CLASS TEST

Sl. : 02SPME_EFGHIJK_17092023

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ENGINEERING MECHANICS

MECHANICAL ENGINEERING

Date of Test : 17/09/2023

ANSWER KEY ➤

1.	(b)	7.	(c)	13.	(c)	19.	(c)	25.	(b)
2.	(d)	8.	(b)	14.	(a)	20.	(d)	26.	(d)
3.	(d)	9.	(c)	15.	(d)	21.	(b)	27.	(c)
4.	(c)	10.	(a)	16.	(b)	22.	(c)	28.	(a)
5.	(c)	11.	(b)	17.	(a)	23.	(a)	29.	(c)
6.	(b)	12.	(c)	18.	(b)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. **(b)**

As per given information,

$$h = 40 \text{ m}, u = 50 \text{ m/s}$$

Let the speed be 'v' when it strikes to the ground

Apply law of conservation of energy,

$$mgh + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

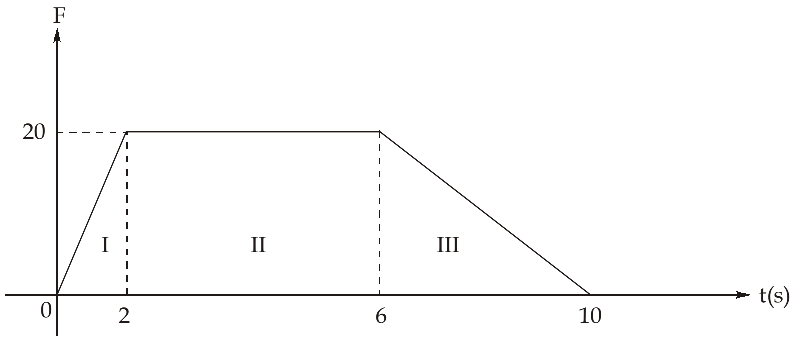
$$m \times 10 \times 40 + \frac{1}{2} \times m \times (50)^2 = \frac{1}{2} \times m \times v^2$$

$$400 + 1250 = \frac{v^2}{2}$$

$$v = 57.44 \text{ m/s}$$

2. **(d)**

As per given information,



$$\text{Impulse} = \int_{t_1}^{t_2} F(t)dt = \text{Area under (F - t) curve}$$

$$\text{Impulse} = \text{Area (I + II + III)}$$

$$= \frac{1}{2} \times 2 \times 20 + 4 \times 20 + \frac{1}{2} \times 4 \times 20$$

$$= 140 \text{ kg.m/s or Ns}$$

3. **(d)**

The block is displaced 2.5 m towards left,

Let the velocity of the body be v at mean position

As dissipative force, (friction = 0)

$$(\text{K.E.})_{max} = (\text{P.E.})_{max}$$

$$\frac{1}{2}mv_{max}^2 = \frac{1}{2} \times k_1 \times x^2 + \frac{1}{2} \times k_2 \times x^2$$

$$mv_{max}^2 = k_1 \times x^2 + k_2 x^2$$

Substituting the given value,

$$100 \times mv_{max}^2 = 40x^2 + 60x^2$$

$$100\, mv_{max}^2 = 100x^2$$

$$v_{max} = x = 2.5 \text{ m/s}$$

4. **(c)**

$$\text{Lagrangian, } L = T - V$$

$$= v^2\dot{u}^2 + 2\dot{v}^2 - u^2 + v^2$$

$$= v^2\left(1+\dot{u}^2\right)+\left(2\dot{v}^2 - u^2\right)$$

The equation of motion, using langrangian (L) for $q = u$,

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}}\right) - \frac{\partial L}{\partial u} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(2v^2\dot{u}\right) - (-2u) = 0$$

$$\Rightarrow \quad 2\left[v^2\ddot{u} + 2v\dot{v}\dot{u}\right] + 2u = 0$$

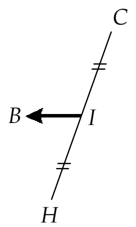
$$\Rightarrow \quad 2v^2\ddot{u} + 4v\dot{v}\dot{u} + 2u = 0$$

5. **(c)**

Force in member AH should be zero, as the AH is corner member with only two members connected to each other at 90°. Hence, in both members AH and GH force is zero.

