CLASS TEST

Sl.:03SK_ECEE_BCDFGHIJK_231023


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# ENGINEERING MATHEMATHICS

## EC & EE

**Date of Test : 23/10/2023**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (a) | 7. | (a) | 13. | (c) | 19. | (b) | 25. | (b) |
| 2. | (a) | 8. | (c) | 14. | (b) | 20. | (b) | 26. | (d) |
| 3. | (a) | 9. | (c) | 15. | (b) | 21. | (d) | 27. | (b) |
| 4. | (d) | 10. | (c) | 16. | (d) | 22. | (c) | 28. | (c) |
| 5. | (d) | 11. | (a) | 17. | (b) | 23. | (b) | 29. | (d) |
| 6. | (b) | 12. | (c) | 18. | (a) | 24. | (b) | 30. | (d) |

## DETAILED EXPLANATIONS

**1. (a)**

For given system of equations to have a non trivial solution,

$$\begin{vmatrix} 1 & K & 3 \\ K & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$1(8-6) - K(4K-4) + 3(3K-4) = 0$$

$\Rightarrow$ $$4K^2 - 13K + 10 = 0$$

$\therefore$ $$K = 2$$

or $$K = 1.25$$

Option (a) is correct.

**2. (a)**

$$\frac{dy}{dx}\sin x = y \log y$$

$$\int \frac{dy}{y\log y} = \int \frac{dx}{\sin x}$$

If $$\log y = t$$

$$\frac{1}{y}dy = dt$$

$\Rightarrow$ $$\int \frac{dt}{t} = \int \frac{dx}{\sin x}$$

$\Rightarrow$ $$\log t = \log \tan\frac{x}{2} + \log C$$

$$\log t = \log\left(C\tan\frac{x}{2}\right)$$

$\Rightarrow$ $$t = C\tan\frac{x}{2}$$

or $$\log y = C\tan\frac{x}{2}$$

$$y = e^{C\tan\frac{x}{2}}$$

$\because$ $$y\left(\frac{\pi}{2}\right) = e$$

$$e = e^{C\tan\pi/4}$$

$\Rightarrow$ $$C = 1$$

Solution : $$y = e^{\tan x/2}$$

Option (a) is correct.

**3. (a)**

$$I = \int_0^{2\pi}\left(\frac{4}{16+\sin^2\theta}\right)d\theta = 4\times\int_0^{\pi/2}\left(\frac{4}{16+\sin^2\theta}\right)d\theta$$

$$= 16\int_0^{\pi/2}\frac{\sec^2\theta}{16\sec^2\theta+\tan^2\theta}d\theta$$

$$= 16\int_0^{\pi/2}\frac{\sec^2\theta\,d\theta}{16+17\tan^2\theta} = \frac{16}{17}\int_0^{\pi/2}\frac{\sec^2\theta d\theta}{\frac{16}{17}+\tan^2\theta}$$

Limits:

Let, $\tan\theta = t$

$\sec^2\theta\, d\theta = dt$

$\theta = 0, t = 0$

$\theta = \frac{\pi}{2};$

$t \to \infty$

$$= \frac{16}{17}\int_0^{\infty}\frac{dt}{t^2+\left(\sqrt{\frac{16}{17}}\right)^2}$$

$$= \frac{16}{17}\times\frac{\sqrt{17}}{4}\left[\tan^{-1}\left(\frac{t\sqrt{17}}{4}\right)\right]_0^{\infty} = \frac{4}{\sqrt{17}}\left[\frac{\pi}{2}-0\right]$$

$$= \frac{2\pi}{\sqrt{17}}$$

**4. (d)**

For eigen value $\lambda = -2$

$$\begin{bmatrix} 3-(-2) & -2 & 2 \\ 0 & -2-(-2) & 1 \\ 0 & 0 & 1-(-2) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or, $$\begin{bmatrix} 5 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $5x_1 - 2x_2 + 2x_3 = 0$

Only (d) satisfies this equation.

