MPSC 2019

Maharashtra Public Service Commission

Assistant Engineer Examination

Civil Engineering

Fluid Mechanics & Fluid Machines

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

MADE EASY Publications

Fluid Mechanics & Fluid Machines

Contents

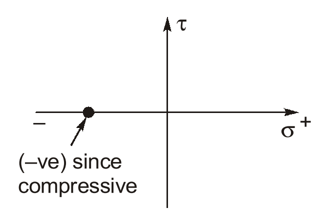
CHAPTER 01

Fluid Properties

1.1 Fluid Mechanics

- Fluid mechanics is the branch of engineering science which involves the study of fluids and forces on them.
- In fluid mechanics we study the fluid behaviour at rest and in motion.
  (i) Study of fluid at rest → fluid statics.
  (ii) Study of fluid in motion when forces responsible for motion are not considered → fluid kinematics.
  (iii) Study of fluid in motion considering the forces responsible for motion → fluid dynamics.
- Fluid mechanics is a branch of continuum mechanics, a subject which models matter without using the information that it is made out of atoms, that is, it models matter from a macroscopic view point rather then from microscopic.

1.2 Fluid



- A substance in liquid or gaseous phase is referred to as fluid, if they are capable of deforming continuously under the action of shear stress, however small the shear stress may be.
- For a static fluid there is no shear force.
- Since there is no shear force in static fluid hence the Mohr's circle is a point.

NOTE: In solids stress is proportional to strain but in fluid stress is proportional to strain rate.

1.2.1 Types of Fluid

1.2.1.1 Ideal Fluid (Perfect Fluid)

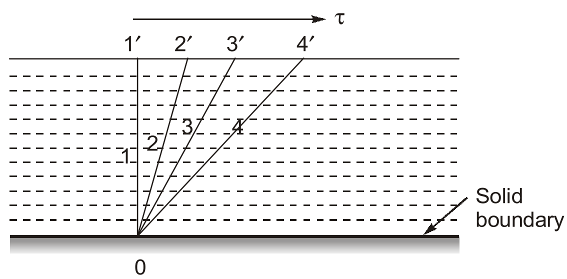
- Non-viscous, friction less and incompressible.
- Does not offer shear resistance against flow.
- Bulk modulus is infinite
- Used in mathematical analysis and flow problems.
- No such fluid exist in practical situation.

1.2.1.2 Real Fluid

- Possess the properties such as viscosity, surface tension and compressibility.
- Offers resistance against flow.

1.3 Fluid Continuous and Continuum Concept

- In a fluid system, the intermolecular spacing between the fluid particles is treated as negligible and the entire fluid mass system is assumed as continuous distribution of mass, which is known as continuum.
- The continuous deformation of fluid under the action of shear stress causes a flow. Figure below shows a shear stress (τ) is applied at any location in a fluid, the element 011′ which is initially at rest, will move to 022′, then to 033′ and to 044′ and soon.



- It is a kind of idealization of the properties of the matter for flow analysis.
- Any matter is composed of several molecules continue concept assumes a continuous distribution of mass within the matter with no empty space or voids.
- **Mean free path:** Statistical average distance, which molecules of the same fluid travel between collisions.
- Mean free path is large in comparison to some characteristics length, gas cannot be treated as continuous medium and instead it is analysed by "molecular theory".
- To describe the degree of departure from continuum, a non-dimensional number known as Knudsen number (K_n) is used

$$K_n = \frac{\lambda}{L} = \frac{\text{Mean free path}}{\text{Characterastics length of flow}}$$

- If $K_n > 0.01$, the concept of continuum does not hold good.
- Fluid can be treated as continuous when $K_n < 0.01$, This holds good for fluid mechanics.

1.4 Properties of Fluids

(i) **Mass density (ρ):** It is the mass of the matter occupied in unit volume at a standard temperature and pressure. It is denoted by 'ρ'.

$$\rho = \frac{m}{V} \cdot (\text{kg/m}^3)$$

Matter	Mass density, P(kg/m^3)
Air	1.2
Water	1000
Mercury	13600
Steel	7850
Wood	600

(ii) Specific weight (γ, W or ρg): Weight of the matter per unit volume

$$W = \gamma = \frac{W}{V} = \frac{mg}{V} = \rho g \cdot \left(\frac{N}{m^3}\right)$$

- It is not absolute quantity and varies from place to place.
- It is also known as weight density.

