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Assistant Executive Engineer

Civil Engineering

Geotechnical & Foundation Engineering

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

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Contents

CHAPTER 7 Seepage Through Soils

7.1 Introduction

Seepage is the flow of water under gravitational forces in a permeable medium. Flow of water takes place from a point of high head to a point of low head. In one dimensional flow, where Darcy's law is valid, the velocity of flow is taken constant at every point over a cross section normal to the direction of flow. However many practical situations (like flow through earth dams, sheet piles) the flow of water is not unidirectional and velocity of water does not remain constant. In such cases, the quantity of seepage and other parameters such as hydraulic gradient and pore water pressure are estimated by the help of flow nets. The concept and construction of flow nets are based on Laplace's equation of continuity.

7.2 Type of Head

There are three types of head available in fluid flow:

(i) Velocity head

- It is equal to $\frac{V^2}{2g}$.
- Since laminar flow occurs during the seepage and velocity is very small during laminar flow. Hence in seepage analysis, velocity head may be neglected.

(ii) Pressure head

- It is equal to $\frac{P}{\gamma_w}$.
- If a piezometer or an open stand pipe is inserted at a point of flow, water would stand at a particular height inside the piezometer.
- The actual height of rise of water column in the piezometer, represents the pressure head.

(iii) Datum or elevation head

- It is represented by 'z'.
- The elevation or datum head at a point is the vertical distance of that point measured from an assumed datum. Generally, datum is assumed at tail water level.

7.3 Total Head

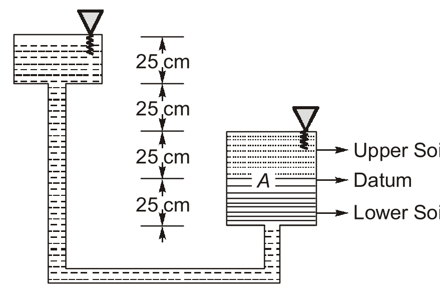
- Total head = velocity head + pressure head + elevation head
- Here, Total head = $0 + \frac{P}{\gamma_w} + z$
- If we insert a stand pipe at a point of flow, the elevation of water level in stand pipe with reference to the datum is equal to total head.

- In seepage analysis, velocity head is neglected. Hence total head and piezometric head both are equal.

7.3.1 Head Loss

- The difference in total head between two points in a soil through which flow is occurring is represented by the head loss during the flow between these points.
- If flow is occurring through the soil sample, the total head is assumed to reduce during flow.

Example - 7.1 Two different granular soils are placed in a permeameter tube and flow is allowed to take place under a constant total head. The total head and pressure head at point *A* in centimeters, are respectively:



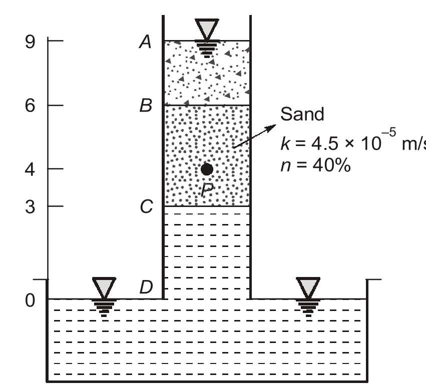
(a) 75, 75
(b) 25, 75
(c) 25, 25
(d) 75, 25

Solution: (c)

Total head at a point is the sum of pressure head and elevation head. It depends upon the datum from which the elevation is measured. Normally datum is fixed at the bottom i.e., at the bottom layer of upper soil at *A*.

Elevation head = 0
Pressure head = 75 cm
Total head = 0 + 75 = 75 cm

Example - 7.2 Determine the pressure head, elevation head and total head at the entering end, exit end and point *P* of the sample. What are the discharge (superficial) and seepage velocity of flow?



Solution:

Assume datum is taken at *EL* 0(*m*).

Elevation of point (*m*)	Elevation Head (*m*)	Pressure Head (*m*)	Total Head = *E.H* + *P.H* (*m*)	Head Loss (*m*)
A	9	0	9	0
B	6	3	9	0
C	3	–3	0	9
D	0	0	0	9
P	4	–1	3	6

It is convenient to first determine the total head loss due to sand specimen which is equal to the difference of initial and final water level. Now determine the elevation and total heads, and then calculate the pressure head by substracting elevation head from the total head. This procedure can be adopted for any point within the soil.

