UPPSC-AE 2020

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Mechanical Engineering

Fluid Mechanics and Hydraulic Machines

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

MADE EASY Publications

Fluid Mechanics and Hydraulic Machines

Contents

CHAPTER 3

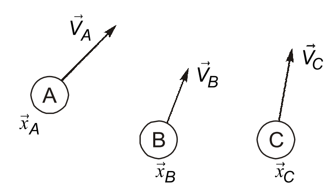
Fluid Kinematics

3.1 Introduction

- In fluid kinematics we study the fluid in motion without considering the forces causing motion hence in fluid kinematics we study about the velocity acceleration, angular velocity of flow. We also study about flow visualisation.
- The fluid motion can be described by two methods:
  (i) Lagrangian method
  (ii) Eulerian method

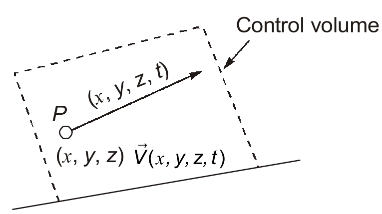
3.1.1 Lagrangian Method

- In Lagrangian approach we concentrate on individual fluid particle and study velocity, acceleration etc.
- Though the approach is very accurate, it is very different to keep track of even a single fluid particle so lagrangian approach to describe the fluid flow is not generally used.



3.1.2 Eulerian Method

In eulerian approach we concentrate on a finite volume through which fluid is flowing this finite volume is called control volume, we concentrate on one point in the control volume and study the velocity, acceleration etc. of any particle occupying that point hence we can describe a velocity as $V(x, y, z, t)$.



3.2 Type of Fluid Flow

Fluids flow may be classified as:

1. Steady flow and unsteady flow.
2. Uniform flow and non-uniform flow.
3. One, two and three dimensional flow.
4. Rotational flow and irrotational flow.
5. Laminar flow and turbulent flow.
6. Compressive flow and incompressible flow.

1. **Steady flow and unsteady flows:**
   - When the fluid properties such as velocity, density acceleration etc. do not change with time at any particular location (but can change from point to point) in a fluid flow, then the fluid flow is known as steady flow.

i.e. $$\frac{\partial V}{\partial t} = 0; \frac{\partial P}{\partial t} = 0; \frac{\partial \rho}{\partial t} = 0$$

Example of steady flow:

(a) Liquid afflux from a vessel in which constant level is maintained.

(b) Flow of water in a pipeline due to centrifugal pump being run at a uniform rotational speed.

- If one or more fluid properties or anyone changes with time, then the fluid flow will be called as unsteady flow.

$$\frac{\partial V}{\partial t} \neq 0; \frac{\partial P}{\partial t} \neq 0$$

Example of steady flow:

(a) Liquid falling under gravity out an opening in the bottom of a vessel.

(b) Liquid flow in the suction and pressure pipes of a reciprocating pump.

(c) Wave motion and cyclic movement of large bodies of water in tidal flow.

2. **Steady flow and non-uniform flow:**

- When velocity of the fluid does not change, both in magnitude and direction from point to point, at any given instant of time, the flow is said to be uniform flow.

i.e. $$\left(\frac{\partial V}{\partial s}\right)_{t=t_0} = 0$$

Example: Fluid flow under pressure through long pipeline of constant diameter is uniform flow.

- If the velocity of fluid changes from point to point at any instant, the flow is said to be non-uniform.

i.e. $$\left(\frac{\partial V}{\partial s}\right)_{t=t_0} \neq 0$$

NOTE

- Types of combination of above flows:

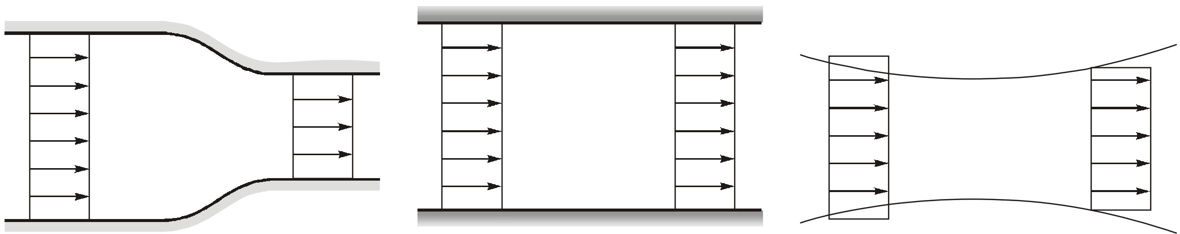
Type of flow	Example
(i) Steady and uniform	Flow through a long pipe of constant diameter at constant rate
(ii) Steady and non-uniform flow	Flow through a tapering pipe at a constant rate
(iii) Unsteady and uniform flow	Constant diameter at either increasing or decreasing rate.
(iv) Unsteady and non-uniform flow	Flow through a tapering pipe at either increasing or decreasing rate.

3. **One, two and three dimensional flow:**

One dimensional flow:

- In one dimensional flow, the fluid parameters (velocity, pressure, temperature, density, viscosity etc.) remain constant throughout any cross-section normal to flow direction but very along longitudinal direction.
- Flow field is represented by stream lines which are essentially straight and parallel.

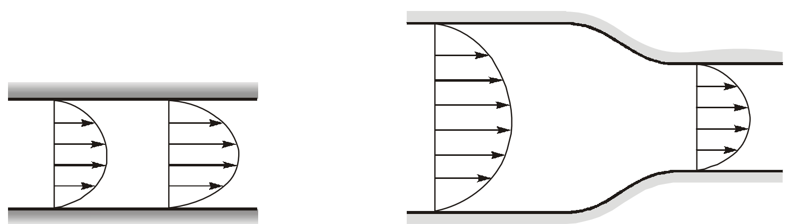
Example:



Incompressible flow in a duct   Flow between paralel plates   Compressible gas flow in a nozzle

Two dimensional flow:

- In two dimensional flow, the flow velocity and other fluid parameters very along two directions (longitudinal and vertical).
- Examples of 2-dimensional flow.