Matter	Specific weight; $\gamma = \rho g$
Air	11.77 N/m^3
Water	9.81 kN/m^3

(iii) Specific volume (V_s):

- Volume occupied by unit mass of fluid.

$$V_s = \frac{1}{\rho}(m^3/kg)$$

- It is reciprocal of mass density.

(iv) Specific gravity (S) or relative density:

- It is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid at a specified temperature.
  (Usually water at 4°C)

$$S = \frac{\rho}{\rho_{water}} = \frac{\gamma}{\gamma_{water}}$$

- Units: No units

Matter	Specific gravity (S)
Air	0.0012
Water	1.0
Wood	0.6

Example-1.1 **Three litres of petrol weighs 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.**

Solution:

Mass density of petrol
$$\rho_p = \frac{\text{Mass}}{\text{Volume}} = \frac{\left(\frac{23.7}{9.81}\right)}{3.0} = 0.805 \text{ kg/litres} = 805 \text{ kg/m}^3$$

Mass density of water
$$(\rho_w) = 1000 \text{ kg/m}^3$$

Specific gravity of petrol,
$$S = \frac{\rho_p}{\rho_w}$$
$$S = \frac{805}{1000} = 0.805$$

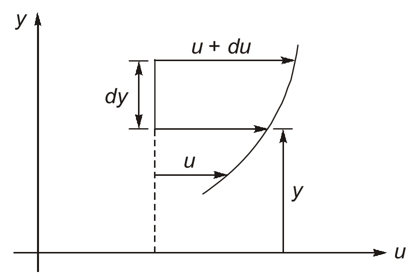
Specific weight of petrol,
$$(\gamma) = \rho g = 805 \times \frac{9.81}{1000} \text{kN/m}^3 = 7.9 \text{ kN/m}^3$$

Specific volume of petrol,
$$V_s = \frac{1}{\rho_p} = \frac{1}{805} = 1.242 \times 10^{-3} \text{ m}^3/\text{kg}$$

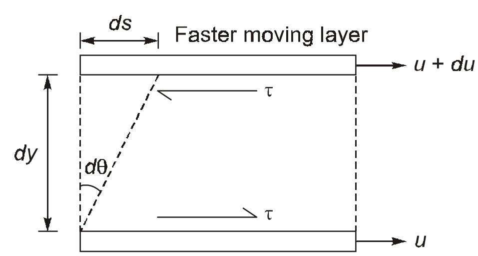
NOTE: If specific gravity, $S < 1 \Rightarrow$ fluid is lighter then water.

1.5 Viscosity

- It is a measure of resistance of fluid to deformation. It is due to cohesion and molecular momentum exchange between fluid layers and as flow occurs, these effects appears as shearing stresses between the moving layers.
- Suppose one layer of fluid is moving with respect to the other layer by a velocity = du and vertical gap between two layers be dy.



- Upper layer which is moving faster tries to draw the lower slowly moving layer along with it. Similarly, as a reaction to this, the lower layer tries to retard the upper one. Thus there exists a shear between the two layers as shown below.



- In time dt, the top layer will move with respect to the bottom layer by a distance $ds = du \cdot dt$.

Hence, $$\frac{ds}{dy} = d\theta = \text{shear strain}$$

$$\frac{du \cdot dt}{dy} = d\theta$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{du}{dy} \quad \text{...(i)}$$

$\Rightarrow$ Rate of change of shear strain $\left(\frac{d\theta}{dt}\right)$ = Velocity gradient $\left(\frac{du}{dy}\right)$

- On the basis of relation between the applied shear stresses and the flow or rate of deformation, fluids can be categorized as Newtonian and Non-Newtonian fluid.
- **Newtonian fluids:** Fluid which obeys Newton's law of viscosity are known as Newtonian fluid.

  Newton's law of viscosity: The fluid for which rate of deformation is linearly proportional to shear stress.

  Thus, for Newtonian fluid

$$\tau \propto \frac{d\theta}{dt}$$

τ = Shear stress opposing the movement of fluid

$$\therefore \quad \tau \propto \frac{du}{dy} \text{ (from equation (i))}$$

$$\therefore \quad \tau = \mu \cdot \frac{du}{dy}$$

μ = Absolute viscosity, 'or' coefficient of viscosity 'or' dynamic viscosity

- Water, air and gasoline are Newtonian under normal conditions.

