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Strength of Materials (Part-1)

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CLASS TEST 2019-2020

CIVIL ENGINEERING

Date of Test : 24/08/2019

ANSWER KEY ➤ Strength of Materials (Part-1)

1.	(c)	7.	(c)	13.	(c)	19.	(d)	25.	(b)
2.	(a)	8.	(d)	14.	(a)	20.	(b)	26.	(a)
3.	(c)	9.	(a)	15.	(b)	21.	(a)	27.	(c)
4.	(d)	10.	(c)	16.	(b)	22.	(a)	28.	(a)
5.	(c)	11.	(b)	17.	(b)	23.	(b)	29.	(c)
6.	(a)	12.	(c)	18.	(a)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (c)

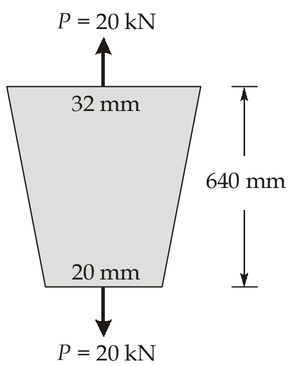
$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{14-(-10)}{2}\right)^2 + 5^2} = \sqrt{12^2 + 5^2}$$

$$= 13 \text{ N/mm}^2$$

2. (a)



Extension of tapered bar which is tapering uniformly is given by

$$\Delta = \frac{4PL}{\pi E D_1 D_2} = \frac{4 \times 20 \times 10^3 \times 640}{\pi E \times 32 \times 20}$$

$$= \frac{80000}{\pi E} \text{ mm}$$

3. (c)
From the given stress tensor,

$$\in_{xy} = 0.002$$

Shear strain, $\phi_{xy} = 2\in_{xy} = 2 \times 0.002 = 0.004$

Shear stress, $\tau_{xy} = G \times \phi_{xy}$

$$= 90 \times 10^3 \times 0.004$$

$$= 360 \text{ MPa}$$

4. (d)

$\because$ $\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$

$\Rightarrow$ $200 + \sigma_2 = 50 + 100$

$\Rightarrow$ $\sigma_2 = -50$ MPa (compressive)

Also, we know that

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad 200 = \frac{50+100}{2} + \sqrt{\left(\frac{50-100}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad (125)^2 = (25)^2 + \tau_{xy}^2$$

$$\Rightarrow \quad \tau_{xy} = 50\sqrt{6} \text{ MPa}$$

5. (c)

As per distortion energy or maximum shear strain energy theory,

$$\sigma_y^2 = \frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

Here, $\sigma_1 = 2\sigma$, $\sigma_2 = \sigma$ and $\sigma_3 = 0$

$$\therefore \quad 2\sigma_y^2 = (2\sigma-\sigma)^2 + (\sigma-0)^2 + (0-2\sigma)^2$$

$$= \sigma^2 + \sigma^2 + 4\sigma^2$$

$$\Rightarrow \quad \sigma = \frac{\sigma_y}{\sqrt{3}}$$

6. (a)

To have balanced moment about the shear centre in this section, shear centre must lie towards left of point B i.e. at point A.

7. (c)

If yielding commences in tension, then as per maximum normal strain theory

$$\Rightarrow \quad \sigma_1 - (-\mu\sigma_2) = \sigma_y$$

$$\Rightarrow \quad 200 + 0.25 \times \sigma_2 = 250$$

$$\Rightarrow \quad \sigma_2 = 200 \text{ N/mm}^2$$

If yielding commences in compression,

$$\Rightarrow \quad -\sigma_2 - \mu\sigma_1 = \sigma_y$$

$$\Rightarrow \quad \sigma_2 = 250 + 0.25 \times 200 = 300 \text{ N/mm}^2 \text{ [Compressive]}$$

Hence, the yielding will commence at $\sigma_2 = 200$ N/mm^2

8. (d)