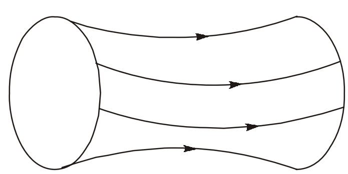


Flow between parallel plates   Viscous flow between converging plates

Three dimensional flow:

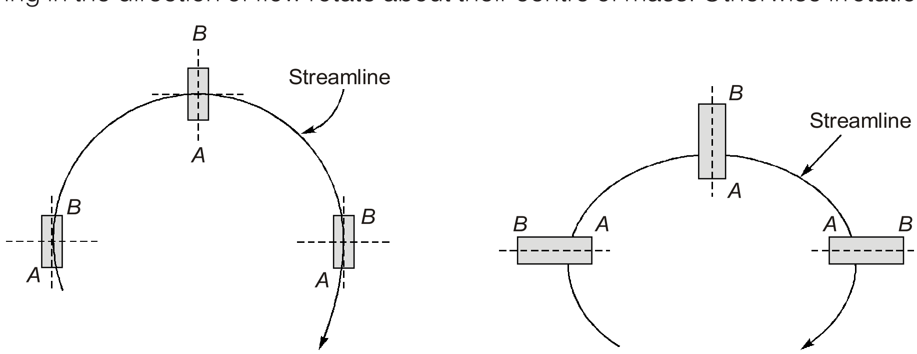
- Flow properties very in all the three directions.

  Example: Flow in a river, flow within fluid machines, flow at inlet to a nozzle.



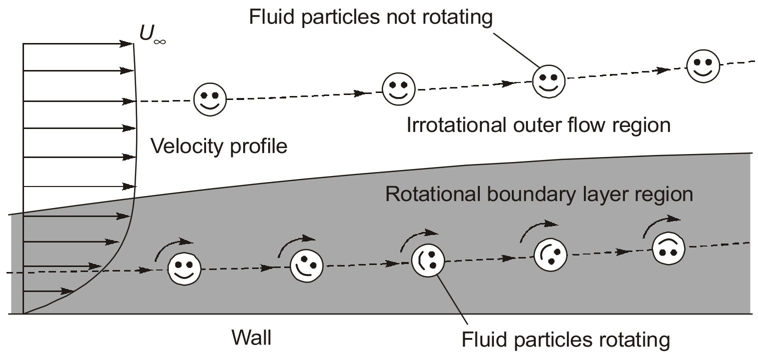
3-dimensional flow

4. **Rotational flow and irrotational flow:** Rotational flow is that type of flow of which fluid particle while moving in the direction of flow rotate about their centre of mass. Otherwise irrotational flow.



Irrotational Flow   ***Rotational Flow***

NOTE: Flow inside the boundary layer is rotational while outside it is irrotational.



Flow inside and outside Boundary Layer

5. Laminar flow and turbulent flow:

   Laminar flow:

   - It is characterized by a smooth flow of one lamina of fluid over another.
   - Fluid elements move in a well defined paths and they retain the same relative positions at successive cross-sections of the flow passage.
   - Laminar flow is also called stream line or viscous flow.
   - It occurs generally in smooth pipes when the velocity of flow is low and in liquids having a high viscosity.

   Turbulent flow:

   - In turbulent flow, the fluid element move in erratic and unpredictable paths.
   - Individual fluid particles are subjected to fluctuating transverse velocities so that the motion is eddying and sinuous rather then rectilinear. The random eddying motion is called turbulence.
   - Turbulent flow is an example of unsteady flow, but vice-versa may not always be true.

6. Compressive flow and incompressible flow:

   Compressive flow:

   - When the density changes in flowing fluid are appreciable, the flow is called compressible flow.
   - Gases are readily compressible fluids.
   - Mach number (ratio of local flow velocity to the sonic velocity in the fluid) is generally taken as a measure of the relative importance of compressibility.
   - Generally for mach number < 0.3, the compressibility effect are ignored.

   Incompressible flow:

   - Flow is incompressible if the density changes due to pressure variations are insignificant in the flow field.
   - Practically, liquids are incompressible.

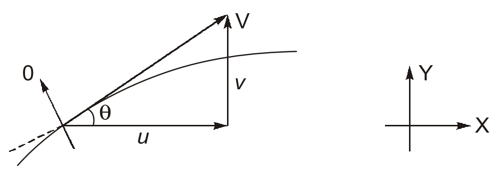
3.3 Flow Visualisation

For visualisation of flow certain lines are drawn in the flow space, these lines are:

- Stream line
- Path line
- Streak line

3.3.1 Stream Line

A stream line is an imaginary curve drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of the instantaneous velocity (V) of flow at that point.



$$\tan\theta = \frac{dy}{dx} = \frac{u}{u}$$

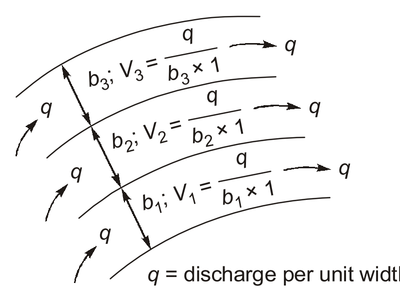
$$= \frac{dx}{u} = \frac{dy}{v} \qquad \ldots(\alpha)$$

Above equation (α) is known as equation of stream line in 2-D.

