UPPSC-AE 2020

Uttar Pradesh Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Civil Engineering

Construction, Technology, Planning and Management

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

MADE EASY Publications

Construction, Technology, Planning & Management

Contents

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CHAPTER 4

Critical Path Method

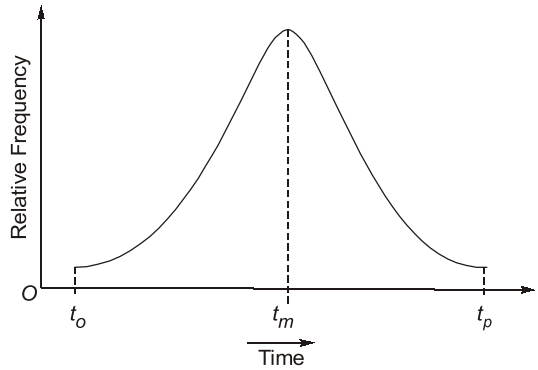
4.1 Introduction

This is based on deterministic approach in which only one time estimate is made for activity completion. Network diagram in CPM is activity oriented.

- It is activity oriented network.
- It is used for repetitive type of work and has deterministic approach.

4.2 Comparison between PERT and CPM

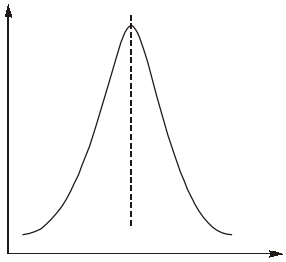
PERT



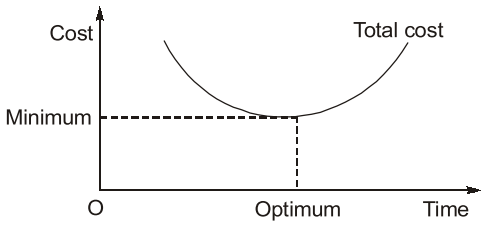
1. It is event oriented.
2. It has probabilistic approach. The probability distribution is of the type of β distribution
3. Three types of times are estimated on the basis of which an expected time t_e is derived.
4. Cost is directly proportional to time. Hence efforts are made to minimize the time so as to result in the minimum cost.
5. It is suitable for newer type of projects which have not been performed in the past and no exact assessment of time and cost are available.

   Examples: Research work, launching of space aircraft, development of missile programme etc.

CPM



1. It is an activity oriented network.
2. It has deterministic approach. Probability value approaches to one here.
3. Only one time is calculated i.e. activity duration *'t'.*
4. Time and cost are related by the following curve given. From this curve optimum time is derived which results in the minimum cost.



Time-cost Relationship

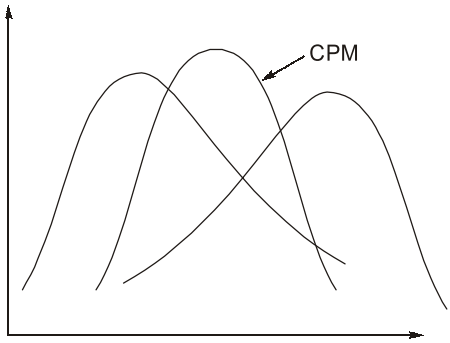
5. It is suitable for repetitive type of work where time and cost can be evaluated with fair degree of accuracy.

Examples:

- Construction work
- Maintenance work
- Civil engineering projects.

4.3 Time Estimates

In PERT analysis, each activity has 3-time estimates which follows β-distribution where difference between t_o and t_p is large due to uncertainty whereas in CPM activities have single time estimate i.e. by deterministic approach and they follow normal distribution because since there is no uncertainty. Variation of time is very small which can be neglected.



4.4 Event Time in CPM

1. **Earliest event occurrence time (T_E):** Time at which an event may occur as early as possible.
2. **Latest allowable occurance time (T_L):** Time at which event may occur as late as possible without delaying the overall project completion time.

   These are similar to PERT and are calculated in the same fashion.

4.5 Activity Time in CPM

1. **Earliest start time (EST):** It is the earliest possible time at which an activity can be started.