Head loss due to sand specimen, $\Delta H = 9 - 0 = 9\text{m}$

For point P, the head loss upto P is calculated proportionately as $\frac{9}{3} \times 2 = 6$ m. Thus the total head at P is equal to total head at B (9 m) minus the head loss upto P (6 m).

i.e. $9 - 6 = 3$ m

The pressure head is computed last; it is the total head – elevation head

i.e. $3 - 4 = -1$ m

We know, discharge (superficial) velocity is given by

$$v = k.\ i = k.\frac{\Delta H}{3} = 4.5 \times 10^{-5} \times \frac{9}{3}$$

$$= 1.35 \times 10^{-4} \text{ m/s or } 0.135 \text{ mm/sec}$$

The seepage velocity, v_S is given by the equation,

$$v_S = \frac{v}{n} = \frac{0.135}{0.4} = 0.3375 \text{ mm/sec}$$

7.4 Seepage Pressure and its Effect on Effective Stress

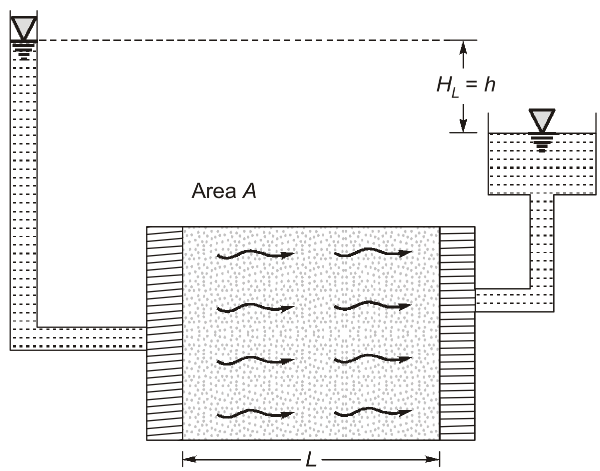
- 

Fig. Seepage pressure in Soil

- **Seepage Pressure:** When water flows through the saturated soil mass, it exerts the pressure over the solids by the virtue of viscous friction and is referred as seepage pressure.

  Hence, seepage pressure is the pressure exerted by the water over the soil solids in the mass through which it percolates.

- If 'h' is the hydraulic head (i.e Head loss) under which flow is taking place, then seepage is given by

$$p_s = h\gamma_w$$

Also, $$p_s = \frac{h}{z}.z\gamma_w$$

or $$p_s = iz\gamma_w$$

If A be the area cross-section, then seepage force is given by

$$P_s = \text{seepage pressure } (p_s) \times A$$
$$= i\ z\ \gamma_w \times A$$
$$= i.\ (z \times A)\ .\ \gamma_w$$
$$= i.\ V.\ \gamma_w$$

Seepage per unit volume is known as "specific seepage force" which is given by

$$P_{ss} = \frac{P_s}{V} = \frac{i.V.\gamma_w}{V} = i\,\gamma_w$$

NOTE

Seepage pressure always acts in the direction of flow. Hence vertical pressure (effective stress) at any given section in flow condition, may either increase or decrease.

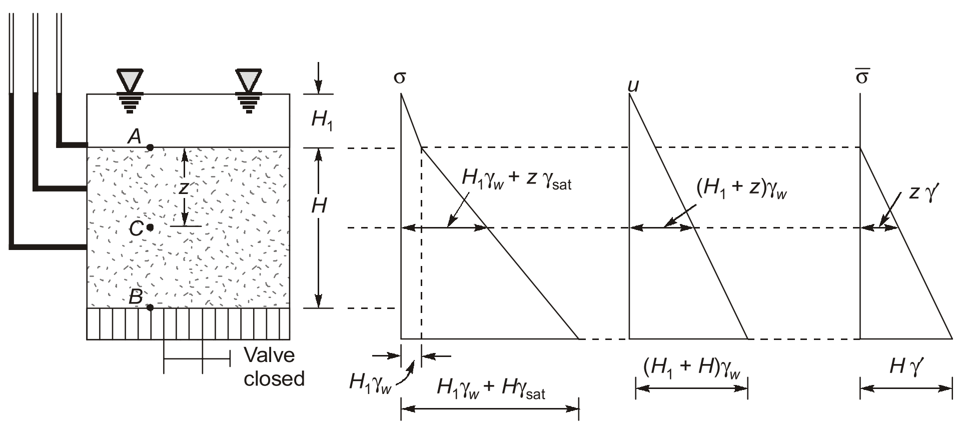
$\therefore \quad \bar{\sigma}' = \bar{\sigma} \pm P_s$

where $\bar{\sigma}$ = effective stress under no flow condition

P_s = seepage pressure under flow condition

7.4.1 No Flow Condition

- The figure shows a tank filled with submerged soil. Since the valve at the base is closed, no seepage will occur.



