_{per} is same)

$$\frac{(T_R)_S}{(T_R)_H} = \frac{(Z_p)_S}{(Z_p)_H} = \frac{D_S^3}{D_H^3(1-k^4)}$$

As weight and lengths are equal.

$$A_S = A_H$$

$$\frac{\pi}{4}D_S^2 = \frac{\pi}{4}D_H^2(1-k^2)$$

$$\frac{D_S}{D_H} = \sqrt{1-k^2}$$

$$\therefore \quad \frac{(T_R)_S}{(T_R)_H} = \frac{(1-k^2)^{3/2}}{(1-k^4)} = \frac{(1-k^2)\sqrt{1-k^2}}{(1-k^2)\sqrt{1+k^2}}$$

$$= \sqrt{1+k^2}$$

$$(T_R)_S < (T_R)_H$$

12. (c)

So, for the given weight Hollow circular shafts are stronger than solid.

$$T = \frac{\pi}{16}d^3\tau_{per}$$

$$T \propto d^3 \qquad \text{(For same material)}$$

$$\frac{T_2}{T_1} = \left(\frac{d_2}{d_1}\right)^3 = \left(\frac{2d}{d}\right)^3 = 8$$

$$T_2 = 8T$$

13. (a)

$$P = T\omega = T \times 2\pi N$$

$$= \frac{\pi}{16}d^3\tau_{per}N$$

$$d \propto \left(\frac{P}{N}\right)^{1/3}$$

14. (c)

When two shaft connected in parallel then

$\theta_1 = \theta_2 = \theta$ and $T_1 + T_2 = T$

15. (d)

$$P = \frac{\pi}{16}d^3{}_{per}N$$

$$\Rightarrow \quad P \propto d^3 \times N$$

$$\Rightarrow \quad \frac{P_2}{P_1} = \frac{d_2^3}{d_1^3}\frac{N_2}{N_1} = (2)^4 = 16$$

$$\Rightarrow \quad P_2 = 16P_1 = 1600 \text{ kW}$$

16. (c)

Given $d_A = 2 \times d_B$; $G_A = G_B$; $N_A = N_B$

$$P \propto \omega d^3$$

$$\frac{P_B}{P_A} = \frac{(\omega d^3)_B}{(\omega d^3)_A} = \left(\frac{d_B}{d_A}\right)^3 = \frac{1}{8}$$

17. (a)

$$\text{Compliance} = \frac{1}{\text{stiffness}} = \frac{1}{\text{spring constant}}$$

18. (b)

$$\text{Stiffness of the spring (S)} = \frac{Gd^4}{8D^3n}$$

$$n_1 = 25$$

$$s_1 = \frac{Gd^4}{8D^3 \times 25}$$

When 5 coils of the spring are cut

$$n_2 = 20$$

$$s_2 = \frac{Gd^4}{8D^3 \times 20}$$

$$\frac{s_2}{s_1} = \frac{25}{20} = 1.25$$

19. (d)

Spring index, $C = \dfrac{D}{d}$ $D \rightarrow$ mean coil dia, $d \rightarrow$ wire dia

20. (c)

For the same maximum shear stress, the average shear stress in a hollow shaft is greater than that in a solid shaft of the same area. Hence the hollow shaft will transmit greater torque than the solid shaft of the same weight.

21. (a)

Power transmitted (P) is $P = 2\pi NT/60$ watts.

$$P = 2\pi \times 150 \times 9000/60$$

$$P = 140 \text{ kW.}$$

■■■■