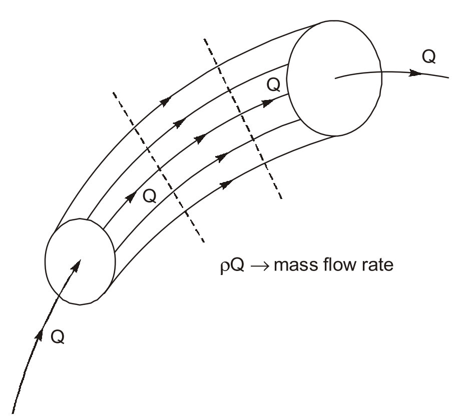
- Integration of this equation gives a constant, for different constant, we get different stream lines.
  In 3-D, equation of stream lines:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \ldots(\beta)$$

u, v and w are the component of velocity in x, y and z direction.



- Since, velocity is always tangential to stream line there is no component of velocity perpendicular to the stream line hence, flow can never cross the stream line thus any discharge getting trap between two stream lines remains same trapped there, thus as the gap between stream line increases, velocity decreases and as the gap decreases velocity increases.
- Converging of the stream lines indicates acceleration flow in that direction.
  Stream tube: It is a bundle of stream lines and flow within the stream tube remains constant at every section i.e. the mass flow rate at every section remain constant.



Stagnation point: In a fluid flow system where the velocity of flow becomes zero, known as staganation point.

Example - 3.1 A steady, two dimensional, incompressible field is represented by

$u = x + 3y + 3$ and $v = 2x - y - 8$

In this flow field, the stagnation point is

(a) (3, 2) (b) (– 3, 2)

(c) (– 3, – 2) (d) (3, – 2)

Ans. (d)

For stagnation point

$$u = 0$$
$$x + 3y + 3 = 0$$
$$x + 3y = 0 \quad \text{...(1)}$$
$$v = 0$$
$$2x - y - 8 = 0$$
$$2x - y = 8 \quad \text{...(2)}$$

Operate eq. (1) + 3 × eq. (2)

$\therefore$
$$7x = 21$$
$$x = 3$$

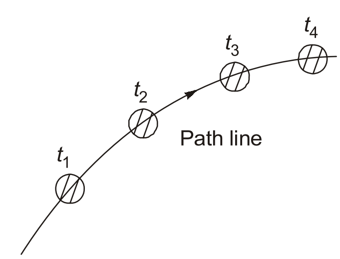
Put this value in eq. (1)

$$x + 3y = -3$$
$$3y = -3 - 3 = -6$$
$$y = -2$$

$\therefore$ Staganation point $\simeq$ (3, – 2)

3.3.2 Path Line

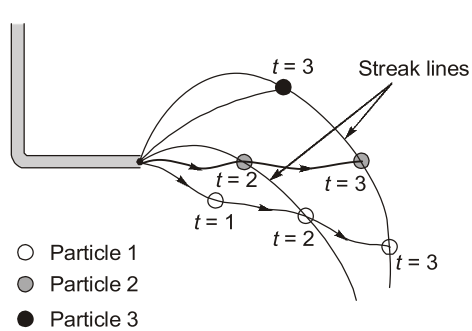
Actual path travelled by any individual fluid particle over some time period is called path line and path line is a lagrangian concept.



- These are the actual lines not imaginary.
- These lines may cut one another.

3.3.3 Streak Line

It is the locus of all fluid particles at an instant, which have crossed through the same point.



- Introduction of dye or smoke from a point in a flow will from streak line.

NOTE

- In physical, all the three lines are different but mathematically they may be same for steady flow.
- Streamline: Direction of motion (velocity).
- Pathline: Motion of a particular fluid particle.
- Streakline: Identification of location of different fluid particle.

Example - 3.2 The equation of the stream line, passing through the origin, in a flow field $u = \cos\alpha$, $v = \sin\alpha$ for a constant 'α' is determined as:

(a) $y = x^3$
(b) $y = x\cot^2\alpha$
(c) $y = x\cdot\tan\alpha$
(d) $y = \alpha\cdot\sin x$

Ans. (c)

Stream line equation,

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{\cos\alpha} = \frac{dy}{\sin\alpha}$$

Integrating both sides

$$x\cdot\sin\alpha = y\cdot\cos\alpha + c$$

At $x = 0,\ y = 0$

$\Rightarrow$ $c = 0$

$\therefore$ $x\sin\alpha = y\cos\alpha$

$$y = x\cdot\tan\alpha$$

Equation of stream line.

3.4 Rate of Flow or Discharge (Q)

It is defined as the quantity of fluid flowing per second through a section of the conduit.

where, A = Cross-sectional area

V = Mean or average velocity

Units: (m^3/sec or cumec) and litres/sec (lps)

3.4.1 Mass Flow Rate ($\dot{m}$)

$$\dot{m} = \rho Q$$

3.5 Continuity Equation

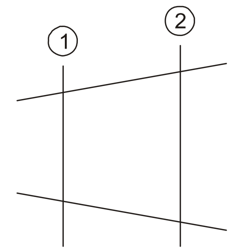
It is based on the principal of conversation of mass. According to this mass inflow in a fixed region should be equal to mass out flow from that fixed region in a particular time.

1-D flow:

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho AV)}{\partial s} = 0 \qquad ...(1)$$

2-D and 3-D:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z} = 0 \quad ...(2)$$



For 1-D steady flow,

$$\rho AV = \text{constant}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For steady and incompressible flow

$$A_1 V_1 = A_2 V_2$$

For 2-D and 3-D

For steady and incompressible flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \quad \text{... 2D flow}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0 \quad \text{... 3D flow}$$

(u, v and w) are the velocity components in x, y and z direction respectively.

NOTE

- For flow to be possible continuity equation must be satisfied.
- $\vec{\nabla}=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}$

$\therefore \quad \vec{V}=u\vec{i}+v\vec{j}+w\vec{k}$

$\vec{\nabla}\cdot\vec{V}=$ Divergence of velocity

From equation (2) we can say that, $\frac{\partial \rho}{\partial t}+\rho\cdot(\vec{\nabla}\cdot\vec{V})=0$

Example - 3.3 In a flow field at any point and at any time, density is 2 kg/m^3 and rate of change of density is 0.5 kg/m^3/sec. Find the divergence of velocity at that point and at that instant.