- Pressure head, datum head and total head are tabulated below:

Point	A	B	C
Pressure head	H_1	$H_1 + H$	$H_1 + Z$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H$	$H_1 + H$

- Total stress, pore water pressure and effective stress and tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_\omega H_1$	$\gamma_{sat} H + \gamma_\omega H_1$	$\gamma_{sat} H + \gamma_\omega H_1$
Pore water pressure (u)	$\gamma_\omega H_1$	$\gamma_\omega (H_1 + H)$	$\gamma_\omega (H_1 + Z)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$\gamma_{sat} H - \gamma_\omega H = \gamma' H$	$\gamma_{sat} Z - \gamma_\omega Z = \gamma' Z$

The variations of σ, u and $\bar{\sigma}$, are shown in figure above:

7.4.2 Downward Flow

- Figure shows a tank filled with submerged soil, since the valve at the base is open and downward seepage is allowed. The downward seepage increases in effective stress.

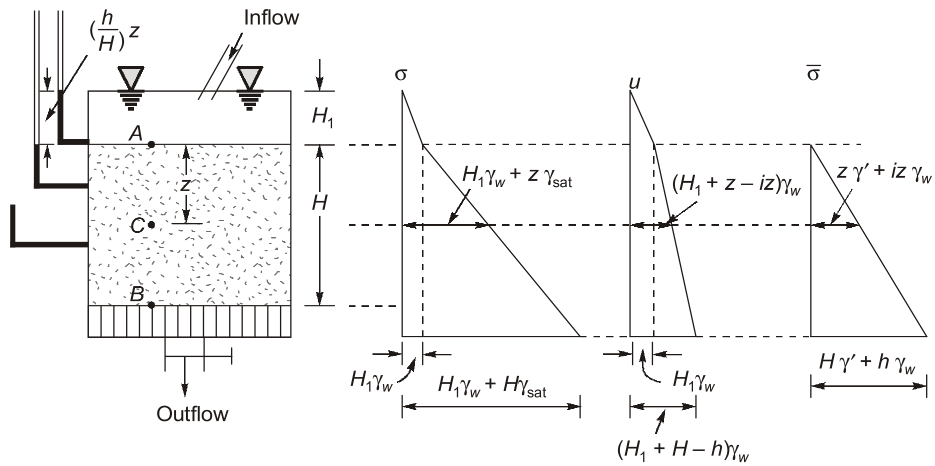


Fig. Variation of σ, u and $\bar{\sigma}$ with depth downward flow

- Pressure head, datum head and total head are tabulated below:

Point	A	B	C
Pressure head	H_1	$H_1 + H - h$	$H_1 + Z - iZ$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H - h$	$H_1 + H - iZ$

where, h = hydraulic head (head loss) under which flow takes place from A to B

Hydraulic gradient, $i = \dfrac{h}{H}$

- The total vertical stress at any point in soil mass is due submerged weight of soil mass and seepage pressure depending upon the direction of flow.
- Total stress, pore water pressure and effective stress are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_\omega H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w (H_1 + H - h)$	$\gamma_w (H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H + \gamma_w h$ $= \gamma H + \gamma_w h$	$= (\gamma_{sat} - \gamma_w) Z + \gamma_w iZ$ $= \gamma Z + \gamma_w iZ$

The variations of σ, u and $\bar{\sigma}$ are shown in figure above

7.4.3 Upward Flow

- Figure shows, valve at the bottom of tank is open and upward seepage is allowed.

Point	A	B	C
Pressure head	H_1	$H_1 + H + h$	$H_1 + Z + iZ$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H + h$	$H_1 + H + iZ$

Where, h = hydraulic head (head loss) under which flow takes place from A to B

Hydraulic gradient, $i = \dfrac{h}{H}$

Total stress, pore water pressure and effective stresses are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w(H_1 + H + h)$	$\gamma_w(H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H - \gamma_w h$ $= \gamma' H - \gamma_w h$ $= \gamma' H - iZ\gamma_w$	$= (\gamma_{sat} - \gamma_w) H - \gamma_w h$ $= \gamma' H - \gamma_w h$ $= \gamma' H - iZ\gamma_w$

The variations of σ, u and $\bar{\sigma}$ are shown in figure.