   For an activity $i - j$, earliest event time of event i, i.e. T_E^i is EST of activity $i - j$.

2. **Earliest finish time (EFT):** It is the earliest possible time by which an activity can be completed.

   For an activity $i - j$

   $$\text{EFT} = \text{EST} + t_{ij} = T_E^i + t_{ij}$$

   t_{ij} = Activity duration

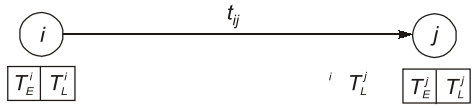


Fig. *EST and EFT*

3. **Latest start time (LST):** This is the latest possible time at which an activity can be started without delaying the overall project

$\therefore$ LST = LFT – Activity duration

$\therefore$ $\text{LST} = T_L^j - t_{ij}$

LFT = Latest finish time of activity $i - j = T_L^j$

4. **Latest finish time (LFT):** This is the latest time by which an operation or activity must be completed without delaying the project.
   For an activity $i - j$, latest allowable time of head event j, i.e. T_{Lj} is LFT of activity $i - j$.

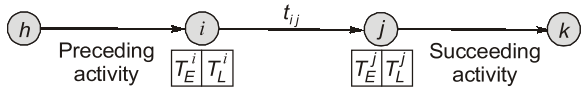
NOTE: LST of an activity is to be calculated on the basis of latest occurrence time of its head event and not on the brass of latest occurrence time of its tail event.

4.6 Float

- It is associated with activity times
- It is analogous to slack of events in PERT.
- It is the range within which start or finish time of an activity may fluctuate without affecting the project completion time.
- Floats are of following types:

1. **Total float:** The time span by which starting or finishing of an activity can be delayed without delaying the completion of the project.
   It is the maximum available time in excess of the activity completion time.
   Total float is given by F_T

$$F_T = (T_L^j - T_E^i) - t_{ij}$$



i = Tail event
j = Head event

Fig. *Total float*

   Total float of an activity affects total float of succeeding as well as preceding activities.

   Do you know? Total float of an activity constrains the finishing of preceding activity and starting of succeeding activity.

2. **Free float (F_F):** The delay which can be made without delaying succeeding activities. It affects only preceding activities. It is denoted by F_F. It is assumed that all activities start as early as possible.
   Free float is given by

$$F_F = (T_E^j - T_E^i) - t_{ij}$$

$$\Rightarrow \quad F_F = F_T - S_j$$

   where S_j is head event slack.

DO YOU KNOW?

In free float, preceding activity is not allowed to occur at its latest time and hence total float of preceding activity is affected. However the succeeding activity can start at its earliest start time and hence its total float is not affected.

3. **Independent Float (F_{ID}):** It is the minimum excess available time which exists without affecting any of succeeding or preceding activities. It is denoted by F_{ID}.
   It is the excess of minimum available time over the activity duration.

$$F_{ID} = (T_E^j - T_L^i) - t_{ij}$$

$$F_{ID} = F_F - S_i$$

   where S_i is tail event slack.

4. Interfering float (F_{INT}): It is similar to head event slack.

$$F_{INT} = S_j = F_T - F_F$$

4.6.1 Type of total Float:

Out of all the four floats, total float is most important as it is involved with overall project duration.
+ve total float → sub-critical activity
zero total float → critical activity
–ve total float → super critical activity

