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COMMUNICATIONS

ELECTRONICS ENGINEERING

Date of Test : 02/10/2023

ANSWER KEY >

1. (c)	7. (b)	13. (c)	19. (a)	25. (b)
2. (a)	8. (a)	14. (d)	20. (b)	26. (c)
3. (a)	9. (d)	15. (c)	21. (c)	27. (b)
4. (a)	10. (c)	16. (a)	22. (b)	28. (b)
5. (c)	11. (b)	17. (c)	23. (a)	29. (a)
6. (a)	12. (a)	18. (b)	24. (a)	30. (c)

Detailed Explanations

1. (c)

The condition required to eliminate the slope-overload distortion is,

$$\frac{\Delta}{T_s} \geq \left| \frac{dm(t)}{dt} \right|_{\max} = |2\pi f_m \sin(2\pi f_m t)|_{\max}$$

$$2f_s \geq 2\pi f_m$$

$$f_s \geq \pi f_m \approx 3.14 f_m$$

$$f_{s(\min)} = 3.14 f_m$$

2. (a)

$$\text{Quality factor, } Q = \frac{f_c}{B}$$

$$\text{Range of } Q, 10 < Q < 100$$

$$10 < \frac{f_c}{B} < 100$$

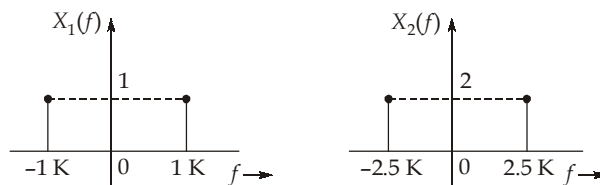
$$\frac{f_c}{B} > 10 \quad \frac{B}{f_c} < 0.1$$

$$\frac{f_c}{B} < 100 \quad \frac{B}{f_c} > 0.01$$

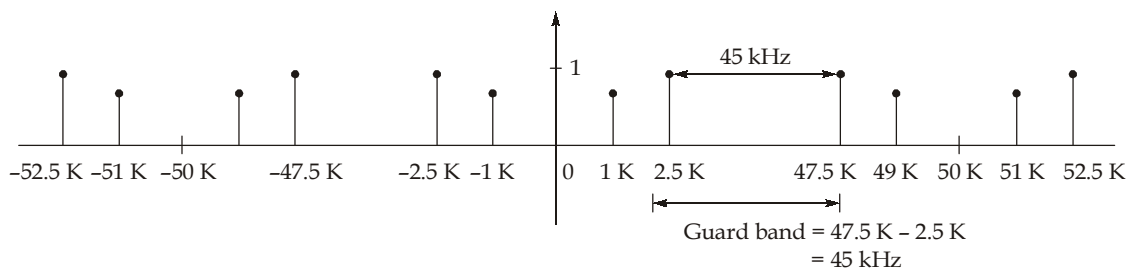
$$0.01 < \frac{B}{f_c} < 0.1$$

3. (a)

Before sampling



After sampling at a frequency of $f_s = 50$ kHz, $X_s(f)_{n=-\infty}^{\infty} = \Sigma X_1(f - nf_s)_{n=-\infty}^{\infty} + \Sigma X_2(f - nf_s)$



4. (a)

Autocorrelation function has maximum value at $\tau = 0$.

$$\therefore R_{XX}(0) \geq R_{XX}(\tau)$$

Autocorrelation function is even function.

$\therefore R_{XX}(-\tau) = R_{XX}(\tau)$
Value of autocorrelation function decreases as τ increases.