**5. (d)**

$Z^2 + 4 = 0$

$\Rightarrow$ $Z^2 = -4$

$\Rightarrow$ $Z = \pm 2i$

**6. (b)**

$$|\vec{a}+\vec{b}|^2 - |\vec{a}-\vec{b}|^2 = 4\vec{a}\cdot\vec{b}$$

$\therefore$ $$4\vec{a}\cdot\vec{b} = 10^2 - 8^2 = 36$$

$\Rightarrow$ $$\vec{a}\cdot\vec{b} = 9$$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$

$$8^2 = 5^2 + |\vec{b}|^2 - 2(9)$$

$$8^2 = 7 + |\vec{b}|^2$$

$$|\vec{b}| = \sqrt{57}$$

Option (b) is correct.

**7. (a)**

$$f_1(z) = z^3$$

if $$z = x + iy$$

$$z^2 = (x+iy)^2 = x^2 - y^2 + 2ixy$$

$$z^3 = (x^2 - y^2 + 2ixy)(x+iy)$$

$$= (x^3 - 3xy^2) + (3x^2y - y^3)i$$

$$u = x^3 - 3xy^2$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial y} = -6\,xy$$

$$v = 3x^2y - y^3$$

$$\frac{\partial v}{\partial y} = 3x^2 - 3y^2$$

$$\frac{\partial v}{\partial x} = 6\,xy$$

$\because$ $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

and $$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$\therefore$ $f_1(z) = z^3$ is analytic for all $z$-values

Now, $$f_2(z) = \log z$$

$$= \log(x+iy)$$

$$= \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{y}{x}$$

$$u = \frac{1}{2}\log(x^2+y^2)$$

$$v = \tan^{-1}\frac{y}{x}$$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{y}{x^2+y^2} = \frac{-\partial v}{\partial x}$$

$\because$ $C$–$R$ equation are satisfied but the partial derivatives are not continuous at (0, 0)

$\Rightarrow$ $f_2(z)$ is analytic everywhere except $z = 0$

$\Rightarrow$ Option (a) is correct.

**8. (c)**

Comparing the given equation with general form of second order partial differential equation

$$\frac{A\partial^2 P}{\partial x^2} + \frac{B\partial^2 P}{\partial y \partial x} + \frac{C\partial^2 P}{\partial y^2} + \frac{D\partial P}{\partial x} + \frac{E\partial P}{\partial y} + FP = g(x, y)$$

$$A = 1$$

$$B = 3$$

$$C = 1$$

$\Rightarrow$ $$B^2 - 4AC = 5 > 0$$

$\therefore$ PDE is hyperbolic.

**9. (c)**

$$\text{Probability of success, } p = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Probability of failure, } q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

$$n = \text{number of throws} = 8$$

$\therefore$ $$\text{Mean, } np = 8\left(\frac{1}{3}\right) = \frac{8}{3}$$

$$\text{Variance} = npq = \frac{8}{3} \times \frac{2}{3} = \frac{16}{9}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

Hence option (c) is correct.

**10. (c)**

$$\frac{\sin z}{z^8} = \frac{z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \frac{z^9}{9!} + \ldots}{z^8}$$

$$= \frac{1}{z^7} - \frac{1}{3!z^5} + \frac{1}{5!z^3} - \frac{1}{7!z} + \frac{1}{9!}\ldots$$

$$\underset{z \to 0}{\text{Res}} \frac{\sin z}{z^8} = \text{Coefficient of } \frac{1}{z} = \frac{-1}{7!}$$

**11. (a)**

In Poisson's distribution,

$$\text{variance} = \text{mean}$$

$$\therefore \quad \text{variance} = 5$$

**12. (c)**

$\bar{E}$ be the event that room is not lighted,

Then $$P(\bar{E}) = \frac{{}^4C_3}{{}^{10}C_3} = \frac{4!\,7!\,3!}{3!\,10!}$$

$$= \frac{4\times3\times2\times7!}{10\times9\times8\times7!} = \frac{1}{30}$$

$\therefore$ $$\text{Required probability} = P(E) = 1 - P(\bar{E})$$

$$= 1 - \frac{1}{30} = \frac{29}{30}$$

Hence option (c) is correct.