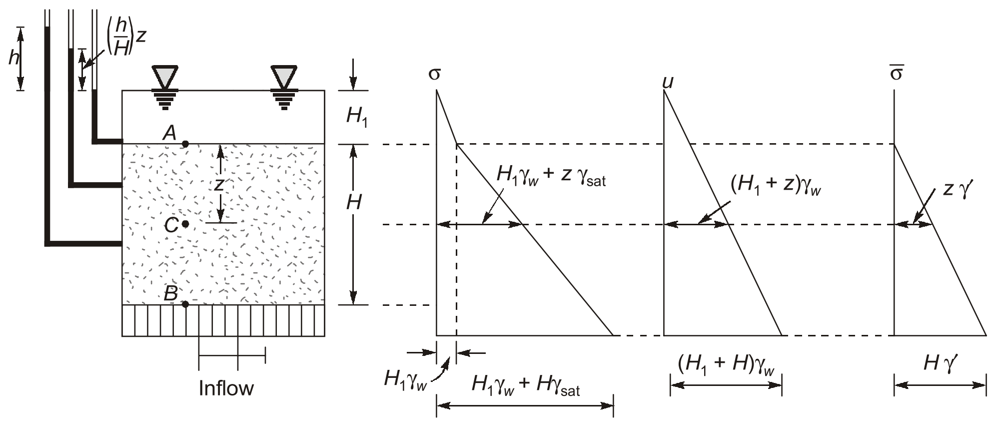
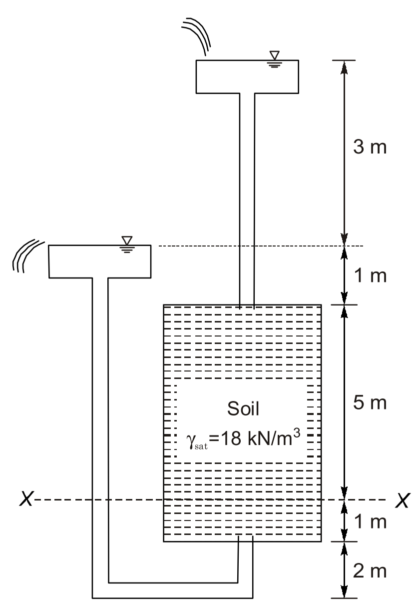


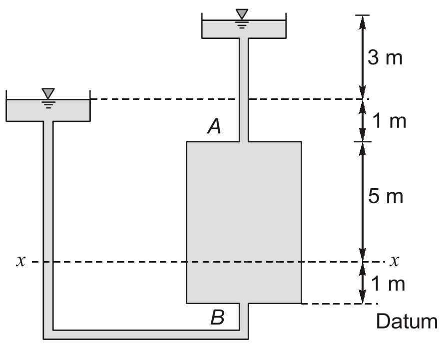
Fig. Variation of σ, u and $\bar{\sigma}$ with depth for upward flow

Example - 7.3 A seepage flow condition is shown in the figure. The saturated unit weight of the soil γ_{sat} = 18 kN/m^3. Using unit weight of water, γ_w = 9.81 kN/m^3, the effective vertical stress (expressed in kN/m^2) on plane *X-X* is ______



(a) 65.475
(b) 72.982
(c) 35.981
(d) 45.864

Solution: (a)



	Datum head	Pressure head	Total head
A	6	4	10
B	0	7	7
x-x	1	6.5	7.5

Hydraulic gradient,

$$i = \frac{\text{Hydraulic head difference}}{\text{Length}} = \frac{H_A - H_B}{6}$$

$$i = \frac{10-7}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Force} = h\gamma_w$$

Head loss at *x-x* $= i \times 5 = 2.5\,\gamma_w$

Effective stress at *x-x* $= 5\,\gamma_{sub} + 2.5\,\gamma_w$

$$= 5 \times (18 - 9.81) + 2.5 \times 9.81$$

$$= 65.475\ \text{kN/m}^2$$

7.5 Quick Sand Condition

For upward flow condition, effective stress at any point within soil mass is given by

$$\bar{\sigma} = \bar{\sigma} - p_s = \gamma' z - p_s$$

It is clear from above equation, upward seepage pressure decreases effective stresses in the soil mass.