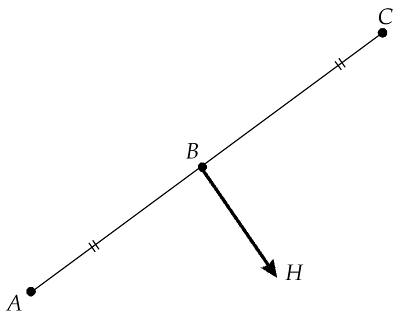
6. **(b)**

As in the given truss,



$$F_{CI} = F_{IH}$$

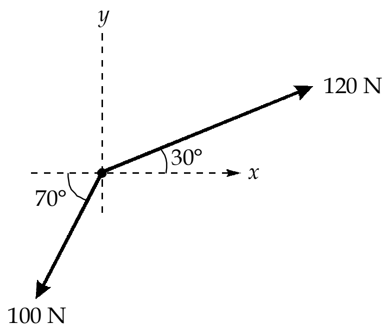
Therefore, $F_{BI} = 0$ (zero force member)



$$F_{CB} = F_{AB}$$

Therefore, $F_{BH} = 0$ (also zero force member)

7. (c)



$$\Sigma F_x = 120\cos 30° - 100\cos 70° = 69.72 \text{ N}$$

$$\Sigma F_y = 120\sin 30° - 100\sin 70° = -33.969 \text{ N}$$

$$\text{Resultant force, } R = \sqrt{(F_x)^2 + (F_y)^2}$$

$$= \sqrt{(69.72)^2 + (-33.969)^2} = 77.55 \text{ N} \simeq 78 \text{ N}$$

8. (b)

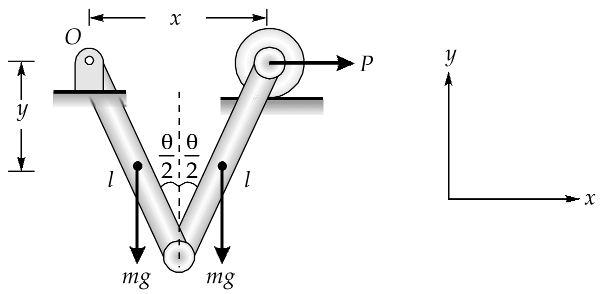
Conservation of linear momentum,

$$(m_A v_A)_i + (m_B \times v_B)_i = (m_A + m_B)v_f$$

$$15000 \times 1.5 + (-12000 \times 0.75) = 27000 \times v_f$$

$$v_f = 0.5 \text{ m/s}$$

9. (c)



$$x = 2l\sin\frac{\theta}{2}$$

$$\partial x = l\cos\frac{\theta}{2}\partial\theta$$

$$y = -\frac{l}{2}\cos\frac{\theta}{2}$$

$$\partial y = +\frac{l}{4}\sin\frac{\theta}{2}\partial\theta$$

$$+P(\partial x) + (-2mg)\cdot\partial y = 0$$

$$P\left(l\cos\frac{\theta}{2}\partial\theta\right) - 2mg\left(\frac{l}{4}\sin\frac{\theta}{2}\partial\theta\right) = 0$$

$$Pl\cos\frac{\theta}{2}\partial\theta = 2mg\times\frac{l}{4}\sin\frac{\theta}{2}\partial\theta$$

$$\tan\frac{\theta}{2} = \frac{2P}{mg}$$

$$\theta = 2\tan^{-1}\left(\frac{2P}{mg}\right)$$

10. (a)

Using energy conservation

$$(PE)_A + (KE)_A = (PE)_B + (KE)_B \qquad \text{[No energy loss due to smooth surface]}$$

$$mg\times 12+\frac{1}{2}m\times 0^2 = mg\times 8+\frac{1}{2}mV_B^2$$

$$\frac{mg\times 4\times 2}{m} = V_B^2$$

$$V_B = \sqrt{8\times 9.81} = 8.859 \text{ m/s}$$

11. (b)

As per given information,

$$m = 30 \text{ kg}; \qquad r = 0.2 \text{ m}$$

$$\omega = 20 \text{ rad/s}; \qquad T = 5 \text{ Nm}$$

$$F = 10 \text{ N}$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2}\times 30\times 0.2^2 = 0.6 \text{ kg.m}^2$$

Let the disk rotate an angle of θ rad.