**13. (c)**

If the function satisfies MVT,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

where, $a < c < b$

$\Rightarrow$ $$f(a) = 0$$

$$f(b) = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{3}{8}$$

$$f(x) = x(x^2 - 3x + 2)$$

$$f(x) = x^3 - 3x^2 + 2x$$

$$f'(x) = 3x^2 - 6x + 2$$

$$f'(c) = 3c^2 - 6c + 2$$

$\therefore$ $$3c^2 - 6c + 2 = \frac{(3/8)-0}{(1/2)-0}$$

$\Rightarrow$ $$12c^2 - 24c + 5 = 0$$

On solving $$c = 1.764;\ 0.236$$

$\because$ $c = 0.236$ lies between 0 and 1/2.

Option (c) is correct.

**14. (b)**

$$\frac{2x}{y^3}dx + \frac{y^2-3x^2}{y^4}dy = 0$$

or $$2xydx + (y^2 - 3x^2)dy = 0$$

$$M = 2xy$$

and $$N = y^2 - 3x^2$$

$$\frac{\partial N}{\partial x} = -6x$$

and $$\frac{\partial M}{\partial y} = 2x$$

$\because \quad \dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

$\therefore$ The given equation is not exact

$\because \quad \dfrac{\partial y}{\partial x} = \dfrac{-2xy}{y^2 - 3x^2}$

$\therefore$ Gives equation is homogeneous equation.

$\Rightarrow$ Option (b) is correct.

**15.** **(b)**

If $z$ were the function of $x$ alone,

$$z = A\sin x + B\cos x$$

But, $z = e^y$ when $x = 0$, $A$ and $B$ can be arbitrary functions of $y$.

$\therefore$ Solution, $z = f(y)\sin x + \phi(y)\cos x$ .. (i)

Differentiating partially w.r.t. $x$,

$$\frac{\partial z}{\partial x} = f(y)\cos x - \phi(y)\sin x \quad \text{.. (ii)}$$

$$\left|\frac{\partial z}{\partial x}\right|_{x=0} = f(y)\cos 0 - \phi(y)\sin 0 = 1$$

Putting $x = 0$ in eq. (i).

If $x = 0$, $z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$

On comparison, $\phi(y) = e^y$ and $f(y) = 1$.

$\therefore$ Required solution in, $z = \sin x + e^y\cos x$

$\Rightarrow$ Option (b) is correct.

**16.** **(d)**

$$n \text{ (sample space)} = \frac{9!}{4!\,3!\,2!}$$

$$n \text{ (event)} = 3!$$

$$\therefore \quad P(E) = \frac{(3!)(4!)(3!)(2!)}{9!} = \frac{1}{210}$$

$\Rightarrow$ Option (d) is correct.

**17.** **(b)**

Given that, $u = x\log(xy)$ ...(i)

and $x^3 + y^3 + 3xy = 1$ ...(ii)

Differentiating (i), with respect to $x$ we get,

$$\frac{du}{dx} = x \times \frac{1}{xy} \times \left(x\frac{dy}{dx} + y\right) + \log(xy) \times (1) \quad \text{...(iii)}$$

Differentiating equation (ii), with respect to $x$,

$$3x^2 + 3y^2\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x^2 + y}{y^2 + x}\right)$$

Substituting in equation (iii),

$$\frac{du}{dx} = (1+\log xy) - \frac{x}{y}\left(\frac{x^2+y}{y^2+x}\right)$$

⇒ Option (b) is correct.