5. (c)

$$\begin{aligned} \text{Bit duration, } T_b &= \frac{1}{R_b} \\ R_b &= n f_s \\ f_s &= 5 \times 2 \times 2 = 20 \text{ kHz} \\ T_b &= \frac{1}{n \times 20 \times 10^3} \\ 5 \times 10^{-6} &= \frac{1}{n \times 20 \times 10^3} \\ n &= \frac{10^3}{20 \times 5} \\ n &= 10 \end{aligned}$$

6. (a)

$$\begin{aligned} P(x_1 x_2 x_1 x_3) &= (0.4)(0.3)(0.4)(0.2) \\ &= 0.0096 \end{aligned}$$

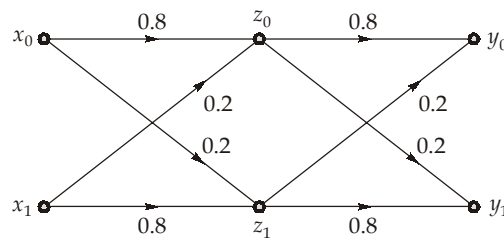
\therefore Information, $I(x_1 x_2 x_1 x_3) = -\log_2(0.0096) = 6.7$ bits

$$\begin{aligned} P(x_4 x_3 x_3 x_2) &= (0.1)(0.2)(0.2)(0.3) \\ &= 0.0012 \end{aligned}$$

$$\begin{aligned} I(x_4 x_3 x_3 x_2) &= -\log_2(0.0012) \\ &= 9.7 \text{ bits} \end{aligned}$$

$$\therefore \text{Ratio} = \frac{6.7}{9.7} = 0.691$$

7. (b)



Crossover probability of overall channel = $P(y_0 | x_1) = P(y_1 | x_0)$

$$\begin{aligned} P(y_0 | x_1) &= P(y_0 | z_0) P(z_0 | x_1) + P(y_0 | z_1) P(z_1 | x_1) \\ &= (0.80 \times 0.20) + (0.20 \times 0.80) = 0.32 \end{aligned}$$

So, the crossover probability of the resultant BSC = 0.32

8. (a)

$$\begin{aligned} H(x) &= -\sum_{i=0}^3 P(x_i) \log_2 P[x_i] \\ &= -\left[\frac{1}{2} \log_2 \left(\frac{1}{2} \right) + \frac{1}{4} \log_2 \left(\frac{1}{4} \right) + \frac{1}{8} \times 2 \log_2 \left(\frac{1}{8} \right) \right] \end{aligned}$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{6}{8}$$

$$H(x) = 1.75 \text{ bits/symbol}$$

9. (d)
For matched filter,

$$(\text{SNR})_{\max} = \frac{2E_s}{N_0}$$

$$E_s = \text{Energy of the signal } s(t)$$

$$= \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_0^2 (4)^2 dt = 32$$

So,

$$(\text{SNR})_{\max} = \frac{2(32)}{N_0} = \frac{64}{N_0}$$

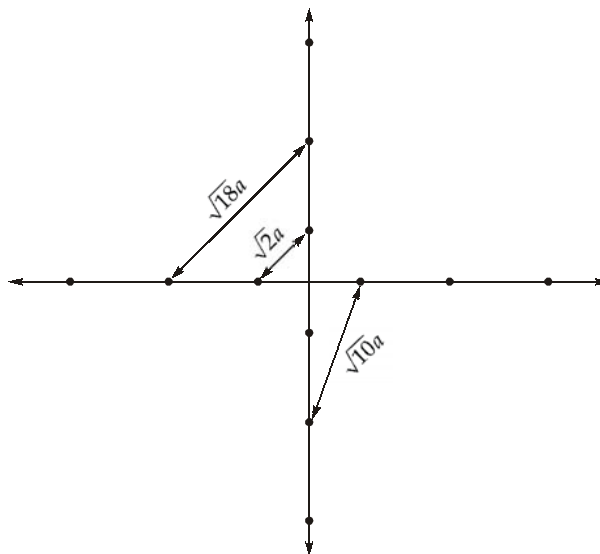
10. (c)
For coherent BPSK,

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{(N_0/2)}}\right)$$

$$= Q\left(\sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-9}}}\right) = Q(\sqrt{10^4}) = Q(100)$$