Deflection of simply supported beam subjected to load P at centre is given by,

$$\delta_1 = \frac{Pl^3}{48EI} = \frac{Pl^3}{48 \times E \times \frac{bd^3}{12}}$$

$$= \frac{Pl^3}{4Ebd^3}$$

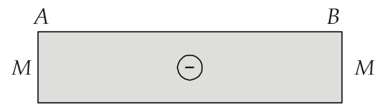
Now,

$$\delta_2 = \frac{(3P)\,l^3}{4E \times b \times \left(\frac{d}{2}\right)^3} = 24\left(\frac{Pl^3}{4Ebd^3}\right)$$

$$\therefore \quad \frac{\delta_1}{\delta_2} = \frac{1}{24}$$

$$\Rightarrow \quad \delta_2 = 24\delta_1$$

9. (a)



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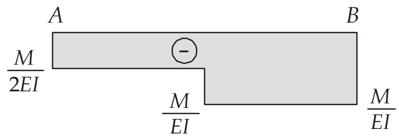


$\frac{M}{EI}$ diagram

$$\text{Change in slope from } A \text{ to } B = \text{Area of } \frac{M}{EI} \text{ diagram between } A \text{ and } B$$

$$\theta_B - \theta_A = -\left(\frac{M}{2EI}\times\frac{L}{2}\right)-\left(\frac{M}{EI}\times\frac{L}{2}\right)$$

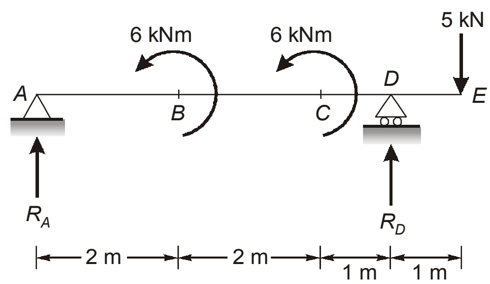
$$\theta_B = \frac{-3\,ML}{4\,EI}$$

10. (c)

$$e = \frac{b^2h^2t}{4I}$$

11. (b)

Horizontal load at J produces a couple of 6 kNm (anticlockwise) and a thrust of 6 kN at A (→), load of 6 kN at H produces 6 kNm couple (anticlockwise) and a thrust of 6 kN at A (←). Therefore, net thrust at A becomes zero.



For support reactions, take moments about A,

$$\Sigma M_A = 0$$

$$\Rightarrow \quad 6 + 6 + 5R_D - 5\times 6 = 0$$

$$\therefore \quad R_D = 3.6 \text{ kN}$$

$$\Rightarrow \quad R_A = 5 - 3.6 = 1.4 \text{ kN}$$

Bending moment diagram:

$$M_A = 0 \text{ kNm}$$

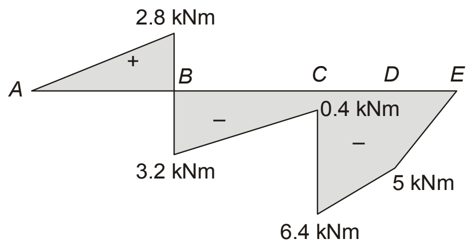
$$M_B = R_A \times 2 = 1.4 \times 2 = 2.8 \text{ kNm}$$

$$M_B' = 2.8 - 6 = -3.2 \text{ kNm}$$

$$M_C = 1.4 \times 4 - 6 = -0.4 \text{ kNm}$$

$$M_C' = -0.4 - 6 = -6.4 \text{ kNm}$$

$M_D = 1.4 \times 5 - 6 - 6 = -5$ kNm
$M_E = 0$ kNm



12. (c)

For no change in length

$$\varepsilon_L = 0$$

i.e. $$\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} = 0$$

$$\sigma_y = \frac{\sigma_x}{\mu} = \frac{60}{0.3} = 200\,\text{MPa}$$

So, option (c) is correct.

13. (c)

From the given Mohr's circle:

Maximum principal strain, $\in_1 = +180\,\mu$

Minimum principal strain, $\in_2 = -80\,\mu$

Radius of Mohr's circle of strain is engineering's stress, shear stress is twice of engineering stress.