**18. (a)**

$$\vec{A} = xy\hat{i} + x^2\hat{j}$$

$$d\vec{l} = dx\hat{i} + dy\hat{j}$$

$$\oint_c \vec{A}\cdot d\vec{l} = \oint_c (xy\hat{i} + x^2\hat{j})\cdot\left(dx\hat{i} + dy\hat{j}\right)$$

$$= \oint_c xy\,dx + x^2dy$$

$$= \int_{1/\sqrt{3}}^{2/\sqrt{3}} x\,dx + \int_{2/\sqrt{3}}^{1/\sqrt{3}} 3x\,dx + \int_1^3 \frac{4}{3}dy + \int_3^1 \frac{1}{3}dy$$

$$= \frac{1}{2}\left[\frac{4}{3}-\frac{1}{3}\right] + \frac{3}{2}\left[\frac{1}{3}-\frac{4}{3}\right] + \frac{4}{3}[3-1] + \frac{1}{3}[1-3] = 1$$

**19. (b)**

The characteristic equation of the matrix $A$ is

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{bmatrix} = 0$$

or, $(2-\lambda)(1-\lambda)(2-\lambda) - 1(0) + 1(\lambda - 1) = 0$

or, $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

According to Cayley - Hamilton theorem, we have

$$A^3 - 5A^2 + 7A - 3I = 0$$

Now, $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$

$$= A^2 + A + I$$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

**20. (b)**

For a matrix containing complex numbers, eigen values are real if and only if

$$A = A^\theta = \left(\bar{A}\right)^T$$

$$A = \begin{bmatrix} 10 & 2+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

$$A^\theta = (\bar{A})^T = \begin{bmatrix} 10 & \bar{x} & 4 \\ 2-j & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

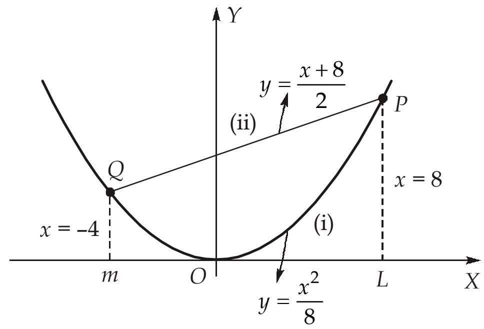
By comparing these, $x = 2 - j$

**21. (d)**

Given parabola is $x^2 = 8y$ ...(i)

and the straight line is

$x - 2y + 8 = 0$ ...(ii)




Substituting the value of $y$ from (ii) in equation (i), we get

$$x^2 = 4(x + 8)$$

or $x^2 - 4x - 32 = 0$

∴ $x = 8$ and $-4$

Thus (i) and (ii) intersect at $P$ and $Q$ where $x = 8$ and $x = -4$.

$$\text{Required area } POQ = \int_{-4}^{8} \frac{x+8}{2}dx - \int_{-4}^{8} \frac{x^2}{8}dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2} + 8x\right|_{-4}^{8} - \frac{1}{8}\left|\frac{x^3}{3}\right|_{-4}^{8} = 36$$

**22. (c)**

We have, $f(x) = 2x^3 - 15x^2 + 36x + 1$

$$f'(x) = 6x^2 - 30x + 36$$
$$= 6(x - 3)(x - 2)$$

Note that $f'(x) = 0$, gives $x = 2$ and $x = 3$

$$f''(x) = 12x - 30$$
$$f''(2) = -6 < 0 \text{ i.e maxima}$$
$$f''(3) = 6 > 0 \text{ i.e. manima}$$

We shall now evalute the value of $f$ at these points and the end points of the interval [1, 5] i.e. at $x = 1$, $x = 2$, $x = 3$ and $x = 5$, so

$$f(1) = 2(1^3) - 15(1^2) + 36(1) + 1 = 24$$
$$f(2) = 2(2^3) - 15(2^2) + 36(2) + 1 = 29$$

$$f(3) = 2(3^3) - 15(3^2) + 36(3) + 1 = 28$$
$$f(5) = 2(5^3) - 15(5^2) + 36(5) + 1 = 56$$

Thus, we conclude that absolute minimum value of $f$ in the interval [1, 5] is 24, which occurs at $x = 1$.