11. (b)
Probability of error in terms of Q-function is given by,

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right)$$



$$d_{\min} = \sqrt{2}a = 2\sqrt{2}$$

$$\frac{N_0}{2} = 4 \text{ W/Hz}$$

$$N_0 = 8 \text{ W/Hz}$$

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{(2\sqrt{2})^2}{2 \times 8}}\right)$$

$$P_e = Q\left(\sqrt{\frac{8}{2 \times 8}}\right)$$

$$P_e = Q(\sqrt{0.5})$$

12. (a)

$$p(0/1) = 0.25$$

$$p(0/1) = 1 - q$$

$$1 - q = 0.25$$

$$q = 0.75$$

$$p(z = 0) = 0.4 = p(0)p\left(\frac{0}{0}\right) + p(1)p\left(\frac{0}{1}\right)$$

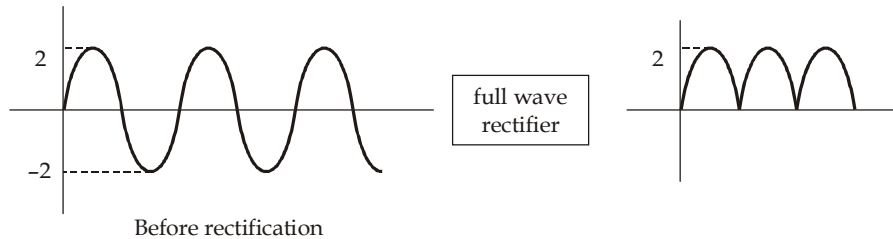
$$0.4 = 0.5p + 0.5 \times 0.25$$

$$0.4 = 0.5p + 0.125$$

$$0.5p = 0.275; \quad p = 0.55$$

Crossover probabilities $\rightarrow (1 - p) = 0.45$ and $(1 - q) = 0.25$

13. (c)



RMS value of signal remains same after rectification but dynamic range decreases therefore step size decreases and Noise power decreases.

$$\text{Signal power} = \frac{A^2}{2} = \frac{2^2}{2} = 2$$

$$\text{Quantization Noise Power} = \frac{\Delta^2}{12} \text{ where } \Delta = \text{step size}$$

$$\Delta = \frac{2 - 0}{2^n} = \frac{2}{2^8} = \frac{1}{2^7}$$

$$\frac{S}{N} = \frac{2}{1} \cdot 2^{14} \times 12 = 393216 = 55.94 \text{ dB}$$

14. (d)

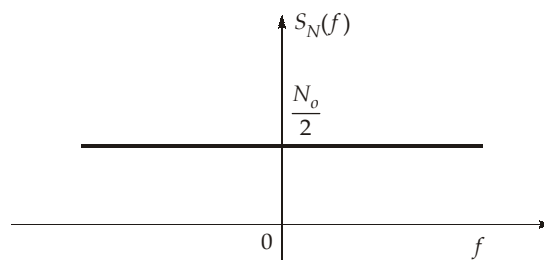
$$\begin{aligned}
 Z &= 2X + Y \\
 E[Z] &= E[2X + Y] \\
 &= 2E[X] + E[Y] \\
 E[X] &= 0, \quad E[Y] = 0 \\
 E[Z] &= 0 \\
 \text{Var}(Z) &= E[Z^2] - E[Z]^2 \\
 &= E[(2X + Y)^2] - 0 \\
 &= E[4X^2 + Y^2 + 4XY] \\
 &= 4E[X^2] + E[Y^2] + 4E[XY] \\
 E[X^2] &= \text{Var}[X] \text{ as } E[X] = 0 \\
 E[Y^2] &= \text{Var}[Y] \text{ as } E[Y] = 0 \\
 E[XY] &= E[X] E[Y] \text{ as } X \text{ and } Y \text{ are independent} \\
 \text{Var}(Z) &= 4 \times 1 + 1 + 4 \times 0 \\
 \text{Var}(Z) &= 5
 \end{aligned}$$

Therefore,

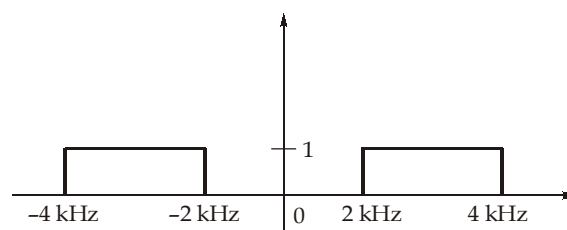
Therefore,

15. (c)