Solution:

Since, flow is taking place. Hence continuity equation is valid

$$\frac{\partial \rho}{\partial t}+\rho\cdot(\vec{\nabla}\cdot\vec{V}) = 0$$

$$\vec{\nabla}\cdot\vec{V} = -\frac{0.50}{2} = -0.25 \text{ sec}^{-1}$$

Example - 3.4 For a flow, the velocity field and variation in density is given as

$\vec{V}$ $(10x + 3y + 2z)\vec{i} + (12x + 4y + 5z)\vec{j} + (8x + 7y + \lambda z)\vec{k}$ and $\rho = \rho_0 e^{-3t}$ respectively.

What is the value of λ if the mass is conserved?

(a) –11
(b) –10
(c) 10
(d) 11

Ans. (a)

Since, mass is conserved continuity equation must be satisfied.

$$\therefore \qquad \frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z} = 0$$

$$-3\cdot \rho_0 e^{-3t}+\rho\cdot(10+4+\lambda) = 0$$

$$-3\rho_0 e^{-3t}+\rho_0 e^{-3t}(14+\lambda) = 0$$

$$\therefore \qquad -3+14+\lambda = 0$$

$$\lambda = -11$$

NOTE: From the given velocity profile comparing with $\vec{V} = u\vec{i}+v\vec{j}+w\vec{k}$, we get u, v and w.

3.6 Acceleration of Fluid Particle

$$\vec{V} = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

$$\frac{d\vec{V}}{dt} = a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$$

$$a_x = \frac{du}{dt}$$

$$= \frac{\partial u}{\partial x}\cdot\left(\frac{dx}{dt}\right)+\frac{\partial u}{\partial y}\cdot\left(\frac{dy}{dt}\right)+\frac{\partial u}{\partial z}\cdot\left(\frac{dz}{dt}\right)+\frac{\partial u}{\partial t}\cdot\left(\frac{dt}{dt}\right)$$

$$a_x = \frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x}+v\cdot\frac{\partial u}{\partial y}+w\cdot\frac{\partial u}{\partial z}$$

Similarly

$$a_y = \frac{\partial v}{\partial t} + u\cdot\frac{\partial v}{\partial x}+v\cdot\frac{\partial v}{\partial y}+w\cdot\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\cdot\frac{\partial w}{\partial x}+v\cdot\frac{\partial w}{\partial y}+w\cdot\frac{\partial w}{\partial z}$$

Local or temporal acceleration

Convective or advective acceleration

$\therefore$ Total acceleration = (Convective 'or' Advective acceleration) + (Local or temporal acceleration)

- For steady flow: Total acceleration = Convective acceleration
- For uniform flow: Total acceleration = Temporal acceleration
- For steady-uniform flow, (total acceleration = 0).

3.7 Tangential and Normal Acceleration

Unlike velocity vector, the acceleration vector has no specific orientation with respect to stream line i.e. it need not always be tangential to stream line. Therefore at any point it may have acceleration components tangential and normal.

- Tangential acceleration is developed when the magnitude of velocity changes w.r.t. time and space.

- Normal acceleration is developed when a fluid particle moves in a curved path i.e. simply due to change in direction of velocity of fluid particle regardless of whether the magnitude of the velocity is changing not, for steady flow local acceleration = 0.

Tangential total acceleration $a_s = \frac{\partial V_s}{\partial t} + V \cdot \left(\frac{\partial V_s}{\partial s}\right)$

Normal total acceleration $a_n = \frac{\partial V_n}{\partial t} + \left(\frac{V_s^2}{r}\right)$

where, r = Radius of curvature of stream line

V_s = Tangential component of velocity V

V_n = Normal component of velocity V, generated due to change in direction

$\frac{\partial V_s}{\partial t}$ = Local tangential acceleration

$\frac{\partial V_n}{\partial t}$ = Local normal acceleration

$V_s \cdot \left(\frac{\partial V_s}{\partial t}\right)$ = Tangential convective acceleration

$\frac{V_s^2}{r}$ = Normal convective acceleration

NOTE

If stream lines are:

(i) Equidistant, tangential convective acceleration is zero.

(ii) Straight (not curved), normal convective acceleration is zero.

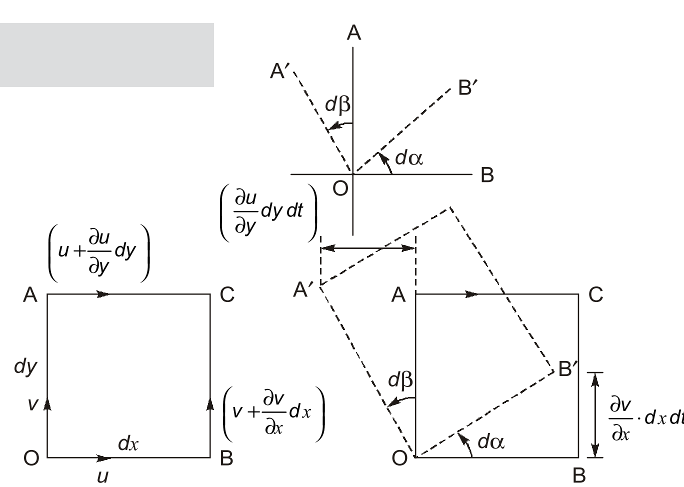
- If the stream lines are straight and parallel to each other, there is no acceleration.
- If the stream lines are curved and equidistant there will be only normal convective acceleration.
- If the stream lines are curved and converging, then both normal and tangential convective accelerations exist.

