OPSC-AEE 2020

Odisha Public Service Commission

Assistant Executive Engineer

Civil Engineering

Fluid Mechanics

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

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Fluid Mechanics

Contents

CHAPTER 4

Fluid Dynamics

4.1 Introduction

- Dynamics of fluid flow is the study of fluid motion with the forces causing the motion.
- Dynamics behaviour of fluid flow is analyzed by conservation of mass, energy and momentum.
- In the fluid flow various forces acting are:

  (a) Gravity force $(\vec{F}_g)$ (b) Pressure force $(\vec{F}_p)$

  (c) Viscous force $(\vec{F}_\nu)$ (d) Force due to compressibility $(\vec{F}_c)$

  (e) Force due to turbulence $(\vec{F}_t)$ (f) Minor forces like surface tension $(\vec{F}_\sigma)$
- When all forces are taken into account the equation of motion is called Newton's equation of motion.
- When compressibility and other minor forces are neglected, it is called Reynold's equation of motion.

$$\vec{F}_g + \vec{F}_p + \vec{F}_\nu + \vec{F}_t = m\bar{a}$$

- When compressibility, turbulence and minor forces neglected and only gravity, pressure and viscosity is taken into account, the equation of motion is called Navier stokes equation.

$$\vec{F}_g + \vec{F}_p + \vec{F}_\nu = m\bar{a}$$

- When only gravity and pressure force are considered the equation of motion is called Euler's equation of motion.

$$\vec{F}_g + \vec{F}_p = m\bar{a}$$

- Dynamics of fluid motion when only pressure and gravity is accounted for, is governed by momentum principle (Euler's equation) and energy principle (Bernoulli's equation).

4.2 Bernoulli's Equation of Motion

Integration of Euler's equation of motion along a stream line under steady incompressible condition yields Bernoulli's equation.

4.2.1 Euler's Equation of Motion

$$\frac{dP}{\rho} + g\,dz + v\,dv = 0 \qquad ...(1)$$

Integrating above equation

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

$$\int \frac{dP}{\rho} + gz + \frac{V^2}{2} = \text{constant} \qquad ...(2)$$

Equation (2) is applicable for steady and compressible flow and the equation of motion is known as Bernoulli's equation for compressible flow.

- For incompressible flow, the equation (2) can be reduced to

$$\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{constant} \quad ...(3)$$

The above equation (3) is known as Bernoulli's equation or energy equation for incompressible flow. The conditions to be satisfied for applicability of Bernoulli's equations are:

(i) Flow is along a stream line.

(ii) Flow is steady and incompressible.

(iii) Effect of friction (viscous forces) is negligible i.e. when fluid is ideal or viscosity has negligible effect.

Following are the conditions for which the Bernoulli's equation cannot be applied to:

(i) Long narrow flow passage → friction is significant.

(ii) Wake region downstream of an object → losses in energy.

(iii) Near solid boundary → viscosity predominates.

(iv) Diverging flow section → chances of flow separation and wake formation.

(v) Flow section that involves fan turbine and any other machines. Such device disrupts the stream line and carries out energy interaction with fluid particles.

(vi) For mach number ≥ 0.3 → (because compressibility becomes predominant).

(vii) Flow sections that involves temperature changes, because as temperature changes density (i.e. when there is heat transfer).

4.2.2 Analysis of Bernoulli's Equation

From equation (2)

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{Constant}$$

where,

$$\frac{P}{\rho} = \text{Pressure energy per unit mass}$$

$$\frac{V^2}{2} = \text{Kinetic energy per unit mass}$$

$$gz = \text{Potential energy per unit mass}$$

Also, equation (2) can be written as

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{Constant}$$

$$\frac{P}{\gamma} = \text{Pressure energy per unit weight or pressure head}$$

$$\frac{V^2}{2g} = \text{Kinetic energy per unit weight or kinetic energy head}$$

$$z = \text{Potential energy per unit weight or elevation head}$$

- For steady incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.

Also,
$$P+\frac{\rho V^2}{2}+\rho gz = \text{Constant}$$

$P \rightarrow$ Static pressure (actual pressure in fluid)

$\frac{\rho V^2}{2} \rightarrow$ Dynamic pressure (pressure rise when fluid in motion is brought to rest)

$\rho gz \rightarrow$ Hydrostatic pressure (effect of fluid weight on pressure)

Statement of Bernoulli's equation: During steady incompressible flow with negligible friction the various forms of mechanical energy are converted to each other but their sum remains constant provided that there is not heat or mass transfer.

- Bernoulli's equation can be applied between any two points in fluid flow if the flow is irrotational. However for rotational flow, Bernoulli's equation can be applied between two points on same stream line.
- Practically we use energy equation which has the same form as Bernoulli's equation but in this case we consider losses.