Maximum shear strain = 2 × Radius of Mohr's circle

$$= 2 \times \frac{180-(-80)}{2} = 260\mu$$

Normal strain on the plain of maximum shear strain

= Center of Mohr circle

$$= \frac{180-80}{2} = 50\,\mu$$

14. (a)

As we know the relation,

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \qquad \sigma = \frac{Ey}{R} = \frac{220 \times 10^9 \times 0.025}{25 \times 10^6} = 220 \text{ MPa}$$

Strain energy stored per meter length,

$$U = \frac{\sigma^2}{6E} \times \text{Volume}$$

$$U = \frac{\left(220 \times 10^6\right)^2}{6 \times 220 \times 10^9} \times 3.2 \times 0.5 \times 1 \times 10^{-6}$$

$U = 58.67 \times 10^{-3}$ Nm or 58.67 Nmm

15. (b)

$$\sigma_1 = \frac{f_1+f_2}{2} + \sqrt{\left(\frac{f_1-f_2}{2}\right)^2 + q^2}$$

$$\sigma_2 = \frac{f_1+f_2}{2} - \sqrt{\left(\frac{f_1-f_2}{2}\right)^2 + q^2}$$

Principal stresses are of opposite nature

$\therefore$ $$\sigma_1 \,.\, \sigma_2 < 0$$

$\Rightarrow$ $$\left[\frac{f_1+f_2}{2} + \sqrt{\left(\frac{f_1-f_2}{2}\right)^2 + q^2}\right]\left[\frac{f_1+f_2}{2} - \sqrt{\left(\frac{f_1-f_2}{2}\right)^2 + q^2}\right] < 0$$

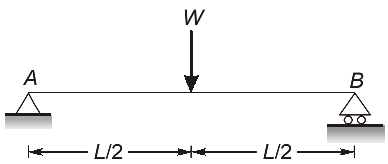
$\Rightarrow$ $$\left(\frac{f_1+f_2}{2}\right)^2 - \left[\left(\frac{f_1-f_2}{2}\right)^2 + q^2\right] < 0$$

$\Rightarrow$ $$\left(\frac{f_1+f_2}{2}\right)^2 - \left(\frac{f_1-f_2}{2}\right)^2 - q^2 < 0$$

$\Rightarrow$ $$\frac{4f_1f_2}{4} - q^2 < 0$$

$\therefore$ $$f_1\, f_2 < q^2$$

16. (b)



The strain energy due to bending in beam is given as

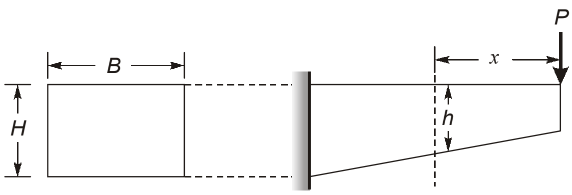
$$E = 2\times\int_0^{L/2} \frac{M_x^2 dx}{2EI}$$

where, $M_x = \frac{W}{2}x$

$\Rightarrow$ $$E = \frac{1}{EI}\int_0^{L/2}\left(\frac{W}{2}x\right)^2 dx = \frac{W^2}{4EI}\int_0^{L/2} x^2 dx$$

$$= \frac{W^2}{4EI}\left[\frac{x^3}{3}\right]_0^{L/2} = \frac{W^2L^3}{96EI}$$

17. (b)



$$\text{Stress at support, } \sigma_1 = \frac{M}{Z} = \frac{P \times L}{\left(\frac{B \times H^3}{12}\right)} \times \frac{H}{2} = \frac{6PL}{BH^2}$$

$$\text{Stress at distance } x,\ \sigma_2 = \frac{M}{Z} = \frac{6P \cdot x}{B \cdot h^2}$$

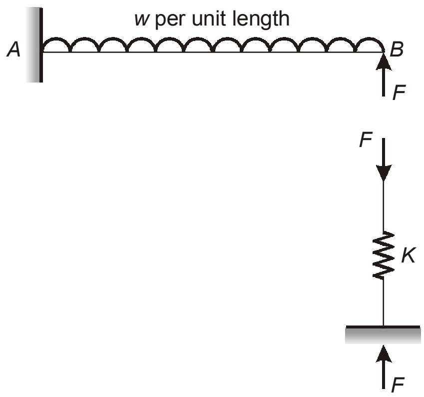
Equating,

$$\sigma_1 = \sigma_2$$

$$\frac{6PL}{BH^2} = \frac{6P \cdot x}{B \cdot h^2}$$

$$h = \sqrt{\frac{x}{L}} \cdot H$$

18. (a)