3.8 Angular Velocity (Ω)

$$\Omega = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

Let two initially perpendicular lines at O, OA and OB. Average rotation rate of these two perpendicular slines is $\frac{d\theta}{dt}$;

where $\frac{d\theta}{dt} = \frac{\left(\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right)}{2}$



Let the velocity at point 0 be u and v and length of OA and OB be dy and dx respectively.

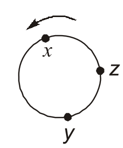
Let velocity in y-direction at B, be $\left(v + \frac{\partial v}{\partial x} dx\right)$ and velocity in x-direction at A be $\left(u + \frac{\partial u}{\partial y} dy\right)$. In time dt, point B goes to B′ and A goes to A′.

Hence,
$$\frac{d\alpha}{dt} = \left(\frac{\partial v}{\partial x}\right)\frac{dx}{dx}$$

$$\frac{d\beta}{dt} = \left(-\frac{\partial u}{\partial y}\right)\frac{dy}{dy}$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{1}{2}\cdot\left[\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right]$$

$$\Rightarrow \quad \omega_z = \frac{1}{2}\cdot\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$



Similarly,
$$\omega_y = \frac{1}{2}\cdot\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_x = \frac{1}{2}\cdot\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\Omega = \frac{1}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

- If angular velocity is zero, flow will be irrotational and for 2-D flow in x-y plane, if flow is irrotational $(w_z = 0)$

  Also,
  $$\vec{\Omega} = \frac{1}{2}(\vec{\nabla} \times \vec{V})$$

3.9 Vorticity

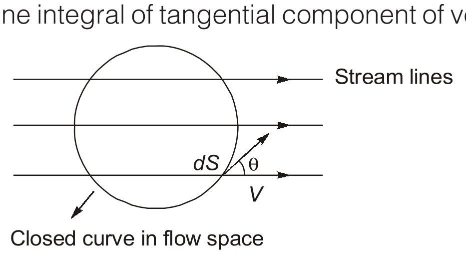
Vorticity is defined as twice of angular velocity

$$\vec{\xi} = 2\vec{\Omega} = \vec{\nabla} \times \vec{V}$$

- If vorticity at any point is zero, the flow is irrotational.

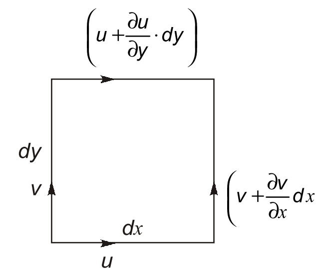
3.10 Circulation

Circulation Γ is defined as line integral of tangential component of velocity vector along a closed curve.



- $$\Gamma = \oint \vec{V} \cdot d\vec{S}$$
- If circulation around a closed curve is zero, flow in that region is irrotational.
- Consider a two dimensional flow and take a closed curve as shown:

$$\Gamma = u\,dx + \left(v + \frac{\partial v}{\partial x}dx\right)\cdot dy - \left(u + \frac{\partial u}{\partial y}dy\right)\cdot dx - v\,dy$$



$$\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\cdot dx\,dy$$

$\therefore$ Circulation = Vorticity × Area

$$\Rightarrow \quad \frac{\text{Circulation}}{\text{Area}} = \text{Vorticity} = 2\omega_z$$

Example - 3.5 Velocity components $u = (\lambda xy^3 - x^2y)$, $v = (xy^2 - 3/4 \cdot y^4)$, then the value of λ for possible flow field involving steady incompressible flow is

Solution:

Given,

$$u = (\lambda xy^3 - x^2y)$$
$$v = (xy^2 - 3/4 \cdot y^4)$$

For possible flow, steady and incompressible, continuity equation must be satisfied

$$\Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(\lambda xy^3 - 2xy) + \left(2xy - \frac{3}{4}\times 4y^3\right) = 0$$

$$y^3(\lambda - 3) = 0$$

$$\Rightarrow \quad \lambda = 3$$

3.11 Velocity Potential Function (ϕ)

Velocity potential (ϕ) is defined as a scalar function of space and time such that its negative derivative w.r.t. any direction gives velocity of flow in that direction.

$$\phi = f(x, y, z, t)$$

$$u = -\frac{\partial \phi}{\partial x},\quad v = -\frac{\partial \phi}{\partial y},\quad w = -\frac{\partial \phi}{\partial z}$$

- Velocity potential function exists only for ideal flow and irrotational flow.
- Velocity potential concept helps in integration of Euler's equation to find out Bernoulli's equation.
- If ϕ is constant in a particular direction, velocity will not exist in that direction.
- Flow always occurs in the direction of decreasing flow potential.
- If velocity potential function satisfies Laplace's equation continuity equation will be satisfied and flow is possible.

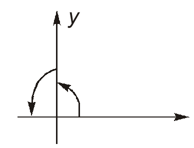
i.e.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

- Along equipotential line, flow will not occur, because $\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial z} = 0$ equipotential line.

3.12 Stream Function (ψ)

- Stream function is a scaler function of space and time such that its partial derivative w.r.t. any direction gives the velocity component at right angles (in anticlockwise direction) to this direction.



$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$

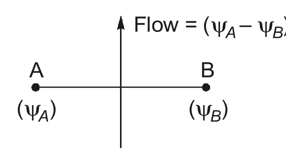
$$\therefore \quad v = \frac{\partial \psi}{\partial x}; \qquad u = -\frac{\partial \psi}{\partial y}$$

- $$w_z = \frac{1}{2}\cdot\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\cdot\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

If $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

If stream function satisfies Laplace's equation the flow will be irrotational.

- Property of stream function is that difference of stream function between two points is equal to flow across the line joining these two points.
- Existence of ψ implies that flow may be rotational or irrotational.
- Equipotential line will be orthogonal to the stream line.