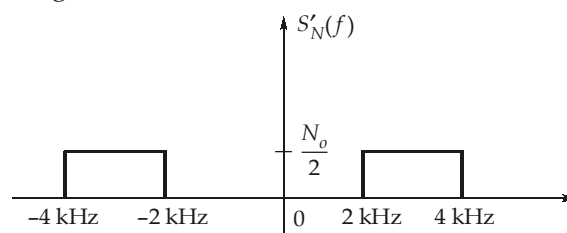
Power spectral density of white noise



Frequency response of filter



Noise after passing through the filter



Output noise power = Area under power spectral density curve

$$\begin{aligned}
 &= 2 \times \frac{N_o}{2} \times 2 \times 10^3 = 2 \times 4 \times 10^{-6} \times 10^3 \\
 &= 8 \times 10^{-3} \text{ W} = 8 \text{ mW}
 \end{aligned}$$

16. (a)

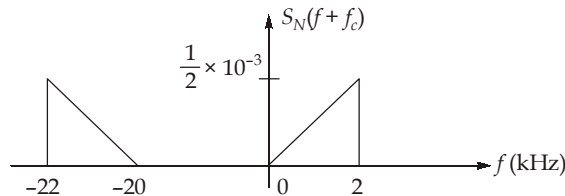
$$\begin{aligned} \text{Local oscillation frequency, } f_{LO} &= f_s + f_{IF} \\ \text{Maximum } f_{LO} &= f_{s_{max}} + f_{IF} \\ f_{LO_{max}} &= 1600 + 455 = 2055 \text{ kHz} \\ \text{Minimum local oscillation frequency, } f_{LO_{min}} &= f_{s_{min}} + f_{IF} \\ f_{LO_{min}} &= 540 + 455 = 995 \text{ kHz} \\ \frac{f_{LO_{max}}}{f_{LO_{min}}} &= \frac{2055}{995} = 2.065 \end{aligned}$$

17. (c)

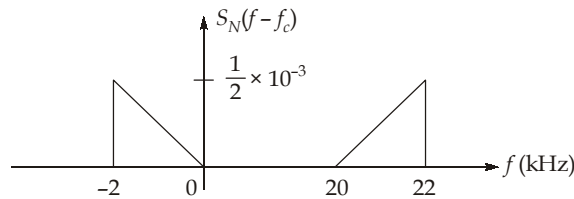
$$S_{NC}(f) = \begin{cases} S_N(f - f_c) + S_N(f + f_c), & -B \leq f \leq B \\ 0 & \text{else} \end{cases}$$

$$f_c = 10 \text{ kHz,}$$

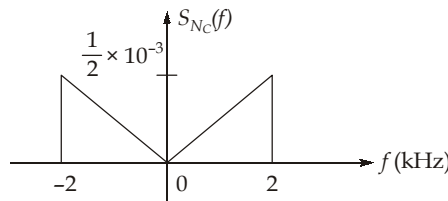
Plotting $S_N(f + f_c)$



Plotting $S_N(f - f_c)$



∴



18. (b)

If X is equally likely to take both positive and negative values then,

$$P(X < 0) = P(X > 0)$$

$$P(X) = \text{Area under curve}$$

$$P(X < 0) = b \times 1 = b$$

$$P(X > 0) = b^2 \times 4 = 4b^2$$

$$b = 4b^2$$

$$b = \frac{1}{4}$$

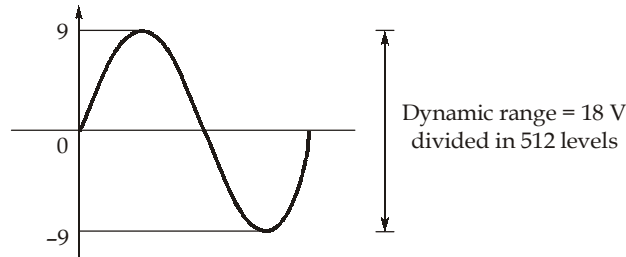
Also, Area of PDF curve = 1

$$b + 4b^2 + a = 1$$

$$\frac{1}{4} + 4 \times \frac{1}{16} + a = 1$$

$$a = 1 - \frac{1}{2} = 0.5$$

19. (a)