Dimensions and Units:

Dynamic Viscosity (μ):

SI system: Pa-sec, or $\frac{\text{N}-\text{sec}}{\text{m}^2}$ or $\frac{\text{kg}}{\text{m-sec}}$.

CGS system:

$$1 \text{ poise} = \frac{\text{Dyne-sec}}{\text{cm}^2} \quad \left\{\text{Dyne} = \frac{\text{gm-cm}}{\text{sec}^2}\right\}$$

$$\therefore \quad 1 \text{ poise} = \frac{\text{gm}}{\text{cm-sec}}$$

Conversion:

$$1 \text{ poise} = \frac{1}{10} \text{ Pa-sec}$$

Dimensions of dynamic viscosity: $[ML^{-1}T^{-1}]$

$$(\mu)_{water} = 10^{-3} \frac{\text{N}-\text{sec}}{\text{m}^2} = 1 \text{ centipoise}$$

$$(\mu)_{air} = 1.81 \times 10^{-5} \frac{\text{N}-\text{sec}}{\text{m}^2}$$

(Both at 20° and at standard atmospheric pressure)

NOTE: Water is nearly 55 times viscous then air.

Kinematic Viscosity (ν):

$$\nu = \frac{\text{Dynamic viscosity}}{\text{Mass density}} = \frac{\mu}{\rho}$$

Units:

SI system - m^2/sec

CGS system - cm^2/sec or stoke

$$1 \text{ stoke} = \text{cm}^2/\text{sec} = 10^{-4} \text{ m}^2/\text{sec}$$

Dimension: $[L^2 T^{-1}]$

At 20°C and at standard atmospheric pressure

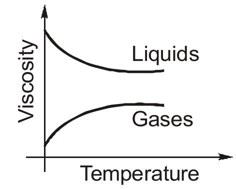
$$\nu_{water} = 1 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$\nu_{air} = 15 \times 10^{-6} \text{ m}^2/\text{sec}$$

NOTE: Kinematic viscosity of air is about is times greater than the corresponding value of water.

Variation of Viscosity with Temperature:

- Increase in temperature cause a decrease in the viscosity of a liquid whereas viscosity of gases increases with temperature growth.



NOTE: In gases, molecular momentum increases and cohesion is negligible.

Example-1.2 A plate 0.05 mm distant from a fixed plate moves at 1.2 m/sec and requires a shear stress of 2.2 N/m^2 to maintain this velocity. Find the viscosity of the fluid between the plates.

Solution:

Let μ be the viscosity of fluid between the plates.

Given, $V = 1.2$ m/sec

$y = 0.05$ mm $= 0.05 \times 10^{-3}$ m

Shear stress $(\tau) = 2.2$ N/m^2, To find $(\mu) = ?$

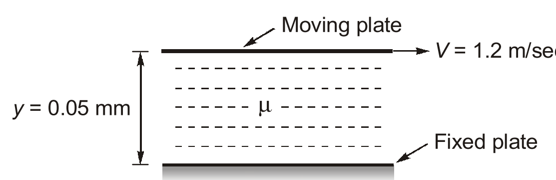
By Newton's law of viscosity we know that

$$\tau = \mu \cdot \frac{V}{y}$$

$$2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$$

$$\mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2}$$

$$\mu = 9.16 \times 10^{-5} \text{ N-sec/m}^2$$



Non-Newtonian Fluids

- These do not follow Newton's law of viscosity. The relation between shear stress and velocity gradient is

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

where A and B are constants depending upon type of fluid and condition of flow.

- The study of Non-Newtonian fluid is knows as Rheology.
  (i) For Dilatant Fluids: $n > 1$ and $B = 0$,
  Ex. Butter, Quick sand, Rice starch, Sugar in H_2O
  (ii) For Bingham Plastic Fluids: $n = 1$ and $B \neq 0$
  Ex. Sewage sludge, Drilling mud, Tooth paste, Gel.
  These fluids always have certain minimum shear stress before they yield.
  (iii) For Pseudoplastic Fluids: $n < 1$ and $B = 0$
  Ex. Paper pulp, Rubber solution, Lipsticks, Paints, Blood, Polymetric solutions, milk, etc.
  (iv) For Thixotropic Fluids: $n < 1$ and $B \neq 0$
  Viscosity increases with time.
  Ex. Printers ink and Enamels.
  (v) For Rheopectic Fluids: $n > 1$ and $B \neq 0$
  Viscosity decreases with time.
  Ex. Gypsum solution in water and Bentonite solution.