Equipotential line:

$$d\phi = 0$$

$$\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

$$-u\,dx - v\,dy = 0$$

$$\frac{dy}{dx} = -\frac{u}{v}$$

$$(m_1) = -\frac{u}{v} \qquad \text{...(1)}$$

Stream line:

$$d\psi = 0$$

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y}\cdot dy = 0$$

$$-v\,dx - u\,dy = 0$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$$m_2 = \frac{v}{u} \qquad \text{...(2)}$$

$$\therefore \quad m_2 \cdot m_2 = -\frac{u}{v} \times \frac{v}{u} = -1$$

Hence, equipotential line and stream line are orthogonal to each other.

Example - 3.6 If the fluid velocity for a potential flow is given by

$\vec{V}(x,y) = u(x,y)\vec{i} + v(x,y)\vec{j}$ with usual notations, then the slope of the potential lines at (x, y) is

(a) $\frac{v}{u}$

(b) $\frac{-u}{v}$

(c) $\frac{-v^2}{u^2}$

(d) $\frac{u}{v}$

Ans. (b)

Example - 3.8 Match List-I (Type of flows) with List-II (Basic ideal flows) and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Flow over a stationary cylinder.	1.	Source + sink + uniform flow
B.	Flow over a half rankine body.	2.	Doublet + Uniform flow
C.	Flow over a rotating body.	3.	Source + Uniform flow
D.	Flow over a Rankine oval.	4.	Doublet + Free vortex + Uniform flow

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	2	4	3	1
(c)	1	3	4	2
(d)	2	3	4	1

Ans. (d)

Student's Assignments

Q.1 Normal acceleration in fluid flow situation exists only when
(a) the stream lines are straight and parallel
(b) the flow is two-dimensional
(c) the stream lines are curved
(d) the flow is unsteady

Q.2 A flownet is a graphical representation of stream lines and equipotential lines such that
(a) these lines indicate the direction and magnitude of velocity vector.
(b) these lines intersect each other orthogonally forming curve linear squares.
(c) these lines intersect each other at various different angles forming irregular shaped nets.
(d) the velocity potential increases in the direction of flow.

Q.3 Stream line, streak lines and path lines are all identical in case of
(a) unsteady flow
(b) steady flow
(c) uniform flow
(d) non-uniform flow

Q.4 In irrotational flow of an ideal fluid
(a) a velocity potential exists
(b) all particles must move in a straight line
(c) the motion must be uniform
(d) the velocity must be zero at a boundary

Q.5 A two dimensional flow is described by velocity component $u = 2x$ and $v = -2y$. The discharge between points (1, 1) and (2, 2) is equal to
(a) 9 units (b) 8 units
(c) 7 units (d) 6 units

Q.6 An imaginary tangent at a point which shows the direction of velocity of a liquid particle at that point is
(a) path line (b) stream line
(c) streak line (d) vortex line

Q.7 A two dimensional flow field is given by stream function $\psi = x^2 - y^2$. The magnitude of absolute velocity at a point (1, 1) is ________.
(a) 2 (b) 4
(c) 8 (d) $2\sqrt{2}$

Q.8 In a converging steady flow there is
(a) no acceleration
(b) no temporal acceleration
(c) only convective acceleration
(d) convective and temporal acceleration

Q.9 The concept of stream function which is based on the principle of continuity is applicable to
(a) three dimensional flow
(b) two dimensional flow
(c) uniform flow cases only
(d) irrotational flow only

Q.10 In a two dimensional flow, the equation of a stream line is given as:
(a) $\frac{dy}{u} = \frac{dx}{v}$ (b) $\frac{dx}{u} = \frac{dy}{v}$
(c) $\frac{dx}{dt} = u;\ \frac{dy}{dt} = v$ (d) $\frac{u}{dx} = \frac{dy}{v}$

Q.11 The Toricelli theorem gives velocity of jet as
(a) $\sqrt{gh}$ (b) $\sqrt{2gh}$
(c) $\frac{1}{\sqrt{2gh}}$ (d) $\sqrt{\frac{\sqrt{2gh}}{3}}$

Q.12 The basic principles of fluid flow are
1. Conservation of mass
2. Conservation of energy
3. Conservation of momentum
Which of these statements are correct?
(a) Both 1 and 2 (b) Both 2 and 3
(c) Both 1 and 3 (d) 1, 2 and 3

Q.13 The mean velocities at two ends of a streams tube 10 cm apart are 2.5 m/sec and 3 m/sec. The convectional tangential acceleration mid-way is
(a) Zero (b) 0.5 m/s^2
(c) 13.75 m/s^2 (d) Not determinable

Q.14 If the flow velocity in x and y directions are given by $u = 2x + 3y$ and $v = -2y$. The circulation around the circle of radius 2 unit will be
(a) -8π (b) -10π
(c) -12π (d) -14π

Q.15 In a two dimensional flow of fluid, if a velocity potential function ϕ exists. Which satisfies the relation $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$, then the flow is
(a) steady incompressible
(b) steady laminar and incompressible
(c) irrotational and incompressible
(d) turbulent and incompressible

Q.16 Equation of continuity is based on the principle of conservation of
(a) Mass (b) Energy
(c) Momentum (d) None of these

Q.17 A one dimensional flow is one which
(a) is uniform
(b) is steady uniform
(c) takes place in straight lines
(d) involves zero transverse component of flow

Q.18 For a flow, the velocity components are given by $(u = \lambda xy^2 - x^3y^2)$ and $(v = x^2y^3 - 3y^3)$ what is the value of λ for the possible flow field which includes steady incompressible flow
(a) 3 (b) 5
(c) 7 (d) 9

Q.19 What is the value of angle (degree) between stream lines and equipotential lines at the point of intersection in the flow net?

(a) 0 (b) 45
(c) 60 (d) 90

Q.20 When the water drawn from the central hole made is wash hand basin, the type of flow of water is ________.

