CLASS TEST

Sl.: 02SKEE_EFGHIJK_16092023

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# ELECTROMAGNETIC FIELDS

## ELECTRICAL ENGINEERING

Date of Test : 16/09/2023

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (a) | 7. | (c) | 13. | (c) | 19. | (c) | 25. | (c) |
| 2. | (c) | 8. | (b) | 14. | (b) | 20. | (b) | 26. | (b) |
| 3. | (b) | 9. | (a) | 15. | (d) | 21. | (c) | 27. | (b) |
| 4. | (a) | 10. | (d) | 16. | (c) | 22. | (a) | 28. | (c) |
| 5. | (a) | 11. | (b) | 17. | (c) | 23. | (a) | 29. | (d) |
| 6. | (c) | 12. | (d) | 18. | (a) | 24. | (a) | 30. | (b) |

**2. (c)**

$$\text{Divergence (Curl } \vec{A}) = 0$$

**3. (b)**

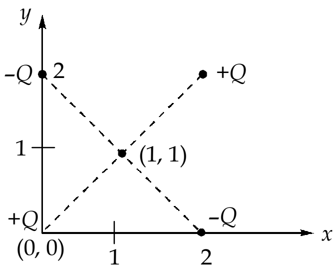
$$\text{Electric flux density, } D = \varepsilon E$$

$$\text{Electric field intensity, } E = \frac{D}{\varepsilon} = \frac{\frac{x}{4}\times 10^{-9}}{6\times\frac{1}{36\pi}\times 10^{-9}}$$

At $x = \frac{1}{2}$,
$$E = \frac{6\pi\times 10^{-9}}{4\times 10^{-9}}\times\frac{1}{2}$$

$$E = \frac{3}{4}\pi\ \frac{\text{V}}{\text{m}}$$

**4. (a)**




The two positive charges $Q$ are diagonally opposite in position and at the same distance from the point (1, 1, 0) fields produced by them are equal and opposite and so their resultant field is zero. Similarly for negative charges.

**5. (a)**

As both the coils are same axis and carrying currents in opposite directions, the field components produced by both the coils are in opposite direction and they cancel out each other. So, the net field at the point on the axis midway between the coils is zero.

**6. (c)**

$$\text{Continuity equation, } \nabla.\vec{J} = \frac{-\partial\rho_v}{\partial t}$$

$$\text{for static fields, } \frac{\partial\rho_v}{\partial t} = 0$$

$$\text{So, for static fields, } \nabla.\vec{J} = 0$$

**7. (c)**

Given, $\phi = 4x^2 + y^2 + cz^2$

In source free region,

$$\nabla\cdot\vec{D} = 0$$

Also $D = \in E$

So, $\in(\nabla\cdot\vec{E}) = 0$

or, $\nabla\cdot\vec{E} = 0$

Also $\vec{E} = -\nabla V = -\nabla\phi$

$$\nabla V = \frac{\partial\phi}{\partial x}\hat{a}_x + \frac{\partial\phi}{\partial_y}\hat{a}_y + \frac{\partial\phi}{\partial z}\hat{a}_z$$

$$-\vec{E} = 8x\,\hat{a}_x + 2y\,\hat{a}_y + 2cz\,\hat{a}_z$$

Again, $\nabla\cdot\vec{E} = 0$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 8 + 2 + 2c = 0$$

$$c = -5$$

**8.** **(b)**

When spheres are brought in contact total charge gets redistributed

$$Q = \frac{Q_1 + Q_2}{2} = \frac{4.5 - 1.5}{2} = 1.5 \text{ nC}$$

After separation of 50 cm between spheres

$$\text{Force, } |F| = \frac{Q_1 \times Q_2}{4\pi \in_0 \times r^2}$$

$$= \frac{(1.5\times10^{-9})\times(1.5\times10^{-9})}{4\pi \in_0 \times(50\times10^{-2})^2}$$

$$= \frac{2.25\times10^{-18}}{4\pi\times8.854\times10^{-12}\times(50\times10^{-2})^2} = 80.89 \text{ nN}$$