Let F be the force between the beam and the spring.

$$\text{Deflection of spring, } \delta = \frac{F}{K}$$

Upward deflection of beam due to F,

$$\delta_1 = \frac{FL^3}{3EI}$$

Downward deflection of beam at B due to w,

$$\delta_2 = \frac{wL^4}{8EI}$$

Now,

$$\delta_2 - \delta_1 = \delta$$

$$\frac{wL^4}{8EI} - \frac{FL^3}{3EI} = \frac{F}{K}$$

$$\therefore \quad F = \frac{\frac{wL^4}{8EI}}{\left(\frac{1}{K} + \frac{L^3}{3EI}\right)} = \frac{\frac{3}{8}wL}{1 + \frac{3EI}{KL^3}}$$

19. (d)

Deflection at free end, $\delta_1 = \dfrac{WL^3}{3EI_1}$

Here, $I_1 = \dfrac{bd^3}{12}$

With doubling of depth and width,

$$I_2 = \frac{(2b)\times(2d)^3}{12} = 16\, I_1$$

$$\delta \propto \frac{1}{I}$$

$$\therefore \quad \delta_2 = \delta_1 \times \frac{I_1}{I_2}$$

$$\Rightarrow \quad \delta_2 = \frac{\delta_1}{16}$$

δ_2 as a percentage of $\delta_1 = \dfrac{1}{16}\times 100 = 6.25\%$

20 (b)

Strain energy in bar *X*,

$$U_x = \frac{P^2L}{2AE}$$

Strain energy in bar *Y*,

$$U_y = \frac{P^2(L/2)}{2\left(\frac{A}{2}\right)E} + \frac{P^2(L/2)}{2AE} = \frac{3}{4}\frac{P^2L}{AE}$$

$$\therefore \frac{U_x}{U_y} = \frac{\frac{P^2L}{2AE}}{\frac{3P^2L}{4AE}} = \frac{2}{3}$$

21. (a)

The vertical displacement at *C* will be due to deflection of point *B*. The beam *AB* is subjected to a moment *Pa* and axial force *P*. So the deflection of point *C* will be $\dfrac{Pa(3a)^2}{2EI} = \dfrac{9Pa^3}{2EI}$ in the vertically upward direction.

22. (a)

$$\sigma_1 = \frac{E}{1-\mu^2}(\epsilon_1 + \mu\,\epsilon_2)$$

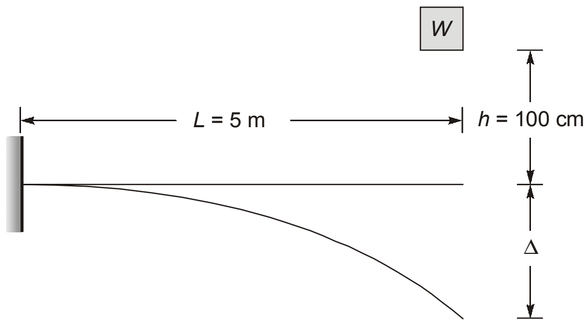
and $$\sigma_2 = \frac{E}{1-\mu^2}(\epsilon_2 + \mu\,\epsilon_1)$$

$$\sigma_1 = \frac{2\times 10^5}{1-0.3^2}(0.00152 + 0.3\times 0.00081) = 387.47 \text{ N/mm}^2$$

and $$\sigma_2 = \frac{2\times10^5}{1-0.3^2}(0.00081+0.3\times0.00152) = 278.24\,\text{N/mm}^2$$

$$\tau_{max} = \left|\frac{\sigma_1-\sigma_2}{2}\right| = \left|\frac{387.47-278.24}{2}\right| = 54.61\,\text{N/mm}^2$$

23. (b)