Now output of resistive network = $\frac{1}{2} V_i = \frac{9}{2} \sin \omega_m t$

Dynamic range = 9 V

Dynamic range reduces to $\frac{1}{2}$

Step size remains same,

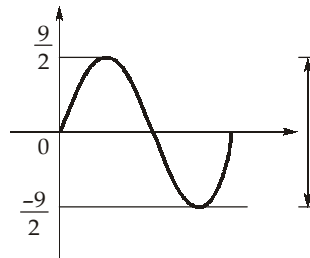
\therefore No. of levels $\frac{1}{2} \times 512 = 256$

As dynamic range reduces, $n = 8$

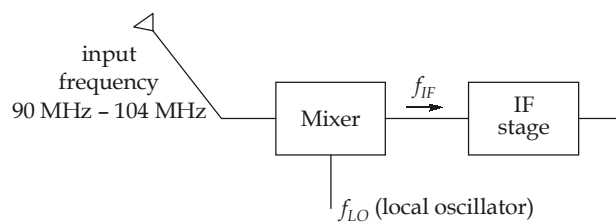
$$\frac{S}{N} = 1.76 + 6n$$

$$= 1.76 + 6 \times 8$$

$$= 49.8 \text{ dB}$$



20. (b)



$$f_{LO} = f_s + f_{IF}$$

$$f_{si} \text{ (image frequency)} = f_s + 2f_{IF}$$

When $f_s = 90 \text{ MHz}$

$$f_{LO} = f_s + f_{IF} \quad f_{si} = f_s + 2f_{IF}$$

We have to select f_{IF} such that f_{si} falls outside the tuned range.

$$f_{si} \geq 104 \text{ MHz}$$

$$104 = 90 + 2f_{IF}$$

$$f_{IF_{min}} = 7 \text{ MHz}$$

21. (c)

Probability of transmitting zero, $P(0) = \frac{2}{3}$

Probability of transmitting one, $P(1) = 1 - \frac{2}{3} = \frac{1}{3}$

$$\begin{aligned} P(\text{at least two bits are zeroes}) &= 1 - P(\text{no bit is zero}) \\ &\quad - P(\text{one bit is zero}). \\ &= 1 - {}^5C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5 - {}^5C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^4 \\ &= 1 - \frac{1}{3^5} - \frac{10}{3^5} \\ &= 1 - \frac{11}{243} = 0.954 \end{aligned}$$

22. (b)

The angle of the modulated signal $s(t)$ can be given as,

$$\theta(t) = 2\pi f_c t + 4 \sin(4000\pi t) + 3 \cos(4000\pi t)$$

The instantaneous frequency of the modulated signal can be given as,

$$f_i = \frac{1}{2\pi} \frac{d[\theta(t)]}{dt}$$

$$\begin{aligned} f_i &= f_c + \frac{1}{2\pi} [4 \times 4000\pi \cos 4000\pi t + 3 \times 4000\pi [-\sin 4000\pi t]] \\ &= f_c + [8000 \cos(4000\pi t) - 6000 \sin(4000\pi t)] \\ &= f_c + 2000 \times 5 [\cos(4000\pi t + \alpha)] \text{ where } \alpha = \tan^{-1} \left(\frac{3}{4} \right) \end{aligned}$$

$$\begin{aligned} f_{i(\max)} &= f_c + 2000 \times 5 \\ &= 100 \text{ kHz} + 10 \text{ kHz} \end{aligned}$$

$$f_{i(\max)} = 110 \text{ kHz}$$

23. (a)

The carrier component of the FM signal will be zero when $J_0(\beta) = 0$.