**9.** **(a)**

We know,

Biot Savart's law, $$H = \int\frac{I\,\vec{dl}\times\hat{a}_r}{4\pi R^2} = \int_0^{2\pi}\frac{IR\,d\phi\,\hat{a}_\phi}{4\pi R^2}(-\hat{a}_\rho) = \frac{I}{4\pi}\int_0^{2\pi}\frac{Rd\phi}{R^2}(\hat{a}_z) = \frac{I}{2R}\hat{a}_z$$

**10.** **(d)**

Force acts in a direction from high flux concentrated area to low flux area when currents are in opposite direction then the force will be repulsive.

**11.** **(b)**

$$V_{AB} = -\int_B^A \vec{E}\cdot\vec{dl}$$

Where, $$\vec{E} = \frac{\rho_l}{2\pi \in_0 r}\hat{a}_r$$

$$V_{AB} = -\int_B^A \frac{10^{-9}}{2(2\pi \in_0 r)}dr = \frac{-10^{-9}\times36\pi}{4\pi\times10^{-9}}\int_B^A\frac{1}{r}dr = -9[\ln r]_4^2$$

$$V_{AB} = 6.24 \text{ V}$$

**12.** **(d)**

Consider the length of the coaxial cable as $L$

Let $V_0$ be the potential difference between the inner and outer conductors so that,

$$V_{(\rho = a)} = 0$$

and $$V_{(\rho = b)} = V_0$$

$$\vec{J} = \sigma\vec{E} = \frac{-\sigma V_0}{\rho \ln \frac{b}{a}} \hat{a}_\rho$$

$$\overrightarrow{dS} = -\rho\, d\phi\, dz\, \hat{a}_\rho$$

$$I = \int \vec{J} \cdot \overrightarrow{dS} = \int_{\phi=0}^{2\pi} \int_{z=0}^{L} \frac{V_0 \sigma}{\rho \ln \frac{b}{a}} \rho\, d\phi\, dZ = \frac{2\pi L \sigma V_0}{\ln \frac{b}{a}}$$

Resistance per unit length, $\frac{R}{L} = \frac{V_0}{I} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma}$

The conductance per unit length is,

$$G = \frac{1}{R} = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)}$$

**13.** **(c)**

According to Gauss's Law,

for region, $2 \text{ m} \le r \le 4 \text{ m}$

$$\pi\rho L\,(r^2 - 4) = D\,(2\pi\, rL)$$

$$D = \frac{\rho}{2r}(r^2 - 4)\hat{a}_r \,(\text{C/m}^2)$$

**14.** **(b)**

According to Ampere's law,

$$I_{\text{enc}} = \oint_{r=r_0} \vec{H} \cdot dl$$

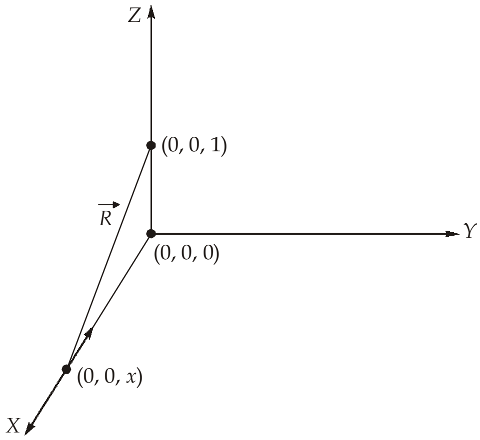
$$= \int_0^{2\pi} \frac{10^4}{r_0}\left(\frac{4r_0^2}{\pi^2}\sin\frac{\pi}{2} - \frac{2r_0^2}{\pi}\cos\frac{\pi}{2}\right) \cdot r_0\, d\phi$$

$$= 10^4 \int_0^{2\pi} \frac{4r_0^2}{\pi^2} d\phi$$

$$I_{\text{enc}} = 10^4 \cdot \frac{4r_0^2}{\pi} \times 2 = \frac{8}{\pi} \text{ Ampere}$$