(a) forced vortex (b) free vortex
(c) tangential flow (d) transitional flow

Q.21 Which of the following is calculated with the help of Moddy's equation

(a) discharge (b) friction factor
(c) pressure (d) velocity of flow

Q.22 If at the particular instant of time, the velocity of flow does not change with location over specific with location over specific region, the flow is called as

(a) steady flow (b) unsteady flow
(c) uniform flow (d) non-uniform flow

Q.23 In which of the following case flow net cannot be drawn?

(a) irrotational flow
(b) steady flow
(c) when flow is governed by gravity
(d) when flow is not governed by gravity

Q.24 The value obtained from dividing limiting value of circulation by area of closed contour is known as

(a) Potential function
(b) Stream function
(c) Vorticity
(d) None of these

Q.25 Which of the following expressions represents the continuity equation in case of steady incompressible flow?

(a) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

(b) $\frac{\partial u}{\partial x}+\frac{\partial p}{\partial y}+\frac{\partial p}{\partial z}=0$

(c) $\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z}=0$

(d) None of these

Q.26 A path line describes

(a) The velocity direction at all points on the line
(b) The path followed by particles in a flow
(c) The path over a period of times of a single particle that has based out at a point
(d) The instantaneous position of all particles that have passed a point

Q.27 If u and v are the component of velocity in the 'x' and 'y' directions of a flow given by $u = ax + by$, $v = cx + dy$ then the flow condition to be satisfied is ________.

(a) $a + c = 0$
(b) $b + d = 0$
(c) $a + b + c = 0$
(d) $a + d = 0$

Q.28 For a fluid with $\vec{V} = 2x\hat{i} + y\hat{j}$ the acceleration at (1, 1) is ________ units.

(a) 5 (b) 9
(c) 17 (d) $\sqrt{17}$

Q.29 In a three dimensional motion of a fluid, the component of rotation about the x-axis (w_n) is ________.

(a) $\frac{1}{2}\cdot\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)$ (b) $\frac{1}{2}\left(\frac{\partial y}{\partial x}-\frac{\partial w}{\partial y}\right)$

(c) $\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ (d) $\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial w}{\partial y}\right)$

Q.30 If flow conditions satisfy "Laplace equation" then

(a) flow is rotational
(b) flow does not satisfy continuity equation
(c) flow is irrotational but does not satisfy continuity equation
(d) flow is irrotational and flow satisfy continuity equation

Q.31 A flow variable like $\vec{V} = \vec{V}(x, y, z, t)$ are used to define in

(a) Langrangian approach
(b) Rankine vortex motion
(c) Eulerian approach
(d) None of these

Q.32 The velocity potential function in a two dimensional flow field is given by $\phi = x^2 - y^2$. The magnitude of velocity at point $P(1, 1)$ is

(a) 0 (b) 2

(c) $2\sqrt{2}$ (d) 8

Q.33 The convective acceleration of fluid in the x-direction is given by

(a) $u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z}$

(b) $\frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} + \frac{\partial w}{\partial t}$

(c) $u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + u\frac{\partial w}{\partial z}$

(d) $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

Q.34 A stream function is given by $(x^2 - y^2)$. The potential function of the flow will be

(a) $2xy + f(x)$ (b) $2xy$ + constant

(c) $2(x^2 - y^2)$ (d) $2xy + f(y)$

Q.35 If for a flow, a stream function ψ exists and satisfies the Laplace equation, then which one of the following is the correct statement?

(a) The continuity equation is satisfied and the flow is irrotational.

(b) The continuity equation is satisfied and the flow is rotational.

(c) The flow is irrotational but does not satisfy the continuity equation.

(d) The flow is rotational.

Q.36 Irrotational flow is characterized as the one in which the

(a) fluid flows along a straight line

(b) fluid does not rotate as it moves along

(c) net rotation of fluid particles about their mass centres remains zero

(d) streamlines of flow are curved and closely spaced

Q.37 The components of velocity in a two dimensional frictionless incompressible flow are $u = t^2 + 3y$ and $v = 3t + 3x$. What is the approximate resultant total acceleration at the point (3, 2) and $t = 2$?

(a) 5 (b) 49

(c) 59 (d) 54

Q.38 A steady, incompressible flow is given by:

$u = 2x^2 + y^2$ and $v = -4xy$

What is the convective acceleration along x-direction at point (1, 2)?

(a) $a_x = 6$ unit (b) $a_x = 24$ unit

(c) $a_x = -8$ unit (d) $a_x = -24$ unit

Q.39 Consider the following remarks pertaining to the irrotational flow:

1. The Laplace equation of stream function $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$ must be satisfied for the flow to be potential.
2. The Laplace equation for the velocity potential $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$ must be satisfied to fulfil the criterion of mass conservation i.e. continuity equation.

Which of the above statements is/are correct?

(a) 1 only (b) Both 1 and 2

(c) 2 only (d) Neither 1 nor 2

ANSWER KEY — STUDENT'S ASSIGNMENTS

1. (d)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (d)
6. (a)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (b)
11. (b)	**12.** (d)	**13.** (c)	**14.** (c)	**15.** (c)
16. (a)	**17.** (d)	**18.** (d)	**19.** (d)	**20.** (b)
21. (b)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (a)
26. (c)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (d)
31. (c)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (a)
36. (c)	**37.** (c)	**38.** (c)	**39.** (b)	

HINTS & SOLUTIONS — STUDENT'S ASSIGNMENTS

5. (d)