We know $J_0(\beta) = 0$ for $\beta = 2.41, 5.52, 8.65, 11.8 \dots$

So, when $A_m = 4 \text{ V}$, the corresponding modulation index is $\beta = 2.41$.

$$\beta = 2.41$$

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

$$k_f = \frac{\beta f_m}{A_m} = \frac{2.41 \times 2 \times 10^3}{4}$$

$$k_f = 1.205 \text{ kHz/V}$$

24. (a)

x_4	0.5 (0)	0.5 (0)	0.5 (0)	$\left. \begin{array}{l} \\ \\ \end{array} \right\} (0)$
x_2	0.25 (10)	0.25 (10)	$\left. \begin{array}{l} \\ \end{array} \right\} (10)$	$\left. \begin{array}{l} \\ \\ \end{array} \right\} (1)$
x_3	0.125 (110)	0.25 (11)	$\left. \begin{array}{l} \\ \end{array} \right\} (11)$	
x_1	0.125 (111)			

$$L = \sum_{i=1}^4 P(x_i)n_i$$

$$= (0.5 \times 1) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3)$$

$$= 0.5 + 0.5 + 0.75$$

$$L = 1.75 \text{ bits/symbol}$$

$$H = -\sum_{i=1}^4 P(x_i)\log_2 P(x_i)$$

$$H = -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{2}{8}\log_2\left(\frac{1}{8}\right) \right]$$

$$H = 1.75 \text{ bits/symbol}$$

$$\eta = \frac{H}{L} = \frac{1.75}{1.75} = 1$$

$$\text{Redundancy } \gamma = 1 - \eta = 1 - 1 = 0$$

25. (b)

Bandwidth of the baseband signal with raised cosine pulse shaping will be,

$$(\text{BW})_{\text{signal}} = \frac{R_b}{2}(1 + \alpha) = \frac{1000}{2}(1 + \alpha) = 500(1 + \alpha) \text{ kHz}$$

For proper transmission of the data,

$$(\text{BW})_{\text{signal}} \leq (\text{BW})_{\text{channel}}$$

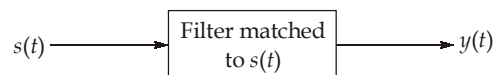
$$500(1 + \alpha) \leq 600$$

$$(1 + \alpha) \leq 1.20$$

$$\alpha \leq 0.20$$

$$\alpha_{\text{max}} = 0.20$$

26. (c)



For a matched filter, peak value of the output will be numerically equal to the energy of the input signal.

So,

$$|y(t)|_{\text{max}} = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

$$s(t) = \begin{cases} \left(3 - \frac{3}{2}|t|\right) \text{ V} & ; 0 \leq |t| \leq 2 \\ 0 & ; \text{ otherwise} \end{cases}$$

So,

$$\begin{aligned}
 |y(t)|_{\max} &= 2 \int_0^2 \left(3 - \frac{3}{2}t\right)^2 dt \\
 &= \frac{9}{2} \int_0^2 (t^2 + 4 - 4t) dt \\
 &= \frac{9}{2} \left[\frac{t^3}{3} + 4t - 2t^2 \right]_0^2 = 12 \text{ V}
 \end{aligned}$$

27. (b)

$$\begin{aligned}
 f_Z(z) &= f_X(z) * f_Y(z) \\
 f_X(z) &= ae^{-az} u(z) \\
 f_Y(z) &= be^{-bz} u(z)
 \end{aligned}$$

$$L\{f_X(z)\} = \frac{a}{s+a} \quad \text{and} \quad L\{f_Y(z)\} = \frac{b}{s+b}$$

$$\begin{aligned}
 f_Z(z) &= L^{-1} \left\{ \frac{ab}{(s+a)(s+b)} \right\} = L^{-1} \left\{ \frac{ab}{(b-a)} \left[\frac{1}{(s+a)} - \frac{1}{(s+b)} \right] \right\} \\
 &= \frac{ab}{(b-a)} [e^{-az} - e^{-bz}] u(z)
 \end{aligned}$$

28. (b)