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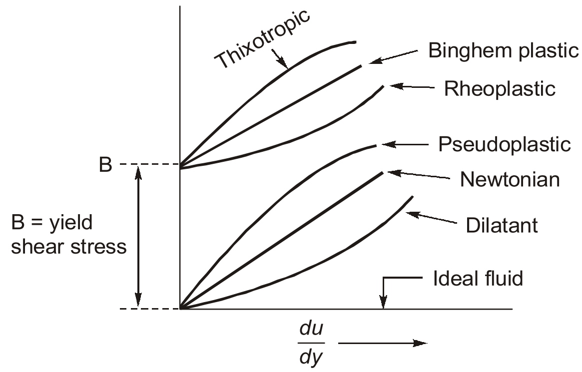
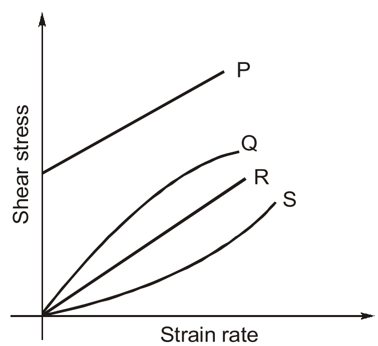


Example-1.3 The Rheological diagram depicting the relation between shear stress and strain rate for different types of fluids is shown in figure below.



The most suitable relation for flow of toothpaste being squeezed out of the tube is given by the curve:

(a) P
(b) Q
(c) R
(d) S

***Solution:* (a)**

Example-1.4 If the shear stress 'τ' and shear strain rate (du/dy) relationship of a material is plotted with τ on the y-axis and du/dy on the x-axis, the behaviour of an ideal fluid is exhibited by:

(a) a straight line passing through the origin and inclined to the x-axis
(b) the positive x-axis
(c) the positive y-axis
(d) a curved line passing through the origin

***Solution:* (b)**

Example-1.5 An oil of kinematic viscosity having 1.25×10^{-4} m^2/sec and a specific gravity of 0.80. What is its dynamic (absolute) viscosity in kg/m-sec?

(a) 0.08
(b) 0.10
(c) 0.125
(d) 1.0

Solution : (b)

Given,

$$\nu = 1.25 \times 10^{-4}\ \text{m}^2/\text{sec}$$
$$S = 0.8$$

$\therefore$
$$\rho = 0.8 \times \rho_w = 0.8 \times 1000 = 800\ \text{kg/m}^3$$

To find μ

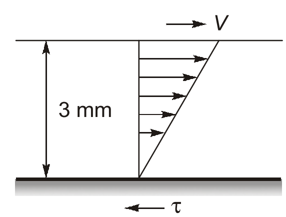
We know that,

$$\nu = \frac{\mu}{\rho}$$

$$1.25 \times 10^{-4} = \frac{\mu}{800}$$

$$\mu = 1.25 \times 10^{-4} \times 800$$
$$\mu = 0.10\ \text{kg/m-sec}$$

Example-1.6 The space between two parallel plates kept 3 mm apart is filled with an oil of dynamic viscosity 0.2 poise. The shear stresses on the fixed plate, if the upper one is moving with a velocity of 90 m/min is ________.



(a) 50 N/m²
(b) 10 N/m²
(c) 15 N/m²
(d) 30 N/m²

Solution : (b)

Given,

$$y = 3\ \text{mm} = 3 \times 10^{-3}\ \text{m}$$
$$\mu_{oil} = 0.2\ \text{poise} = 0.02\ \text{N-sec/m}^2$$
$$V = 90\ \text{m/min} = \frac{90}{60} = 1.5\ \text{m/sec}$$

By Newton's law of viscosity

$$\tau = \mu \cdot \frac{V}{y} = 0.02 \times \frac{1.5}{3 \times 10^{-3}}$$
$$\tau = 10\ \text{N/m}^2$$

Example-1.7 Classify the substances for which du/dy and τ variation are as given below:

(a)

$\frac{du}{dy}$ (rad / sec)	0	1	3	5
τ(kPa)	15	20	30	20

(b)

$\frac{du}{dy}$ (rad / sec)	0	0.5	1.1	1.8
τ(kPa)	0	2	4	6

Solution : (b)

We know that general relationship of τ and $\frac{du}{dy}$ is given as:

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

At
$$\frac{du}{dy} = 0;\ \tau = B$$

$$\frac{d\tau}{d\left(\frac{du}{dy}\right)} = A \cdot n\left(\frac{du}{dy}\right)^{n-1}$$

Thus, slope of $\left(\tau - \frac{du}{dy}\right)$ curve will increase with increase in if $\frac{du}{dy}$ $n > 1$ and will decreases if $n < 1$.