- If the seepage pressure is such that it equals the submerged weight of the soil mass, then effective stresses at that location reduces to zero. Under such condition, cohesionless soil mass loses all shear strength. Now soil mass has a tendency to move also with the flowing water in the upward direction.
- This process in which soil particles are lifted over the soil mass is called quick sand condition. It is also known as 'boiling of sand' as the surface of sand looks it is boiling.
- At quick sand condition, net effective stress is reduced to zero. i.e.

$$\bar{\sigma} = 0$$

$\therefore$ $\gamma' z - p_s = 0$ [where p_s = seepage pressure]

or $\gamma' z - iz\gamma_w = 0$

$$i = \frac{\gamma'}{\gamma_w} = i_{cr}$$

- The hydraulic gradient under which quick sand condition occurs is termed as critical hydraulic gradient. If void ratio and specific gravity of soil is known, then i_{cr} may be given as

$$i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{\frac{(G-1)\gamma_w}{1+e}}{\gamma_w}$$

$\therefore$ $$\boxed{i_{cr} = \frac{G-1}{1+e}}$$

- For fine sand and silts for which specific gravity $\simeq$ 2.65 and void radio $e \simeq 0.65$.

$$i_{cr} = \frac{2.65-1}{1+0.65} \simeq 1$$

- At quick sand condition, cohesionless particles of fine sand may start flowing with the water which may result in piping failure below the hydraulic structure.
- In order to prevent quick sand or piping failure, the hydraulic gradient should be less then critical hydraulic gradient. Hence factor of safety against quick sand failure or piping failure is

$$\boxed{\text{FOS} = \frac{i_{cr}}{i}} \qquad \text{where,} \quad i = \frac{h_L}{L}$$

NOTE

- Quick sand is not a type of sand. It is a flow condition which exists in cohesionless soil mass where effective stresses are reduced to zero in upward flow conditions.
- Quick sand conditions is found only in fine sand and coarse silts and it is not being observed in the case of gravels, coarse sands and clays.
- In cohesive soils which posses inherent cohesion, shear strength is not reduced to zero even when effective stresses are reduced to zero.

  For cohesive soils,

$$\tau_f = c' + \bar{\sigma}\tan\phi'$$

If $p_s = \gamma z$

then, $\bar{\sigma} = \gamma z - p_s = 0$

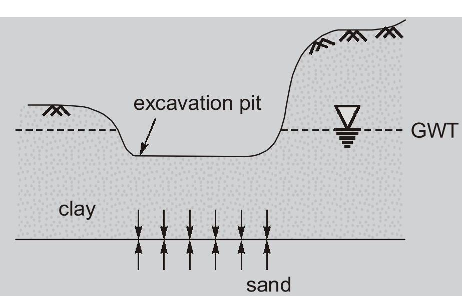
or $\tau_f = c' + 0$

- Quick sand condition is not observed in gravels and coarse sands which are highly permeable soils. As per Darcy's law a large discharge is required to generate critical hydraulic gradient.

$$Q = k.i.A; \qquad i = \frac{Q}{k.A}$$

Remember

Quick sand condition is generally observed when excavation is done below the GWT and water is pumped out to keep the excavated area free from it or it is observed when sand is under artesian pressure and overlain by a clay layer.



Example - 7.4 A soil sample (sand) obtained from trench has water content of 38%. The specific gravity of solid is 2.65. Determine the critical hydraulic gradient for the soil. Also determine the maximum permissible upward gradient, if a factor of safety of 3 is considered.

Solution:

Given, $w = 38\%$, $G = 2.65$

For saturated soil,

$$\Rightarrow \quad 1 \times e = wG$$

$$e = 0.38 \times 2.65 = 1.007$$

Now, critical hydraulic gradient, $i_{cr} = \dfrac{G-1}{1+e}$

$$\therefore \quad i_{cr} = \frac{2.65-1}{1+1.007} = 0.822$$

If the factor of safety is 3, then

Maximum permissible upward gradient, $i = \dfrac{i_{cr}}{F.O.S} = \dfrac{0.822}{3} = 0.274$

Example - 7.5 A 3 m thick soil stratum has coefficient of permeability 3×10^{-7} m/sec. A separate test have porosity 40% and bulk unit weight 21 kN/m^3 at a moisture content of 31%. Determine the head at which upward seepage will cause quick sand condition. What is the flow required to maintain critical conditions?