**15. (d)**

According to the Biot-Savart's law,




$$\vec{H} = \int \frac{I\,\overrightarrow{dL} \times \vec{R}}{4\pi R^3}$$

$$\vec{R} = -x\,\hat{a}_x + \hat{a}_z$$

$$R = \sqrt{1+x^2}$$

$$\vec{H} = \int_{\infty}^{0} \frac{10\,dx(-\hat{a}_x) \times (-x\hat{a}_x + \hat{a}_z)}{4\pi(x^2+1)^{3/2}}$$

$$\vec{H} = \frac{10}{4\pi}\int_{\infty}^{0} \frac{dx}{(x^2+1)^{3/2}}\,\hat{a}_y$$

$$\vec{H} = \frac{10}{4\pi}\left.\frac{x}{\sqrt{1+x^2}}\right|_{\infty}^{0} \hat{a}_y = \frac{10}{4\pi}\hat{a}_y$$

**16. (c)**

The flux in the circuit is,

$$\Psi = \frac{\text{ö}}{\text{U}} = \frac{N_i\, i_1}{l/\mu S} = \frac{N_1\, i_1\, \mu S}{2\pi\rho_0}$$

ö = magneto motive force

U = reluctance

$l$ = mean length

$S$ = cross-sectional area of magnetic core

According to Faraday's Law, the emf induced in the second coil is,

$$V_2 = -N_2\frac{d\Psi}{dt} = -\frac{N_1 N_2\, \mu S}{2\pi\rho_0}\frac{di_1}{dt}$$

$$V_2 = -\frac{100 \times 200 \times 500 \times (4\pi \times 10^{-7}) \times 10^{-3} \times 300\pi\cos 100\pi t}{2\pi(10 \times 10^{-2})}$$

$$= -6\pi\cos 100\pi t \text{ V}$$

**17. (c)**

The magnetic flux density is,

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\,\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{d\theta} & \frac{\partial}{\partial\phi} \\ 0 & r(10\sin\theta) & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[-\frac{\partial}{\partial\phi}(10r\sin\theta)\right]\hat{a}_r - \frac{r}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(0)\right]\hat{a}_\theta + \frac{10r\sin^2\theta}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r)\right]\hat{a}_\phi$$

$$= \frac{10\sin\theta}{r}\,\hat{a}_\phi$$

$$\vec{B}\left(2, \frac{\pi}{2}, 0\right) = \frac{10\sin\pi/2}{2}\hat{a}_\phi = 5\,\hat{a}_\phi \text{ Wb/m}^2$$

**18. (a)**

Let,
$$E = E_1,$$
$$\text{Energy } E_1 = \frac{Q_1^2}{2C_1}$$

Electrically isolated

$$\Rightarrow \quad Q_2 = Q_1$$
$$d_2 = 2d_1$$
$$\Rightarrow \quad C_2 = \frac{C_1}{2}$$
$$E_2 = \frac{Q_2^2}{2C_2} = \frac{Q_1^2}{\frac{2C_1}{2}} = 2\left(\frac{Q_1^2}{2C_1}\right)$$
$$= 2E_1 = 2E$$

**19. (c)**

Given,

Voltage distribution across capacitance is in the ratio 2 : 3 : 4 and applied voltage is 135 V.

Then,
$$V_{C1} = 30 \text{ V}$$
$$V_{C2} = 45 \text{ V}$$
$$V_{C3} = 60 \text{ V}$$

Hence,
$$C_1 = \frac{4500}{30} = 150\ \mu\text{F}$$
$$C_2 = \frac{4500}{45} = 100\ \mu\text{F}$$
$$C_3 = \frac{4500}{60} = 75\ \mu\text{F}$$
$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$C_{eq} = 33.33\ \mu\text{F}$$

**20. (b)**

According to Gausss's law, the electric flux leaving the surface with $R$ = 5 m is equal to the total flux enclosed by the surface.