Let P denote the force exerted by the weight on the beam at the time of maximum deflection workdone by the weight, W = Strain energy stored in beam

$$W(h+\Delta) = \frac{P\Delta}{2}$$

$$\Rightarrow \quad P = \frac{2W}{\Delta}(h+\Delta)$$

Deflection at the tip of cantilever beam due to force P,

$$\Delta = \frac{PL^3}{3EI}$$

$$\Delta = \frac{2W}{\Delta}(h+\Delta)\frac{L^3}{3EI} \quad \ldots(i)$$

Deflection due to weight W, if it was statically applied is

$$\Delta_{st} = \frac{WL^3}{3EI} \quad \ldots(ii)$$

$$\Delta_{st} = \frac{1\times10^3\times5^3}{3\times30\times10^6} = 1.389\,\text{mm}$$

From equation (i) and (ii), we get

$$\Delta^2 - 2\Delta_{st}\,\Delta - 2h\,\Delta_{st} = 0$$

$$\Rightarrow \quad \Delta = \Delta_{st} + \sqrt{\Delta_{st}^2 + 2h\Delta_{st}}$$

$$\therefore \quad \Delta = 1.389 + \sqrt{(1.389)^2 + 2\times1000\times1.389}$$

$$= 54.1\,\text{mm}$$

24. (b)

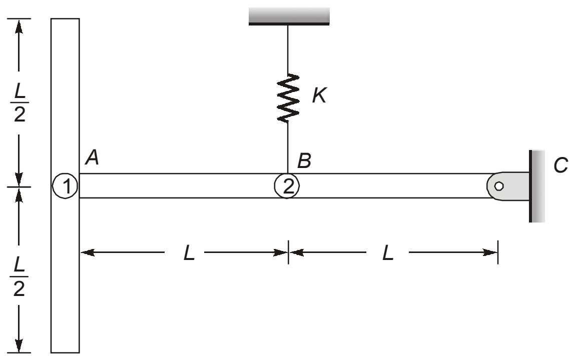
Stiffness of spring system,

$$K_{eq} = 0.5\,K + K + K = 2.5\,K$$

Stiffness of beam, $K_{Beam} = \dfrac{3EI}{l^3}$ $\left[\because \text{ Deflection, } \Delta = \dfrac{Pl^3}{3EI} \Rightarrow \dfrac{P}{\Delta} = \text{Stiffness} = \dfrac{3EI}{l^3}\right]$

$\therefore$ Total stiffness of system $= 2.5K + \dfrac{3EI}{l^3}$

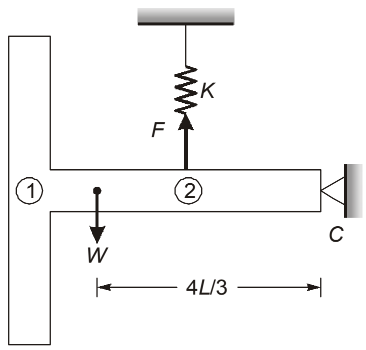
25. (b)



Center of gravity of T-shape is at $\bar{x}$ distance from C, calculated as

$$\bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1 + A_2}$$

$$= \frac{\left(\frac{L}{2}+\frac{L}{2}\right)\times 2L + 2L\times L}{L+2L} = \frac{4}{3}L$$



Taking moment about C

$$\Rightarrow \quad W\times\frac{4}{3}L = F\times L$$

$$\Rightarrow \quad F = Kx = \frac{4}{3}W$$

$$\Rightarrow \quad x = \frac{4}{3}\frac{W}{K}$$

26. (a)

For rectangular rosette, $\phi_{xy} = 2\in_{45^\circ} - (\in_{0^\circ} + \in_{90^\circ})$

$\Rightarrow \quad \phi_{xy} = 2\times 200 - (-500 + 300) = 600\ \mu\text{m/m}$

$$\text{Principal strains, } \in_1/\in_2 = \frac{\in_{0°} + \in_{90°}}{2} \pm \sqrt{\left(\frac{\in_{0°} - \in_{90°}}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= \frac{-500+300}{2} \pm \sqrt{\left(\frac{-500-300}{2}\right)^2 + \left(\frac{600}{2}\right)^2}$$

$$= -100 \pm \sqrt{(-400)^2 + (300)^2}$$

$$= -100 \pm 500$$

$$\in_1 = 400\ \mu\text{m/m}$$

and $\in_2 = -600\ \mu\text{m/m}$

27. (c)

Joints B and C are rigid, hence can act as fixed supports. Hence BA will act as a cantilever with point load at A.

$$\therefore \quad \delta_P = \frac{P(2a)^3}{3EI} = \frac{8Pa^3}{3EI}$$

Now at ends B and C, moments of magnitude 2 Pa will act respectively.