From work energy principle

$$T\cdot\theta + F\times r\cdot\theta = \frac{1}{2}\times I\times\omega^2 \qquad [\because \text{ Workdone = change in energy}]$$

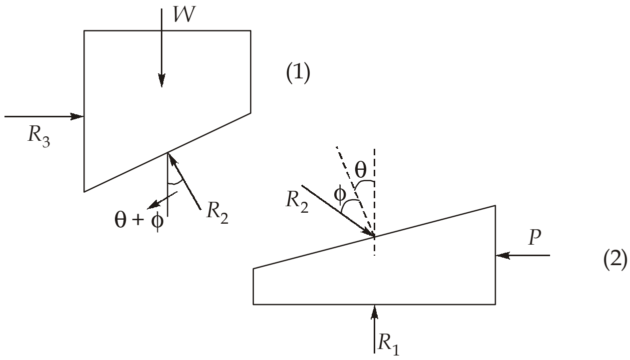
$$5\cdot\theta + 10\times 0.2\times\theta = \frac{1}{2}\times 0.6\times(20)^2$$

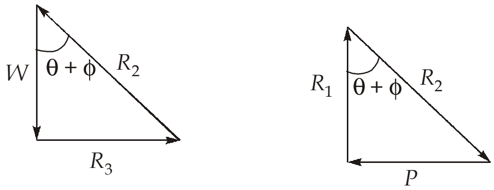
$$7\cdot\theta = 120$$

$$\theta = 17.14 \text{ rad}$$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{17.14}{2\pi} = 2.73 \text{ rev}$$

12. (c)





$$\cos(\theta + \phi) = \frac{W}{R_2}, \quad \sin(\theta + \phi) = \frac{P}{R_2}$$

$$\frac{P}{W} = \tan(\theta + \phi)$$

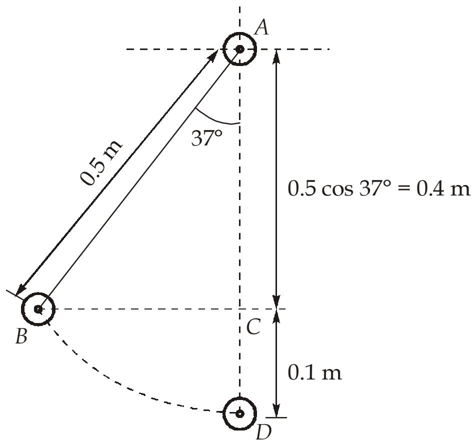
$$P = W \tan(\theta + \phi)$$

$$= 500 \tan(\theta + \phi)$$

$$\phi = \tan^{-1} 0.20 = 11.309$$

$$P = 500 \tan(15 + 11.309) = 247.21 \text{ N}$$

13. (c)



$$CD = AD - AC$$

$$= 0.5 - 0.5 \cos 37°$$

$$= 0.1 \text{ m}$$

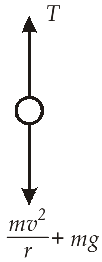
Applying energy conservation between B and D

$$mg \times CD = \frac{1}{2}mv^2 \qquad \because V = \text{Velocity at } D$$

Let $g = 10\ m/s^2$

$$10 \times 0.1 = 0.5v^2$$

$$v^2 = 2$$



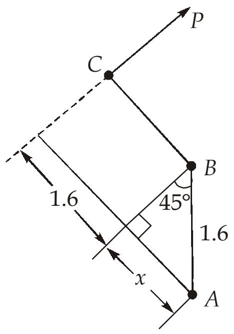
At point '*D*'

$$\text{Tension, } T = \frac{mv^2}{r} + mg$$

$$T = m\left(\frac{v^2}{r} + g\right) = 0.1\left(\frac{2}{0.5} + 10\right)$$

$$T = 1.4\ N$$

14. (a)



$$\text{Moment of P about A} = P \times (1.6 + x)$$

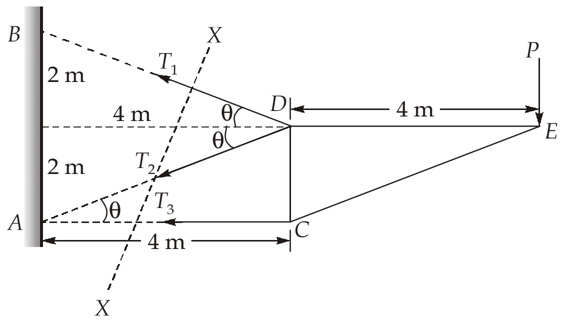
$$\sin 45° = \frac{x}{1.6}$$

$$x = 1.6 \sin 45° = 1.1314\ m$$

$$M_A = 30 \times (1.6 + 1.1314) = 81.9\ N\text{-}m$$

15. (d)

At the section X-X through 3 members of truss as shown below:



Let force along bar BD is T_1

Now taking moment balance about A.