**23. (b)**

The augmented matrix for the system of equation is

$$[A \mid B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right] \qquad [R_3 \to R_3 - R_1]$$

$$[A \mid B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

If $\lambda = 5$

and $\mu \neq 9$

then, $\text{Rank } [A \mid B] = 3$ and rank $[A] = 2$

$\because$ $\text{Rank } [A] < \text{Rank } [A \mid B]$

$\therefore$ Given system of equation has no solution for,

$\lambda = 5$

and $\mu \neq 9$

**24. (b)**

$$\arg(Z_1) = \theta_1 = \tan^{-1}\left(\frac{5\sqrt{3}}{5}\right)$$

$\Rightarrow$ $\theta_1 = 60°$

$$\arg(Z_2) = \theta_2 = \tan^{-1}\left(\frac{2\sqrt{3}}{6}\right)$$

$\Rightarrow$ $\theta_2 = 30°$

$$\arg\left(\frac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2)$$
$$= 60° - 30° = 30°$$

Hence, option (b) is correct.

**25. (b)**

$$P(\text{none dies}) = (1-p)(1-p) \ldots n \text{ times} = (1-p)^n$$
$$P(\text{at least one dies}) = 1 - (1-p)^n$$
$$P(A_{25} \text{ dies}) = \frac{1}{n}\{1-(1-p)^n\}$$

$\Rightarrow$ Option (b) is correct.

**26. (d)**

$$A = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2 - R_3$ we get

$$A = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\text{Determinant of } A = 0\begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix} - (-2c)\begin{vmatrix} b & b \\ c & a+b \end{vmatrix} + (-2b)\begin{vmatrix} b & c+a \\ c & c \end{vmatrix}$$

$$= 2c\,(ab + b^2 - bc) - 2b(bc - c^2 - ac)$$

$$= 2\,abc + 2cb^2 - 2bc^2 - 2b^2c + 2bc^2 + 2\,abc$$

$$= 4\,abc$$

**27. (b)**

It is given that $A$ and $B$ are symmetric matrices

Therefore $\quad A' = A$ and $B' = B \quad$ ...(i)

Now, $\quad (AB - BA)' = (AB)' - (BA)'$

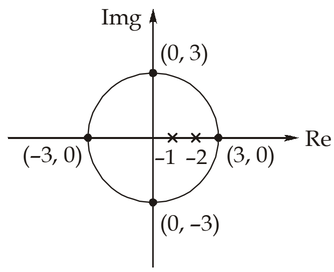
$= B'A' - A'B' \quad$ ...(ii)

Putting the value of equation (i),

$$(AB - BA)' = BA - AB = -(AB - BA)$$

Thus $(AB - BA)$ is a skew-symmetric matrix.

**28. (c)**




Given,

$$f(z) = \frac{3z+4}{(z+1)(z+2)}$$

$$= \frac{1}{z+1} + \frac{2}{z+2}$$

$$\oint_c \frac{3z+4}{(z+1)(z+2)}dz = \oint_c \frac{1}{(z+1)}dz + \oint_c \frac{2}{(z+2)}dz$$

$$= 2\pi i + 2\pi i\,(2) = 6\pi i$$

**29. (d)**

Taylor series expansion of a function $f(x)$ about $x = 0$ is given by

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0)$$

Here, $\quad f(x) = \dfrac{x}{1+x}; \qquad f(0) = 0$

$$f'(x) = \frac{(1+x)-x}{(1+x)^2} = \frac{1}{(1+x)^2}\ ;\ f'(0) = 1$$

$$f''(x) = \frac{-2}{(1+x)^3}\ ;\ f''(0) = -2$$

$$f'''(x) = \frac{6}{(1+x)^4}\ ;\ f'''(0) = 6$$

Therefore,

$$f(x) = 0 + \frac{x}{1!}\times 1 + \frac{x^2}{2}\times(-2) + \frac{x^3}{6}\times 6$$

$$f(x) = x - x^2 + x^3 + \ldots.$$

**30. (d)**

$$\int \frac{dx}{e^x - 1} = \int \frac{dx}{e^x\left(1 - \frac{1}{e^x}\right)}$$

Let

$$u = 1 - \frac{1}{e^x}$$

$$du = e^{-x}\, dx$$

$$\int \frac{dx}{e^x\left(1 - \frac{1}{e^x}\right)} = \int \frac{du}{u} = \ln u + C$$

$$= \ln\left(1 - \frac{1}{e^x}\right) + C = \ln(1 - e^{-x}) + C$$

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