If $n = 1$, slope of $\left(\tau - \frac{du}{dy}\right)$ curve will be constant.

Thus,

(a)

$\frac{du}{dy}$	0	1	3	5
τ	15	20	30	40
$\frac{d\tau}{d\left(\frac{du}{dy}\right)}$	$\frac{5}{1}$	$\frac{10}{2}$	$\frac{10}{2}$	

$\Rightarrow$ At $\frac{du}{dy} = 0;\ \tau = 15$

$\Rightarrow$ $B = 15 \neq 0$

Slope of $\left(\tau - \frac{du}{dy}\right)$ curve is constant $\Rightarrow (n = 1)$

∴ The fluid must be ideal plastic or Bingham plastic.

(b)

$\frac{du}{dy}$	0	0.5	1.1	1.8
τ	0	2	4	6
$\frac{d\tau}{d\left(\frac{du}{dy}\right)}$	$\frac{2}{0.5}$	$\frac{2}{0.6}$	$\frac{2}{0.7}$	

$$A + \frac{du}{dy} = 0;\ \tau = 0$$

$\Rightarrow$ $B = 0$

$\Rightarrow$ Slope of $\left(\tau - \frac{du}{dy}\right)$ curve is decreasing $\Rightarrow (n < 1)$

∴ The fluid must be pseudoplastic.

1.6 Surface Tension

- The property of the liquid surface film to exert tension is called the surface tension.
- Surface tension is a measure of liquid tendency to take a spherical shape, caused by the mutual attraction of the liquid molecules.
- **Cohesion:** Force of attraction between the molecules of the same liquid.
- **Adhesion:** Force of attraction between the molecules of different liquids (or) between the liquid molecules and solid boundary containing the liquid.

- Cohesion enables a liquid to resist very small tensile stress while adhesion enables a liquid to adhere to another body.
- Surface tension is due to cohesion between particles at the surface of liquid.
- A liquid forms an interface with a second interface behaves like a membrane under tension.
- Surface tension is the force exerted by the free surface of the liquid per unit length.
  Units: Newton per metre (N/m)
  Dimension: $[MT^{-2}]$
- The surface energy per unit area of interface is called surface tension.
- It is also expressed as work done per unit surface area.

$$\sigma = \frac{\text{W (or) E}}{A}\,\text{J/m}^2$$

- As temperature increases $\rightarrow$ surface tension decreases (because cohesion decreases).
- A 'tensiometer' and 'stalagmometer' are the experimental instruments used to measure the surface tension of liquid.
- Due to surface tension, pressure changes occurs across a curved interface.

Increase of pressure of inside and outside are:

(i) Liquid droplet:

$$\Delta P = \frac{4\sigma}{d}$$

where, d = diameter of droplet

(ii) Soap bubble:

$$\Delta P = \frac{8\sigma}{d}$$

where, d = diameter of bubble

(iii) Liquid jet:

$$\Delta P = \frac{2\sigma}{d}$$

where, d = diameter of jet

NOTE: Air bubble raise in a liquid treated as air droplet, $\Delta P = 4\sigma/d$.

Example-1.8 What is the pressure within a 1 mm diameter spherical droplet of water relative to the droplet of water relative to the atmospheric pressure outside? Assume surface tension for pure water to be 0.073 N/m.

Solution:

For spherical liquid drop.

$$\Delta P = \frac{4\sigma}{d}$$

Where, ΔP = difference between the pressure inside and outside the drop

$$\Delta P = \frac{4\sigma}{d} = \frac{4\times 0.073}{1\times 10^{-3}}$$

$$\Delta P = 292\ \text{N/m}^2$$

Example-1.9 A 20 mm diameter soap bubble has an internal pressure of 27.576 N/m^2 greater than the outside atmospheric pressure, then the surface tension of soap air bubble is in (N/m) ________.