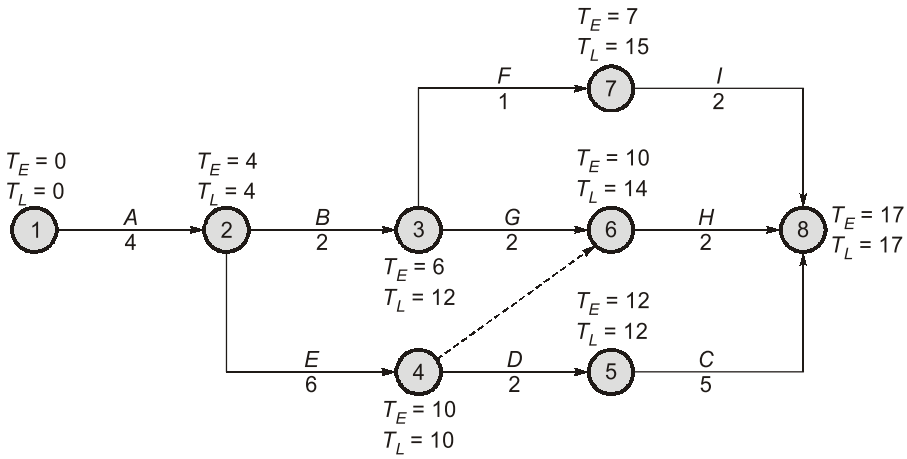
4.7 Critical Path

- Critical path is the longest path timewise in project, this time also gives project duration.
- In CPM, critical path passes through the critical activities i.e. activities having total float is equal to zero.

Example - 4.1 A network is formed by the following activities. The duration of the activities are given below: Draw the network, calculate the project duration and identify the critical path:

Activity	Preceded by	Duration (Days)
A	Starting	4
B	A	2
C(Terminal)	D	5
D	E	2
E	A	6
F	B	1
G	B	2
H (Terminal)	E, G	3
I (Terminal)	F	2

Solution:
The network diagram:



Calculation of float has been done in table below:

$$EST = T_E^i, \qquad EFT = T_E^i + t_E^{ij}$$

$$LST = T_L^j - t^{ij}$$

$$LFT = T_L^j$$

$$F_T = LST - EST = LFT - EFT$$

Activity	t_{ij}	*EST*	*EFT*	*LST*	*LFT*	F_T
A	4	0	4	0	4	0
B	2	4	6	10	12	6
C	5	12	17	12	17	0
D	2	10	12	10	12	0
E	6	4	10	4	10	0
F	1	6	7	14	15	8
G	2	6	8	12	14	6
H	3	10	13	14	17	4
I	2	7	9	15	17	8

Total project duration = 17 days

Critical path, A - E - D - C

Example - 4.2 A network of seven activities is shown in the diagram given below. The respective activity durations are shown beside the arrows. Which one of the following is the total float in AB, the total float in CE and free float in EF, respectively?

(a) 2, 3, 4 (b) 3, 3, 2
(c) 3, 2, 2 (d) 2, 3, 2

Solution: (c)

Solving the network will give earliest occurrence time (T_E) and latest finish time (T_L) as below:

Event	*A*	*B*	*C*	*D*	*E*	*F*
T_E	0	5	6	12	9	18
T_L	0	8	6	12	11	18

Total float = T_L for head event – T_E for tail event – duration of activity

Total float *AB* = 8 – 0 – 5 = 3

Total float, *CE* = 11 – 6 – 3 = 2

Free Float = T_E for head event – T_E for tail event – duration of activity

Free float ,*EF* = 18 – 9 – 7 = 2

4.8 CPM System

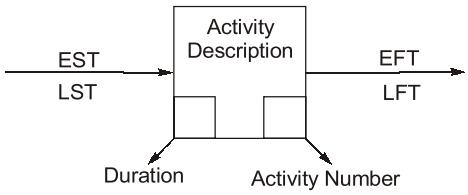
Mainly two systems are used in CPM analysis:

1. **A-O-A system** (Activity on arrow system): An activity is graphically represented by an arrow. The tail end and head end of arrow represent start and finish of an activity respectively.

   Do you know? Dummies have been used to represent the constraint and consequent interdependence. But number of dummies must be minimum for an efficient network.

2. **A-O-N System** (Activity on node system or precedence diagram). Activity is represented by a circle or a node. Events have no places. Arrows are used only to show the dependency relationship between activity nodes.

When two or more activities start parallel then an activity called DEBUT (D_O) is provided at the beginning. Like wise a finish activity (F_O) is provided at the end when more than one activities finish parallel. Activities *D* and *F* has zero duration.