Solution:

Given $k = 3 \times 10^{-7}$ m/s, $n = 0.40$

$\gamma = 21$ kN/m^3, $w = 0.31$

Using, $n = \dfrac{e}{1+e}$

$$0.40 = \frac{e}{1+e},$$

$$\therefore \quad e = 0.667$$

Using, $\gamma_d = \dfrac{\gamma}{1+w}$,

We get $\gamma_d = \dfrac{21}{1+0.31} = 16.03$ kN/m^3

Also, $\gamma_d = \dfrac{G.\gamma_w}{1+e}$

$$\therefore \quad 16.03 = \frac{G \times 9.81}{1+0.667}$$

or $G = \dfrac{16.03 \times 1.667}{9.81} = 2.724$

Now, we can use, $i_{cr} = \dfrac{G-1}{1+e} = \dfrac{2.724-1}{1+0.667} = 1.034$

Also, $i_{cr} = \dfrac{H_L}{L}$

$$\therefore \quad \frac{H_L}{3} = 1.034$$

or $H_L = 1.034 \times 3 = 3.1$ m

Hence, 3.1 m of head is sufficient to cause quick sand condition.
The flow required to cause quick sand condition can be obtain by Darcy's equation,

$$Q = k.i.A$$
$$= 3 \times 10^{-7} \times 1.034 \times 1 \qquad \text{(Assume unit area)}$$
$$= 3.1 \times 10^{-7}\ \text{m}^3/\text{sec/m}^2$$

Example - 7.6 At a given location, 8 m thick saturated clay; natural water content = 30%, G_S = 2.7 is under-lain by sand. The sand layer is under artesian pressure equivalent to 3 m of water head. It is proposed to make an open excavation in the clay. How deep can this excavation be made before the bottom heaves?

Solution:

Let the safe depth of excavation = x m

∴ Thickness of clay overlaying sand at excavation site

$$= (8 - x)\ \text{m}$$

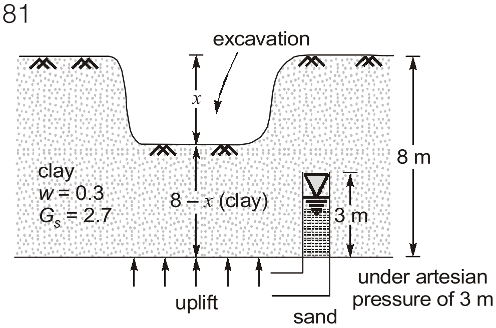
The bottom of excavation will remain stable as long as the downward stresses due to clay stratum are more than upward pressure due to artesian head in sand.
For maximum depth of excavation, the downward and upward pressure should be just equal

$$\therefore \quad (8 - x)\gamma_{sat,clay} = 3 \times \gamma_w \qquad ...(i)$$

where $$\gamma_{sat,clay} = \frac{G+e}{1+e}\gamma_w$$

where $$e = wG = 0.3 \times 2.7 = 0.81$$

$$\therefore \quad \gamma_{sat,clay} = \frac{(2.7+0.81)}{1+0.81} \times 9.81$$
$$= 19.02\ \text{kN/m}^3$$



On substituting values of $\gamma_{sat,clay}$ and γ_w, we get

$$(8 - x) \times 19.02 = 3 \times 9.81$$
$$8 - x = 1.55$$
$$x = 8 - 1.55 = 6.45\ \text{m}$$
$$\therefore \quad \text{Maximum depth of cut} = 6.45\ \text{m}$$

Alternative approach:

Head loss during flow, $$H_L = 3 - (8 - x)$$

∴ Hydraulic gradient, $$i = \frac{3-(8-x)}{(8-x)}$$

For maximum depth of cut, $$i = i_{cr}$$

$$\therefore \quad \frac{3-(8-x)}{(8-x)} = \frac{G-1}{1+e}$$

$$\frac{3-(8-x)}{(8-x)} = \frac{2.7-7}{1+0.81} = 0.939$$

$$x - 5 = 7.514 - 0.939\,x$$
$$1.939\,x = 12.514$$
$$x = 6.45\ \text{m}$$