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3$$
$$\Psi_1 = \text{Electric flux leaving the spherical surface with } R = 1\text{m}$$
$$= 20 \times 10^{-9} \times (4\,\pi R^2)$$
$$= 20 \times 10^{-9} \times 4\,\pi = 80\,\pi\text{nC}$$
$$\Psi_2 = -9 \times 10^{-9} \times (4\,\pi\,(2)^2)$$
$$= -144\,\pi\text{nC}$$
$$\Psi_3 = 2 \times 10^{-9} \times (4\pi \times 9) = 72\,\pi\text{ nC}$$
$$\Psi = 8\,\pi\text{ nC}$$

**21. (c)**

$$\text{The total dielectric flux} = \Psi = Q = CV$$
$$= 2 \times 10^{-4} \times 10^{-6} \times 20 \times 10^{3} = 4\ \mu C$$
$$\text{Potential gradient} = \frac{V}{t} = \frac{20 \times 10^{3}}{2 \times 10^{-3}}$$
$$= 10\ \text{MV/m} = 100\ \text{kV/cm.}$$

**22. (a)**

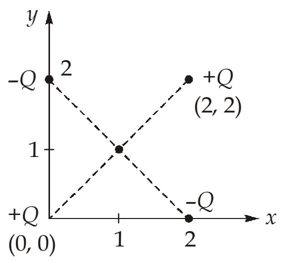
$$E = \frac{\Delta V}{\Delta z} = \frac{250 - 100}{5 \times 10^{-3}} = 3 \times 10^{4}\ \text{V/m}$$
$$\vec{E} = -\nabla \vec{V} = -3 \times 10^{4}\ \text{V/m}$$
$$D = \epsilon_0 \epsilon_r \vec{E} = \frac{10^{-9}}{36\pi} \times 2.4 \times (-3 \times 10^{4})\hat{a}_z$$
$$= -6.37 \times 10^{-7}\hat{a}_z\ \text{C/m}^2$$

As $D$ is constant between the disks and $D_n = \rho_s$ at a conductor surface

$$\rho_s = \pm 6.37 \times 10^{-7}\ \text{C/m}^2$$

Positive sign on upper plate and negative sign on lower plate.

**23. (a)**




Two positive charges $Q$ are diagonally opposite in position and at the same distance from the point (1, 1, 0) fields produce by them are equal and opposite and so their resultant field is zero. Similarly for negative charges.

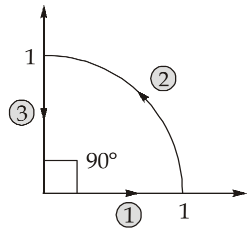
**24. (a)**

We know,

$$I = \int_s \vec{J} \cdot \vec{ds} = \int_{\phi=0}^{2\pi} \int_{0}^{5} \frac{40}{\rho^2 + 1} \rho\, d\rho\, d\phi\ \text{mA}$$
$$= 80\pi \int_0^5 \frac{\rho}{\rho^2 + 1} d\rho\ \text{mA} = \frac{80\pi}{2} \ln(\rho^2 + 1)\Big|_0^5\ \text{mA}$$
$$= 40\pi[\ln(26) - \ln(1)]$$
$$= 40\pi \times 3.258 = 409.42\ \text{mA}$$

**25. (c)**

$$\nabla \times \vec{E} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 4\rho\sin\phi & 0 & 0 \end{vmatrix}$$




$$= \frac{1}{\rho}\left(-\frac{\partial}{\partial \phi}(4\rho\sin\phi)\right)\hat{a}_z$$

$$= -\frac{1}{\rho}4\rho\cos\phi\,\hat{a}_z = -4\cos\phi\,\hat{a}_z$$

$$\oint \vec{E}\cdot d\vec{l} = \int \nabla \times \vec{E}\cdot d\vec{s} \qquad \text{Where, } d\vec{s} = \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$= \int -4\cos\phi\,\rho\, d\rho\, d\phi$$

$$= -4\int_0^{\pi/2}\cos\phi\, d\phi \int_0^1 \rho\, d\rho = -4\sin\phi\Big|_0^{\pi/2} \times \frac{\rho^2}{2}\Big|_0^1 = -4\times 1\times\frac{1}{2} = -2$$