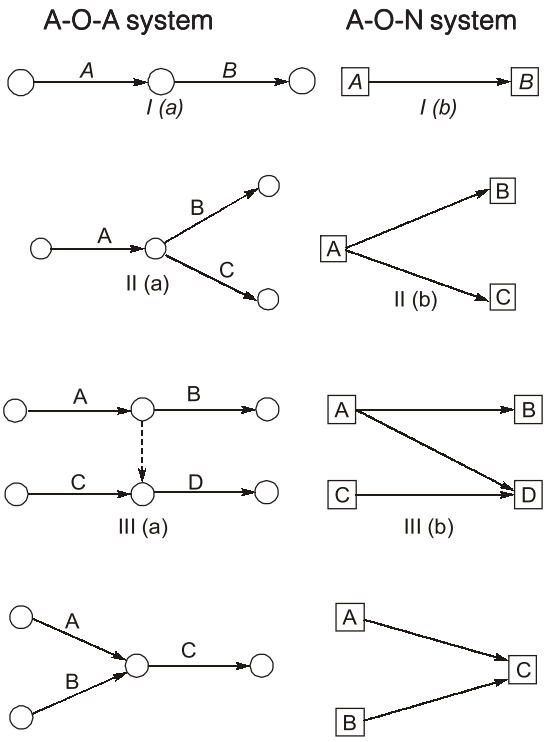


Debut Activity

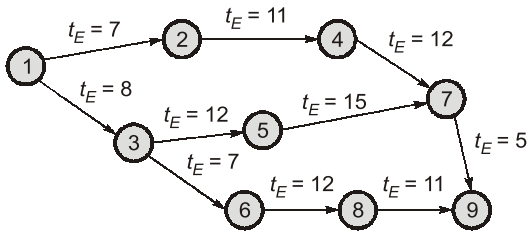
4.8.1 Advantages of A-O-N System over A-O-A System

1. A-O-N system eliminates the use of dummy activities.
2. It is more helpful for projects having more overlapping activities.
3. It is a self sufficient and self -explanatory. All activity times (EST, EFT, LST, LFT) are represented on the diagram.
4. Revision and modifications are easier.
5. Pre-operations and post-operations of activities under consideration are distinctly visible.

Examples:



Example - 4.3 For the network shown below, find the earliest expected time of the network for the completion of work



Time estimates are given in days.

Solution:

Time of completion along path 1 – 2 – 4 – 7 – 9

$$t_E = 7 + 11 + 12 + 5 = 35 \text{ days}$$

Time of completion along path 1 – 3 – 5 – 7 – 9

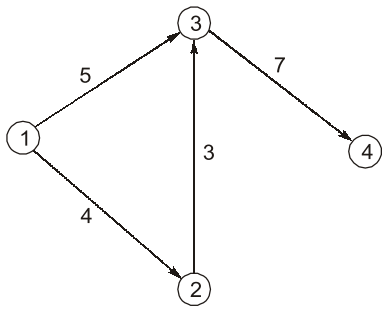
$$t_E = 8 + 12 + 15 + 5 = 40 \text{ days}$$

Time of completion along with path 1 – 3 – 6 – 8 – 9

$$t_E = 8 + 7 + 12 + 11 = 38 \text{ days}$$

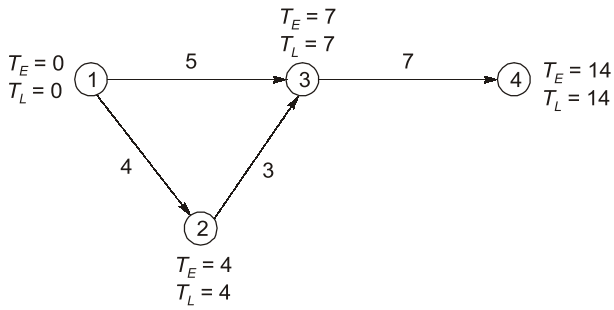
Therefore, earliest expected time of completion of project = 40 days.

Example - 4.4 Find out the project completion time and critical path for the network shown below. The activity durations are in days.