The transmission efficiency of an AM signal can be given by,

$$\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$$

Here,

$$k_a = \text{amplitude sensitivity of the modulator} = 0.25 \text{ V}^{-1}$$

$$P_m = \text{Power of the message signal}$$

For the given message signal,

$$P_m = A^2 = (2)^2 = 4$$

So,

$$\eta = \frac{(0.25)^2 (4)}{1 + (0.25)^2 (4)} = \frac{0.25}{1 + 0.25} = \frac{1}{5} = 0.20 \text{ (or) } 20\%$$

29. (a)

The rule to decide an optimum threshold value using MAP criteria is as follows:

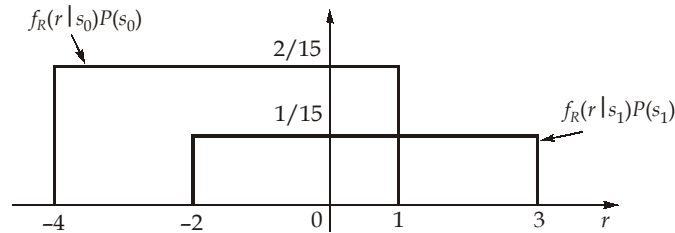
$$\begin{array}{c}
 H_0 \\
 f_R(r | s_0)P(s_0) > f_R(r | s_1)P(s_1) \\
 H_1
 \end{array}$$

The above expression says that,

- Decision is made in favour of "0", if $f_R(r | s_0)P(s_0)$ is greater than $f_R(r | s_1)P(s_1)$.
- Decision is made in favour of "1", if $f_R(r | s_1)P(s_1)$ is greater than $f_R(r | s_0)P(s_0)$.

Given that $P(s_0) = \frac{2}{3}$ and $P(s_1) = \frac{1}{3}$.

The optimum threshold can be decided by using MAP criteria, by plotting the functions $f_R(r | s_1)P(s_0)$ and $f_R(r | s_1)P(s_1)$ as follows:



It is clear from the above diagram that,

For $r < 1$, $f_R(r | s_0)P(s_0) > f_R(r | s_1)P(s_1)$ and for $r > 1$, $f_R(r | s_1)P(s_1) > f_R(r | s_0)P(s_0)$.

So, the optimum threshold value is, $r_{th} = 1$.

30. (c)

The output of the narrowband FM modulator can be given by,

$$x(t) = A \cos[2\pi f_0 t + \phi(t)] ; |\phi(t)|_{\max} = 0.10 \text{ radians}$$

The signal at the output of upper frequency multiplier can be given by,

$$y(t) = A \cos[2\pi n_1 f_0 t + n_1 \phi(t)]$$

After mixing $y(t)$ with the output signal of the lower frequency multiplier, we get,

$$\begin{aligned} z(t) &= A^2 \cos[2\pi n_1 f_0 t + n_1 \phi(t)] \cos[2\pi n_2 f_0 t] \\ &= \frac{A^2}{2} \cos[2\pi(n_1 + n_2)f_0 t + n_1 \phi(t)] + \frac{A^2}{2} \cos[2\pi(n_1 - n_2)f_0 t + n_1 \phi(t)] \end{aligned}$$

It is given that the mixer is designed for up-conversion. So, the signal $s(t)$ can be given by,

$$s(t) = \frac{A^2}{2} \cos[2\pi(n_1 + n_2)f_0 t + n_1 \phi(t)] \quad \dots(i)$$

It is given that, $f_c = 104 \text{ MHz}$ and $\Delta f_{\max} = 75 \text{ kHz}$ for $s(t)$.

So, the modulation index of the wideband signal $s(t)$ will be,

$$\begin{aligned} \beta &= \frac{\Delta f_{\max}}{f_{m(\max)}} = n_1 |\phi(t)|_{\max} \\ n_1(0.10) &= \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5 \\ n_1 &= \frac{5}{0.10} = 50 \\ f_c &= (n_1 + n_2)f_0 = 104 \text{ MHz} \\ (n_1 + n_2) \times 100 &= 104 \times 1000 \\ n_2 &= 1040 - n_1 = 1040 - 50 = 990 \end{aligned}$$