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+h_L$$

h_L = Net head loss between section (1) and (2)

4.2.3 Various Forms of Bernoulli's Equation

- Since from Euler's equation of motion

$$\frac{dP}{\rho}+gdz+Vdv = 0$$

By integration of above equation we get Bernoulli's equation as

$$\int\frac{dP}{\rho}+gz+\frac{V^2}{2} = \text{Constant}$$

Case-1: Incompressive flow (ρ = Constant)

$$\frac{P}{\rho}+gz+\frac{V^2}{2} = \text{Constant}$$

Divide by 'g' both sides

$$\frac{P}{\rho g}+z+\frac{V^2}{2g} = \text{Constant}$$

Case-2: Isothermal process (Temp. = Constant)

$$\Rightarrow \quad \frac{P}{\rho} = \text{Constant} = C_1$$

$$\int\frac{dP}{\rho} = \int\frac{dP}{(P/C_1)} = C_1\int\frac{dP}{P}$$

$$= (\ln P)\frac{P}{\rho} = \frac{P}{\rho}\ln P$$

$$\therefore \qquad \frac{P}{\rho}\ln P + \frac{V^2}{2} + gz = \text{Constant}$$

Divided both side by g

$$\frac{P}{\rho g} imP + \frac{V^2}{2g} + z = \text{Constant}$$

Case-3: Adiabatic Process

$$\frac{P}{\rho^\gamma} = \text{Constant} = C_2$$

$$\int \frac{dP}{\rho} = \int \frac{dP}{(P/C_2)^{1/\gamma}} = (C_2)^{1/\gamma} \int \frac{dP}{P^{1/\gamma}}$$

$$= \left[\frac{P}{\rho^\gamma}\right]^{1/\gamma} \frac{P^{-\frac{1}{\gamma}+1}}{-\frac{1}{\gamma}+1} = \frac{(P)^{\frac{1}{\gamma}-\frac{1}{\gamma}+1}}{\rho\left(\frac{\gamma-1}{\gamma}\right)} = \frac{\gamma P}{(\gamma-1)\rho}$$

$$= \frac{P}{\rho}\left(\frac{\gamma}{\gamma-1}\right)$$

Now, $$\frac{\gamma}{\gamma-1}\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{Constant}$$

Divide by g both side

$$\frac{\gamma}{\gamma-1} \times \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}$$

NOTE

- Sum of static and dynamic pressures is called stagnation pressure.

$$P_{stag.} = P + \frac{\rho V^2}{2}$$

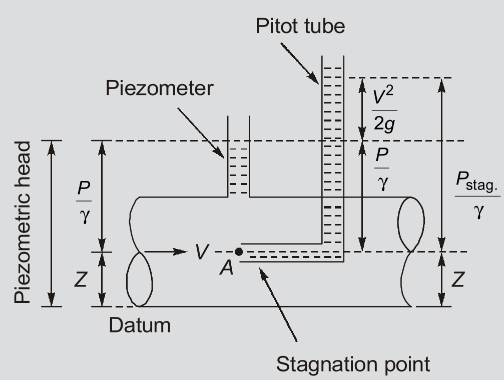
or $$\text{stagnation head} = \frac{P}{\gamma} + \frac{V^2}{2g}$$

- Sum of static pressure and hydrostatic pressure is called piezometric pressure.

$$P_{piez.} = P + \rho_{gz}$$

or $$\text{piezometric head} = \left(\frac{P}{\gamma} + z\right)$$

- If a piezometer is inserted at a point in a fluid flow, the rise of fluid in piezometer is upto the level of piezometric head. The actual rise is however, equal to pressure head.



$$\frac{P_{stag.}}{\gamma} = \frac{P}{\gamma} + \frac{V^2}{2g}$$

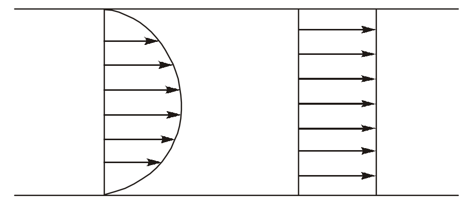
$$\frac{P_{stag.}}{\gamma} - \frac{P}{\gamma} = \frac{V^2}{2g}$$

- Pitot tube is a device used for measuring the velocity of flow at any pivot in a pipe or a channel.

4.3 Kinetic Energy Correction Factor (α)

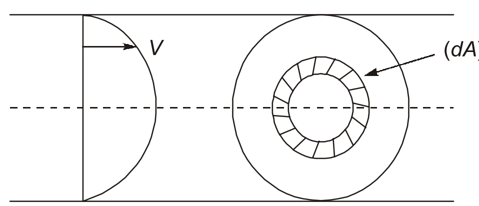
- In Bernoulli's equation, the velocity head is computed on the basis of the assumption that velocity is uniform over the entire cross-section of the stream tube. But in the case of flow of real fluid, the velocity distribution across any cross-sectional area of the flow passage is not uniform.

  So, the actual kinetic energy possessed by the fluid is different from that computed by using the mean velocity.