Solution:

The soap bubble has two surfaces with the air the thinner and the outer and almost the same radius since, the soap film is very thin,

$$\Delta P = \frac{8\sigma}{d}$$

$$27.576 = \frac{8\times\sigma}{20\times10^{-3}}$$

$$\sigma = 0.0689 \text{ N/m}$$

Example-1.10 A small circular jet of water of 2 mm diameter issues forms an opening. What is the pressure difference between inside and outside of the jet?

(Take surface tension of water = 0.0735 N/m)

Solution:

$$\Delta P = \frac{2\sigma}{d} = \frac{2\times0.0735}{2\times10^{-3}}$$

$$\Delta P = 73.5 \text{ N/m}^2$$

Example-1.11 Air is introduced through a nozzle into a tank of water to form a stream of bubbles. If the process requires 2.5 mm diameter air bubbles to be formed, the air pressure at the nozzle must exceed that of surrounding water, [σ_{water} = 0.0735 N/m] is (Posed) is ________.

Solution:

$$\Delta P = \frac{4\sigma}{d} = \frac{4\times0.0735}{2.5\times10^{-3}} = 117.6 \text{ N/m}^2$$

Example-1.12 The diameter of droplet is 0.075 mm. What is the intensity of the pressure (N/cm^2) developed in the droplet by surface tension of 0.000075 N/mm?

(a) 0.4
(b) 0.6
(c) 0.8
(d) 1

Solution: (a)

For air water interface $\sigma = 0.073$ N/m

For water glass interface, contact angle $\theta \simeq 0$

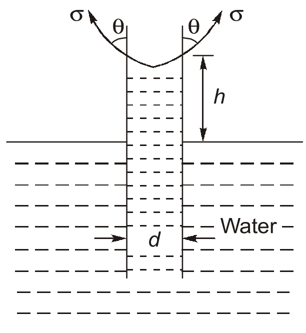
For air mercury interface, $\sigma = 0.480$ N/m

For mercury glass interface contact angle, $\theta \simeq 130°$

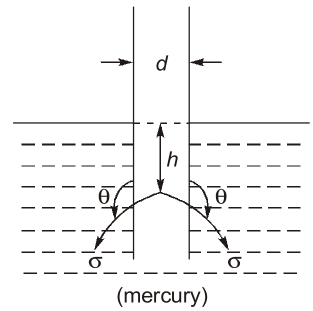
1.7 Capillary

- The phenomenon of rise or fall of a liquid surface relative to the adjacent general level of liquid in small diameter tubes. The rise of liquid surface is designated as capillary rise and lowering is called capillary depression.

- Capillarity is due to both cohesion and adhesion.



Capillary rise



Capillary depression

- **Capillary rise:** For a liquid in contact with the surface, if adhesion predominates cohesion than the liquid will wet the surface with which it is in contact and tend to rise at the point of contact.
- The free surface of the fluid will be concave upward and the contact angle (θ) will be less then 90°.
  Example: Immersion of a glass tube in water.
- **Capillary fall:** If for any liquid, in contact with a surface. Cohesion predominates the liquid will not wet the surface and the liquid surface will be depressed at the point of contact.
- The liquid surface will be concave downward and the angle of contact θ will be greater than 90°.
- Such a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid is known as capillarity.
- $$h = \frac{4\sigma\cos\theta}{\gamma \cdot d}$$

  h = Capillary height (rise/fall)
  σ = Surface tension (N/m)
  d = Diameter of tube (m)
  γ = Specific weight of the liquid (N/m^3)
  θ = Angle of contact between liquid and boundary
  θ = 0° (water and glass) = 130° (mercury and glass)
- **Assumptions in deriving the above equation:**
  (a) The meniscus of the curved liquid surface is a section of sphere.
  (b) The liquid and tube surfaces are extremely clean.

NOTE

- With increase in diameter of the tube, capillary rise decreases. For tube of diameter more than 6 mm (radius > 3 mm) the capillary rise is negligible.
- At 20°C for water.

$$h = \frac{0.30}{d} m$$

where d = diameter of the tube (in cms)

- If an annular tube, is immersed in a liquid, with outer radius r_o and inner radius r_i, then capillary rise is given by

$$h = \frac{4\sigma\cos\theta}{2(r_o - r_i)S\gamma_w} = \frac{2\sigma\cos\theta}{(r_o - r_i)S\gamma_w} = \frac{2\sigma\cos\theta}{(r_o - r_i)\cdot\gamma}$$