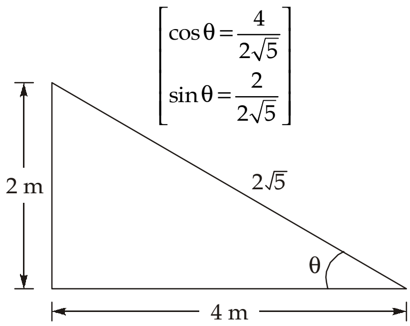
$\Rightarrow \Sigma M_A = 0$

$$\Rightarrow \quad P \times 8 = T_1 \cos\theta \times 2 + T_1 \sin\theta \times 4$$

$$= T_1\left[\frac{4\times 2}{2\sqrt{5}} + \frac{2\times 4}{2\sqrt{5}}\right]$$

$$P \times 8 = T_1 \times \frac{8}{\sqrt{5}}$$

$$T_1 = P\sqrt{5} = 125\times\sqrt{5} = 279.5\text{ kN (T)}$$



16. (b)

Given: $W = 6000$ N, $r = \frac{50}{2} = 25$ mm, $p = 10$ mm

$$\tan\theta = \frac{p}{\pi d} = \frac{10}{\pi\times 50} = 0.06366$$

$$\theta = 3.643°$$

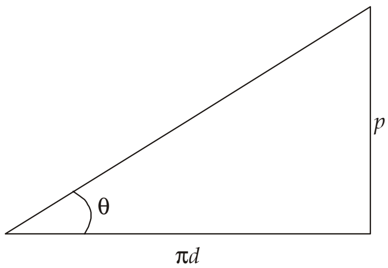
$$\tan\phi = \mu = 0.05$$

$$\phi = \tan^{-1}0.05 = 2.862°$$

$$T = Wr\tan(\theta + \phi)$$

$$= 6000 \times 0.025 \tan(3.643 + 2.862)$$

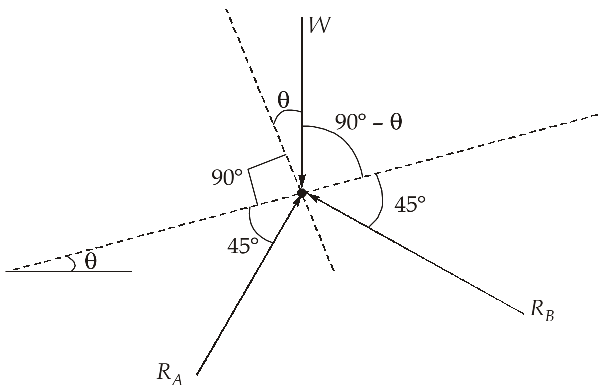
$$= 17.1036 \text{ Nm}$$



tangential force at end of lever effort

$$= \frac{17.1036}{0.3} = 57.01\text{ N}$$

17. (a)



Apply Lami's theorem:

$$\frac{R_A}{\sin(135° - \theta)} = \frac{R_B}{\sin(135° + \theta)}$$

Given: $R_A = 2R_B$

$$\Rightarrow \quad 2\sin(135 + \theta) = \sin(135 - \theta)$$

$$\Rightarrow 2[\sin135° \cos\theta + \cos135° \sin\theta] = [\sin135° \cos\theta - \cos135° \sin\theta]$$

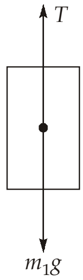
$$\Rightarrow \quad 3\cos135° \sin\theta = -\sin135° \cos\theta$$

$$\Rightarrow \qquad \tan\theta = -\frac{1}{3}\tan 135°$$

$$\theta = 18.43°$$

18. (b)

Cylinder

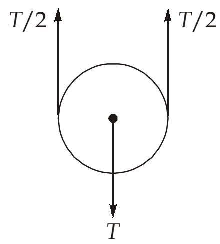


From Newton's first law,

$$m_1g - T = 0$$

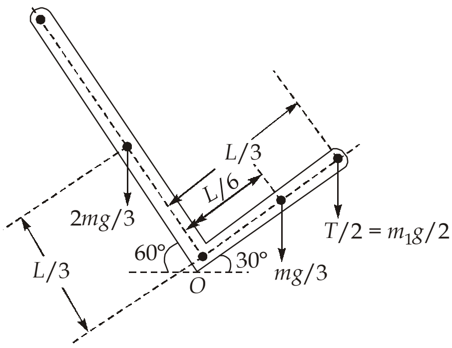
$$T = m_1g$$

Pulley



$$\frac{T}{2} = \frac{m_1g}{2}$$

To cause loss of contact at A, reaction at A will be zero.