Solution:

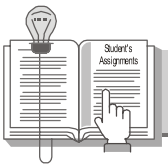
The network can be redrawn as shown below:



Activity (*i-j*)	Duration (t^{ij}, days)	Earliest Time		Latest Time		Total float (F_T)
		Start (EST)	Finish (EFT)	Start (LST)	Finish (LFT)	
1-2	4	0	4	0	4	0
1-3	5	0	5	2	7	2
2-3	3	4	7	4	7	0
3-4	7	7	14	7	14	0

$\therefore$ Project completion time = 14 days.

Critical is the one which is having zero total floats i.e., path 1 → 2 → 3 → 4 is critical.



Student's Assignment

Q.1 Consider the following statements:

1. PERT is activity-oriented and adopts deterministic approach.
2. CPM is event oriented and adopts probabilities approach.
3. PERT is event oriented and adopts probabilities approach.

Which of there statements is/are correct?

(a) 1 only (b) 1 and 2 only
(c) 2 and 3 only (d) 3 only

Q.2 Match **List-I** (Description of activity floats) with **List-II** (Names of the floats) and select the correct answer using the codes given the lists:

List-I

A. Earliest start time of successor activity minus earliest activity in question minus the duration
B. Time available for an activity performance minus the duration of the activity
C. Excess of minimum available time over the required activity duration
D. Difference between total and free float of an activity

List-II

1. Total
2. Free
3. Interfering
4. Independent float

Codes :

	A	B	C	D
(a)	1	2	3	4
(b)	1	2	4	3
(c)	2	1	3	4
(d)	2	1	4	3

Q.3 Consider the implicit details between, before and after successive steps in/within the order while doing time computations on a *CPM* network (*FP* ⇒ Forward Pass; *BP* ⇒ Backward Pass; *LET* ⇒ Late Event Time; *EET* ⇒ Early Event Time; *TF* ⇒ Total Float; *PD* ⇒ Project Duration; *AD* ⇒ Activity Duration)

The correct sequence of computation would be

(a) *EET*, *PD*, *LET*, *TF* (b) *EET*, *PD*, *LET*, *AD*
(c) *AD*, *EET*, *BP*, *PD* (d) *FP*, *EET*, *TF*, *PD*

Q.4 Consider the following pairs:

1. Difference between total float and free float : **Interfering float**
2. Sum of independent float and tail slack : **Free float**
3. Sum of independent float, tail slack and interfering float : **Total float**

Which of these pairs are correctly matched?

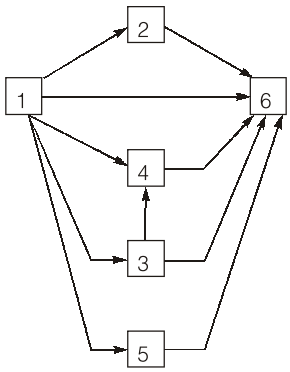
(a) 1, 2 and 3
(b) 1 and 2
(c) 2 and 3
(d) 1 and 3

Q.5 What is the duration by which the completion time of any activity can be delayed without affecting the start of any of the succeeding activities?
(a) Interfering float (b) Free float
(c) Independent float (d) Total float

Q.6 Which one of the following is associated with a critical activity in a A-O-A network?
(a) Maximum float (b) Minimum float
(c) Zero float (d) Free float

Q.7 The amount of time by which the start of the activity may be delayed without interfering with the start of any succeeding activity is called
(a) Activity float (b) Free float
(c) Total float (d) Interfering float

Q.8 A-O-N network is suggested as shown below. The number of errors/incompatibilities in this network is

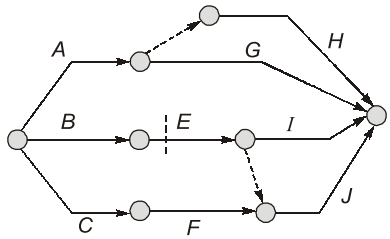


(a) 1 (b) 2
(c) 3 (d) 4

Q.9 Consider the following statements:
1. Total float can affect all activities in the chain.
2. Free float can affect only the preceding activities.
3. Independent float affects only the particular concerned activity.

Which of these statements/are correct?
(a) 1 only (b) 1 and 2 only
(c) 2 and 3 only (d) 1, 2 and 3

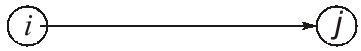
Q.10 Consider the AOA diagram as below:
What is the number of dummy links required to convert it into a most concise AON diagram?