(Velocity variation in real fluid) (Velocity variation in ideal fluid)

- To obtain actual kinetic energy, kinetic energy correction factor (α) is multiplied to kinetic energy obtained through mean velocity.



$$\alpha = \frac{\text{Actual kinetic energy}}{\text{K.E. calculated using average velocity}}$$

$$\text{Actual kinetic energy} = \int_A \frac{1}{2}\cdot(\rho V dA)\cdot V^2 = \int_A \frac{1}{2}\rho V^3 dA$$

Kinetic Energy (K.E.) calculated using average velocity

$$= \frac{1}{2}\cdot(PQ)\cdot V_{aV}^2 = \frac{1}{2}(\rho A V_{aV})\cdot V_{aV}^2 = \frac{1}{2}\rho A V_{aV}^3$$

$$\therefore \qquad \alpha = \frac{\int_A V^3 dA}{A\cdot V_{aV}^3}$$

$$V_{aV} = \frac{Q}{A} = \frac{\int_A V\, dA}{A}$$

- For ideal flow profile ($\alpha \simeq 1.0$).
- For turbulent flow in circular pipe, α = 1.03 to 1.06.
- For laminar flow is circular pipe, $\alpha = 2$ and for parallel plates ($\alpha = 1.543$).
- Value of α for laminar flow is greater than for turbulent flow because in laminar flow velocity gradient across the section is greater than the turbulent flow.
- Accordingly the modified energy equation between any two section is

$$\frac{P_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

4.4 Momentum Correction Factor (β)

A similar parameter momentum correction factor is defined in momentum equation as

$$\beta = \frac{\int_A V^2 dA}{A \cdot V_{aV}^2}$$

$$V_{aV} = \frac{Q}{A} = \frac{\int_A V\, dA}{A}$$

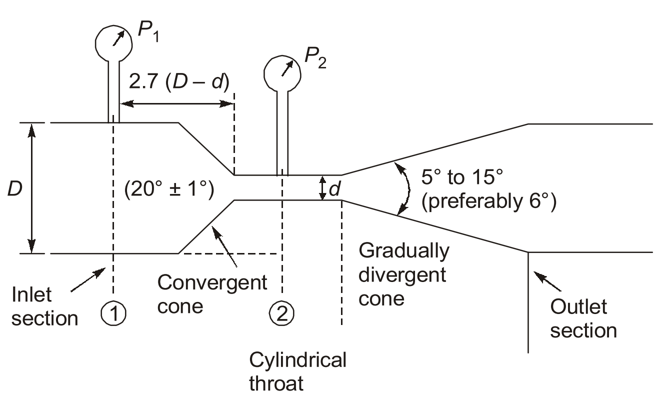
- For circular pipe in laminar flow, $\beta = \frac{4}{3}$.

4.5 Applications of Bernoulli's Equation

4.5.1 Venturimeter

- It is a device used for measuring the rate of flow of fluid through pipe.

  Principle: Reduction in area at throat section results in increase in velocity in steady flow and this increase in velocity results in decrease in pressure. The decrease in pressure is noted and the discharge in found out by applying Bernoulli's equation.



$$\text{Area ratio} = \left(\frac{a_2}{a_1}\right)$$

$$\text{Diameter ratio} = \frac{D_2}{D_1} = \frac{d}{D}$$

Applying Bernoulli's equation between (1) and (2)

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$\left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) = \frac{V_2^2 - V_1^2}{2g} = h$$

$$h = \frac{V_2^2 - V_1^2}{2g} = \text{Piezometric head difference} = \frac{Q^2\left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right)}{2g}$$

$$\Rightarrow \quad Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh} \quad ...(1)$$

The above discharge is a theoretical discharge or ideal discharge as we have neglected losses.

$$\therefore \quad Q_{act} = C_d \cdot \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh} \quad ...(2)$$

- In venturimeter as we have gradual contraction and gradual expansion, there is no flow separation and hence a_2 can be taken as complete area of throat.
- In venturimeter as flow separation does not occur, hence $C_d = 0.98$ can be achieved.
- Cross-sectional area of throat cannot be reduced much otherwise pressure may fall below vapour pressure and cavitation may start.
- As separation of flow may occur in divergent cone of venturimeter, this portion is not used for discharge measurement.
- To avoid flow separation angle of convergence and angle of divergence are taken approx. 22° and 6° respectively. Note that as the chances of flow separation is more in the divergent portion, the angle of divergence is taken smaller so that gradual expansion occurs and flow does not separate.

$$d \le \left(\frac{1}{3} \text{ to } \frac{3}{4}\right) \cdot D$$

Normally, $d = \frac{D}{2}$

Where, d = Diameter of throat, D = Diameter of pipe

Variation of C_d with losses:

$$\left(\frac{P_1}{\gamma} + z_1\right) + \frac{V_1^2}{2g} = \left(\frac{P_2}{\gamma} + z_2\right) + \frac{V_2^2}{2g} + h_L$$

$$h = \frac{V_2^2 - V_1^2}{2g} + h_L$$

$$(h - h_L) = \frac{V_2^2 - V_1^2}{2g} = \frac{Q^2}{2g} \cdot \left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right)$$

$$\therefore \quad Q_{act} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2g(h - h_L)} \quad ...(3)$$

Compare equation (2) and (3)

$$C_d \cdot \sqrt{h} = \sqrt{h - h_L}$$

$$C_d = \sqrt{\frac{h - h_L}{h}} = \sqrt{1 - \frac{h_L}{h}}$$