$\Sigma M_o = 0$

$$\frac{2mg}{3}\times\frac{L}{3}\cos 60° - \frac{mg}{3}\times\frac{L}{6}\cos 30° - \frac{T}{2}\times\frac{L}{3}\cos 30° = 0$$

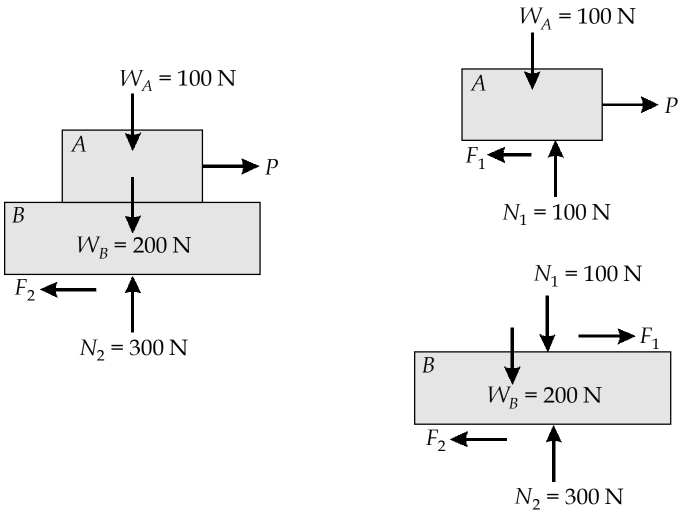
$$\Rightarrow \qquad \frac{2mg}{9}L\cos 60° = \frac{m}{18}gL\cos 30° + \frac{m_1g}{2}\times\frac{L}{3}\cos 30°$$

$$\Rightarrow \qquad \frac{2m}{9}\cos 60° = \frac{m}{18}\cos 30° + \frac{m_1}{6}\cos 30°$$

$$m_1 = 0.436 \text{ m}$$

19. (c)

Given,



Assume impending sliding at surface, 1

$$F_1 = (F_1)_{max} = (\mu_s)N_1 = 0.2 \times 100 = 20 \text{ N}$$

From FBD of block 'A'

$\Sigma F_x = 0,$ $\quad P - F_1 = 0$

$$P = 20 \text{ N}$$

Assume impending sliding at surface force 2 only

$$F_2 = (F_2)_{max} = \mu_s N_2 = 0.1 \times 300 = 30 \text{ N}$$

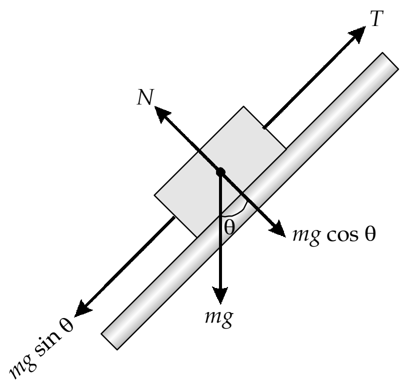
From FDB,

$$P - F_2 = 0$$

$$P = 30 \text{ N}$$

P = 20 N will cause motion to impend at surface 1 and that P = 30 N will cause motion to impend at surface 2, therefore the largest force that can be applied without causing either block to move is P = 20 N.