**26. (b)**

Let the potential division between the two dielectrics be given by $V_1$ and $V_2$ and the respective field intensities by $E_1$ and $E_2$.

$$\because \qquad V_1 = E_1 t_1 = \frac{D_1}{\varepsilon_0\,\varepsilon_{r1}}\times t_1$$

and
$$V_2 = \frac{D_2}{\varepsilon_0\,\varepsilon_{r2}}\times t_2$$

At the interface, from boundary relation

$$D_1 = D_2$$

$$\therefore \qquad \frac{V_1}{V_2} = \frac{t_1/\varepsilon_{r1}}{t_2/\varepsilon_{r2}} = \frac{3/3}{2/5} = \frac{5}{2} \qquad \text{...(i)}$$

Also,
$$V_1 + V_2 = 100\text{ V} \qquad \text{...(ii)}$$

$$\therefore \qquad V_1 = 100\times\frac{5}{7} = 71.43\text{ V}$$

$$V_2 = \frac{2}{7}\times 100 = 28.57\text{ V}$$

**27. (b)**

An electrostatic field with electric field $\vec{E}$ is said to be conservative, if the closed line integral of the field is zero

i.e.
$$\oint \vec{E}\cdot d\vec{l} = 0$$

Applying stokes theorem,

$$\oint \vec{E}\cdot d\vec{l} = \int_S (\nabla\times\vec{E})\cdot d\vec{s}$$

Equation becomes $\nabla\times\vec{E} = 0$, i.e, the curl of the field $\vec{E}$ is equal to zero.

**28. (c)**

We know, $$\rho_v = \nabla \cdot D = \frac{\partial D_z}{\partial z} = \rho \cos^2 \phi$$

The total charge enclosed in cylindrical section,

$$Q = \int_v \rho_v \, dV = \int_v \rho \cos^2 \phi \rho \, d\phi \, d\rho \, dz$$

$$= \int_{z=-2}^{2} dz \int_{\phi=0}^{2\pi} \cos^2 \phi \, d\phi \int_{\rho=0}^{2} \rho^2 \, d\rho = (4)\times(\pi)\times\frac{(2)^3}{3} = \frac{32\pi}{3}\text{ C}$$

**29. (d)**

For boundary between two ideal dielectrics:

We can use conditions,

$$E_{t1} = E_{t2} = (100\hat{a}_y - 400\hat{a}_z)\text{ V/m}$$

and $$D_{n1} = D_{n2}$$

$$\epsilon_{r1} E_{n1} = \epsilon_{r2} E_{n2}$$

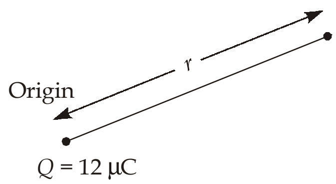
$$E_{n2} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \cdot \epsilon_{n1} = \frac{4}{6} \times 300 = 200\text{ V/m}$$

$$\therefore \quad \vec{E}_2 = 200\,\hat{a}_x + 100\,\hat{a}_y - 400\,\hat{a}_z\text{ V/m}$$

Hence (d) option is correct.

**30. (b)**

Electric flux density, $$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r\text{ C/m}^2$$




For electric flux crossing spherical defined by

$$r = 10\text{ cm},\ 0 < \theta \le \pi,\ 0 < \phi \le \frac{\pi}{2}$$

$$\Psi = \iint \vec{D} \cdot \vec{ds}$$

$$\Psi\big|_{r=0.1\text{m}} = \iint \frac{Q}{4\pi r^2}\hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= \frac{Q}{4\pi}\int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{\pi/2} d\phi = \frac{120\times10^{-6}}{4\pi}\times 2 \times \frac{\pi}{2} = 30\ \mu\text{C}$$

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