(a) 8 (b) 7
(c) 6 (d) 5

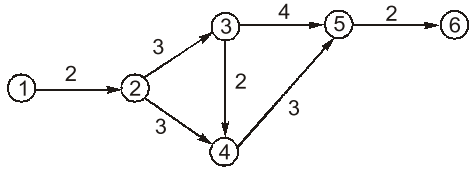
Q.11 In a CPM network the activity is non critical if
(a) EST = LST & EFT = LFT
(b) EST < LST & EFT < LFT
(c) EST > LST & EFT > LFT
(d) EST < LST & EFT > LFT

Q.12 $T_L^j - T_E^i - t_{ij}$ represent



(a) Total Float
(b) Free Float
(c) Independent Float
(d) Interfering Float

Q.13 In the network shown in figure total float for the activity 2-4 will be



(a) 3 (b) 2
(c) 1 (d) zero

Q.14 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Total float	1. $T_E^j - T_E^i - t_{ij}$
B. Independent float	2. $T_L^j - T_E^i - t_{ij}$
C. Free float	3. $T_E^j - T_L^i - t_{ij}$
D. Interfering float	4. S_j

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	2	4	1	3
(c)	2	3	1	4
(d)	2	1	3	4

Q.15 Free float is mainly used to
(a) identify the activities which can be delayed without affecting the total float of preceding activity
(b) identify the activities, which can be delayed without affecting the total float of succeeding activity
(c) identify the activities which can be delayed without affecting the total float of preceding activity
(d) establish priorities

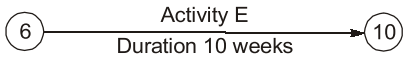
Q.16 For an activity *i-j*, the early event times at *i* and *j*, respectively are 5, 24, 9 and 29. (EST = 5, LST = 9, EFT = 24, LFT = 29. The activity duration is 6. Match **List-I** (Float) with **List-II** (Duration) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Free	1. 5
B. Total	2. 9
C. Interfering	3. 13
D. Independent	4. 18

Codes:

	A	B	C	D
(a)	4	3	1	2
(b)	3	4	1	2
(c)	3	4	2	1
(d)	4	3	2	1

Q.17 Earliest Date = 20 weeks
Latest Date = 40 weeks



$T_E = 20$ $T_E = 45$
$T_L = 25$ $T_L = 50$

Activity *E* is connecting both the events and its duration is 10 weeks. The independent float of the activity is
(a) 5 weeks (b) 10 weeks
(c) 15 weeks (d) 20 weeks

Q.18 In a project logic, four activities M, N, O and P are required to be completed before starting activity Q. If the finish time of M, N, O and P are 12 days, 14 days, 15 days and 17 days respectively, the earliest event occurrence time for the activity Q is:
(a) 12 days (b) 14 days
(c) 15 days (d) 17 days

ANSWER KEY // STUDENT'S ASSIGNMENT

1. (d)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)
6. (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (*)
11. (b)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (b)
16. (b)	**17.** (b)	**18.** (d)		

HINTS & SOLUTIONS // STUDENT'S ASSIGNMENT

1. (d)

CPM is activity oriented and adopts deterministic approach.
PERT is event oriented and it adopts probabilities approach.

2. (d)

Total float is the excess of maximum available time over the activity time.

$$F_T = \left(T_L^j - T_E^i\right) - t^{ij} = LST - EST$$
$$= LFT - EFT$$

Free float is the excess of available time over the activity time when all jobs start as early as possible.

$$F_F = \left(T_E^j - T_E^i\right) - t^{ij} = F_T - S_j$$

Independent float is the excess of minimum available time over the activity time.

$$F_{ID} = \left(T_E^j - T_L^i\right) - t^{ij} = F_F - S_i$$

Interfering float is the difference between total float and free float

$$F_{IN} = F_T - F_F = S_j$$

3. (a)

The sequence is
AD – FP – EET – PD – BP – LET – TF

4. (a)

Time scale for an activity:

Independent Float (*IF*) is the excess of minimum available time over activity time.