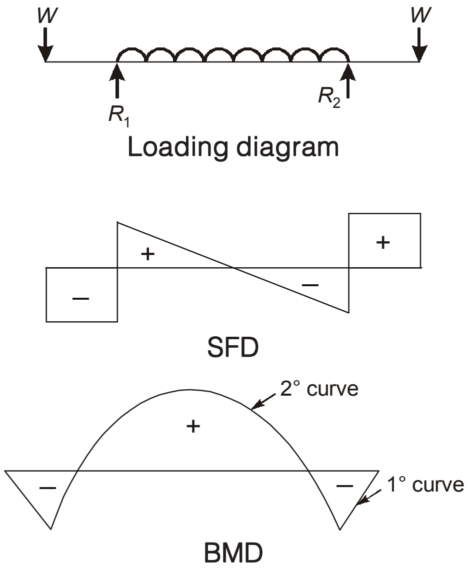
$$\therefore \quad \theta_B = \theta_C = \frac{2Pa \times a}{2EI} = \frac{Pa^2}{EI}$$

and $\delta_M = \theta_B \times 2a = \dfrac{Pa^2}{EI} \times 2a = \dfrac{2Pa^3}{EI}$

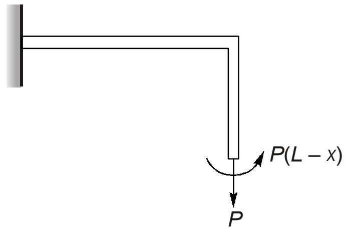
$\therefore$ Total deflection $= 2(\delta_P + \delta_M)$

$$= 2\left(\frac{8Pa^3}{3EI} + \frac{2Pa^3}{EI}\right) = \frac{28Pa^3}{3EI}$$

28. (a)



29. (c)



$$\text{Deflection due to load} = \frac{PL^3}{3EI}\text{(downward)}$$

$$\text{Deflection due to moment} = \frac{ML^2}{2EI} = \frac{P(L-x)L^2}{2EI}\text{ (upward)}$$

Now for the displacement at A equal to zero, we get

$$\therefore \quad \frac{P(L-x)L^2}{2EI} = \frac{PL^3}{3EI}$$

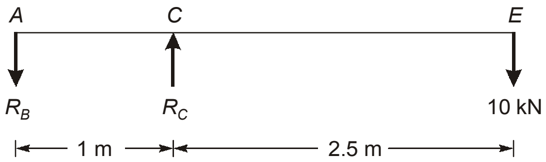
$$\text{or} \quad \frac{L-x}{2} = \frac{L}{3}$$

$$\text{or} \quad 3L - 3x = 2L$$

$$\text{or} \quad x = \frac{L}{3} = 0.33L$$

$$\therefore \quad a = 0.33$$

30. (a)



$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_B - R_C + 10 = 0 \quad \text{...(i)}$$

$$\Sigma M_C = 0$$

$$\Rightarrow \quad R_B \times 1 = 10 \times 2.5$$

$$\Rightarrow \quad R_B = 25 \text{ kN}$$

Putting the value of R_B in equation (i), we get

$$R_C = 35 \text{ kN}$$

$$\text{Stress in each of the two bolts } BD = \frac{R_B}{2A} = \frac{25 \times 10^3}{2 \times \frac{\pi}{4} \times 16^2} = 62.17 \text{ MPa}$$

$$\text{Bearing stress at } C = \frac{R_C}{A'} = \frac{35 \times 10^3}{200 \times 200} = 0.875 \text{ MPa}$$

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