Given, $u = 2x, \; v = -2y$

$$v = \frac{\partial\psi}{\partial x} = -2y \quad \text{...(1)}$$

$$u = \frac{\partial\psi}{\partial y} = +2x \quad \text{...(2)}$$

Integrating equation (1)

$$\psi = -2xy + f(y) \quad ...(3)$$

Differentiating equation (3) w.r.t y

$$\frac{\partial \psi}{\partial y} = -2xy + f'(y) \quad ...(4)$$

Equating equation (2) and equation (4)

$$f'(y) = 0$$

∴ Integrating

$$f(y) = \text{constant}$$

$$\therefore \quad \psi = -2xy + c$$

$$\therefore \quad \psi_A(1, 1) = -2 + c$$

$$\psi_B(2, 2) = -8 + c$$

$$\therefore \quad |\psi_A - \psi_B| = |(-2 + c) - (-8 + c)|$$

$$= 6 \text{ units}$$

7. (d)

$$\psi = x^2 - y^2$$

$$u = -\frac{\partial \psi}{\partial y} = -(-2y) = 2y$$

$$v = \frac{\partial \psi}{\partial x} = 2x$$

$$\therefore \quad \vec{V} = 2y\vec{i} + 2x\vec{j}$$

At (1,1)

$$\vec{V} = 2\vec{i} + 2\vec{j}$$

$$\therefore \quad |\vec{V}| = \sqrt{(2)^2 + (2)^2} = \sqrt{8}$$

$$= 2\sqrt{2} \text{ units}$$

13. (c)

Convectional tangential acceleration is given by

$$V \cdot \frac{dV}{dS} = \bar{V} \cdot \frac{\Delta V}{\Delta S}$$

$$= \left(\frac{2.5 + 3.0}{2}\right)\left(\frac{3.0 - 2.5}{0.1}\right) = 13.75 \text{ m/s}^2$$

14. (c)

$$w_z = \frac{1}{2} \cdot \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2} \cdot (0 - 3) = -\frac{3}{2}$$

$$\therefore \quad \xi(\text{vorticity}) = 2w_z = 2 \times \left(-\frac{3}{2}\right) = -3$$

$$\therefore \quad \frac{\text{Circulation}}{\text{Area}} = -3$$

$$\therefore \quad \text{Circulation} = -3 \times \pi \times (2)^2 = -12\pi$$

18. (d)

For steady incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(\lambda y^2 - 3x^2y^2) + (3x^2y^2 - 9y^2) = 0$$

$$\lambda = 0$$

27. (d)

$$u = ax + by$$

$$v = cx + dy$$

For flow to be satisfied,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(a + d) = 0$$

28. (d)

$$\vec{V} = 2x\hat{i} + y\hat{j}$$

$$u = 2x; \; v = y$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial v}{\partial y} = 2x \times 2 = 4x$$

$$a_y = u \cdot \frac{\partial v}{\partial x} + y \cdot \frac{\partial v}{\partial y} = y \times (1) = y$$

At (1, 1) $\quad a_x = 9; \; a_y = 1$

$$\therefore \quad \vec{a} = 4\vec{i} + \vec{j}$$

$$(\vec{a}) = \sqrt{16 + 1} = \sqrt{17} \text{ units}$$

31. (c)

In eulerian method, we take a finite volume called control volume through which fluid flows in and out.

The flow variable at a particular location, at a particular time, is the value of variable for whichever fluid particle happens to occupy that location at that time,

$$p = p(x, y, z, t) \qquad \boxed{\begin{array}{c}\cdot A \\ (x, y, z, t)\end{array}}$$

$$\vec{V} = \vec{V}(x, y, z, t)$$

$$\vec{a} = \vec{a}(x, y, z, t)$$

32. (c)

$$\phi = x^2 - y^2$$

$$u = -\frac{\partial \phi}{\partial x} = -2x$$

$$v = -\frac{\partial\phi}{\partial y} = 2y$$

$$\vec{V} = u\hat{i} + v\hat{j} = -2x\hat{i} + 2y\hat{j}$$

At (1, 1)

$$\vec{V} = -2\hat{i} + 2\hat{j}$$

$$|\vec{V}| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8}$$

$$= 2\sqrt{2} \text{ units}$$

33. (d)

$$a_x = u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}$$

Where $\frac{du}{dt}$ = Local or temporal acceleration

Remaining terms are called "convective acceleration".

34. (b)

$$\frac{d\psi}{dx} = 2x = -\frac{d\phi}{dy} \qquad \therefore \frac{d\phi}{dy} = -2x$$

and $\frac{d\psi}{dy} = -2y = \frac{d\phi}{dx} \qquad \therefore \frac{d\phi}{dx} = -2y$

Now, $\frac{d\phi}{dy} = -2x$

$\therefore \quad \phi = -2xy + f(x)$

Differentiating w. r. to x

$$\frac{d\phi}{dx} = -2y + f'(x) = -2y$$

$\therefore \; f'(x) = 0$

$\therefore \; f(x) = \text{constant}$

$\therefore \; \phi = -2xy + \text{constant}$

35. (a)

Stream function always satisfies continuity equations since in this case it satisfies Laplace equation therefore it is a case of irrotational flow.

35. (c)

Mathematically, fluid flow in which the curl of velocity function i.e. vorticity is zero everywhere, so that the circulation of the velocity about any closed curve vanishes. Irrotational motion is also known as **acyclic motion**.

37. (c)

$$a_x = v\frac{du}{dy} + \frac{du}{dt} = (3t + 3x)3 + 2t = 49 \text{ units}$$

$$a_y = u\frac{dv}{dx} + \frac{dv}{dt} = (t^2 + 3y)3 + 3 = 33 \text{ units}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(49)^2 + (33)^2} = \mathbf{59 \text{ units}}$$

38. (c)

Convective acceleration along x direction at point (1, 2)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= (2x^2 + y^2)(4x) + (-4xy)(2y)$$

$$= (2 + 4)(4) + (-4)(1)(2)(2)(2)$$

$$= 24 - 32 = \mathbf{-8 \text{ unit}}$$

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