Free Float (*FF*) is the excess of available time over the activity time when all jobs start as early as possible.

So $FF = IF$ + Tail slack.

Interfering Float = $TF - FF$ = Head slack

$\therefore$ TF = Int. F + FF = Int. F + IF + Tail slack

Or $TF = IF$ + Head slack + Tail slack

5. (b)

Free float is the amount of time that an activity can be delayed without delaying the Early Start of its successor activity. Free float can be calculated by subtracting the Early Finish date of the activity from the Early Start date of next activity (ES of next Activity – EF of current Activity).

6. (c)

Critical path activities are the project tasks that must start and finish on time to ensure that the project ends on schedule. If the Early Start date and Late Start date for an activity are the same, the activity is said to have zero float. Activities that have zero float start on time and prevent the schedule from slipping.

7. (b)

Time by which the start of the activity can be delayed without affecting start of succeeding activity is called as free float.

8. (b)

In A-O-N network, there is no place for dummy activity and events.

1-6 and 3-6 is logically redundant in the network.

9. (c)

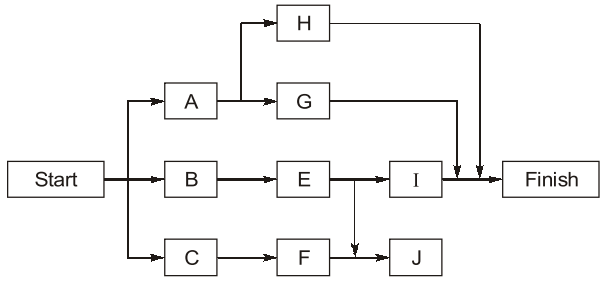
Total Float is the amount of time that an activity can be delayed from its early start date without delaying the project finish date. It affects only preceding and succeeding activities.

10. (*)

In **Activity-On-Node (A-O-N)** system called as precedence diagram, the nodes represent the activities and the arrow, their interdependence or precedence relationships.

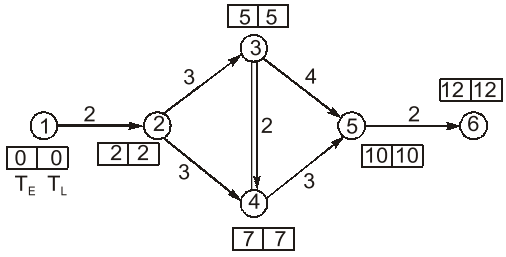
Salient Features:

- A-O-N network completely eliminates the use of dummies.
- It can show activities which should be allowed to overlap each other or must be separated by a time delay.
- This system is self-sufficient as it contains all the activity time (EST, LST, EFT, LFT) on the diagram itself. This facilitates efficient scheduling and controlling.
- Revisions and modifications can be carried out easily and can also be easily understood by non-specialists.
- Pre-operations and post-operations of the activity under consideration are distinctly visible.



A-O-N Diagram

13. (b)



Since $F_T = T_L^j - T_E^i - t_{ij}$

F_T for activity 2 – 4

$$F_T = 7 - 2 - 3 = 2$$

15. (b)

The free float for activity i - j is the difference between its earliest finish time and the earliest finish time for its successor activity.

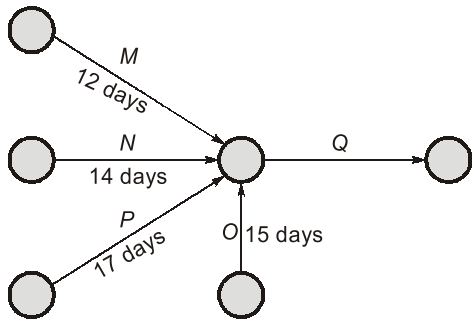
17. (b)

Independent float

$$= \left[T_E^j - T_L^i\right] - t_{ij} = [45 - 25] - 10$$

$$= 20 - 10 = 10 \text{ weeks}$$

18. (d)



We know that, $T_E^j = \left(T_E^i + t_{ij}\right)_{max}$

$\therefore$ Earliest event occurrence time of activity, Q = 17 days.

■■■■