20. (d)



$$T = \mu mg \cos\theta + mg \sin\theta$$

$$= 0.2 \times 10 \times \frac{1}{\sqrt{2}} + \frac{10}{\sqrt{2}}$$

$$= 1.2 \times \frac{10}{\sqrt{2}} = 8.485 \text{ N}$$

$$P_{min} = 8T = 8 \times 4.855 = 67.88$$

21. (b)

$$\text{Velocity, } v = (3t^2 - 6t) \text{ m/s}$$

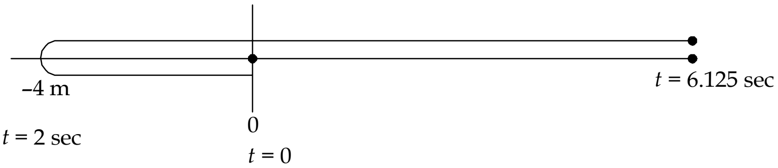
$$\frac{ds}{dt} = v$$

$$s = (t^3 - 3t^2) \text{ m}$$

At $t = 0$, $s = 0$

$t = 2$, $s = -4$ m

$t = 3.5$, $s = 6.125$



So, total distance travelled by the particle

$$s = 4 + 4 + 6.125 = 14.125 \text{ m}$$

22. (c)

Moment about the point 'C'

$$\text{(Vector method), } M_C = \vec{r} \times \vec{F}$$

$$\text{Force vector, } \vec{F} = 500\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

$$= 500\left(\frac{2\hat{i} - 4\hat{j} + 3k}{\sqrt{(2)^2 + (-4)^2 + (3)^2}}\right)$$

$$= 92.847\left(2\hat{i} - 4\hat{j} + 3k\right)$$

$$\text{Position vector, } r_{CA} = -2\hat{i} - 0\hat{j} + 0\hat{k}$$

$$M_C = \vec{r}_{CA} \times \vec{F}$$

$$= \left(-2\hat{i}\right)[92.847\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)]$$

$$M_C = 92.847\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 0 \\ 2 & -4 & 3 \end{vmatrix}$$

$$= 92.847\left(6\hat{j} + 8\hat{k}\right)$$

$$= 557.086\hat{j} + 742.776\hat{k}$$

$$\text{Magnitude, } M_C = \sqrt{(557.086)^2 + (742.776)^2}$$

$$M_C = 928.47 \text{ Nm}$$

23. (a)

Let momentum, $P = at^2 + bt + c$

force, $F = \frac{dP}{dt}$ at $t = 0$ is 80 N

$$F = 2a \times 0 + b$$
$$80 = b$$
$\Rightarrow$ $b = 80$

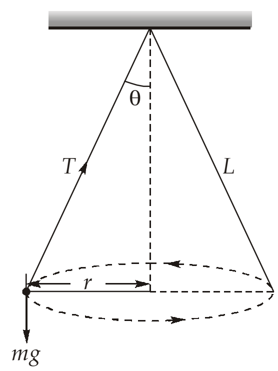
Now, at $t = 2$ sec

$$F = 2a \times 2 + 80$$
$$\frac{480-80}{4} = a$$
$$a = 100$$
$$F = 2at + 80$$

Now, $F = 200t + 80$

$$\text{Acceleration} = \frac{F}{m} = \frac{200t+80}{5} = 40t + 16$$
$$\text{Acceleration} = \frac{dv}{dt}$$
$$dv = (40t + 16)dt$$
$$\int_0^v dv = \int_0^5 (40t+16)\,dt$$
$$v - 0 = \left[40\times\frac{t^2}{2}+16t\right]_0^5$$
$$v = 40\times\frac{5^2}{2}+16\times 5$$
$$v = 25 \times 20 + 80$$
$$v = 580 \text{ m/s}$$

24. (b)



$$\sin\theta = \frac{r}{L}$$

(a) Tension T along the string
(b) The weight mg vertically downwards.
In radial direction,

$$T\sin\theta = \frac{mv^2}{r}$$

In vertical direction, $T\cos\theta = mg$

Equation (i) and (ii)

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{(rg\tan\theta)} = \sqrt{(rg)\times\left(\frac{r}{\sqrt{L^2-r^2}}\right)}$$

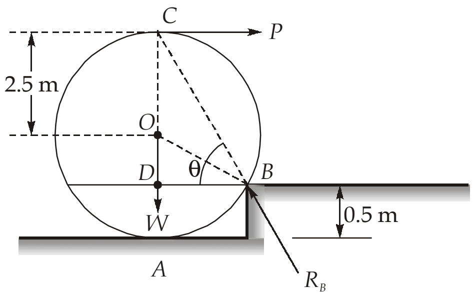
$$v = \frac{r\sqrt{g}}{(L^2-r^2)^{1/4}}$$

By equation (ii), $$T = \frac{mg}{\cos\theta} = \frac{mgL}{(\sqrt{L^2-r^2})}$$

$$T = \frac{mgL}{\left(L^2-r^2\right)^{1/2}}$$

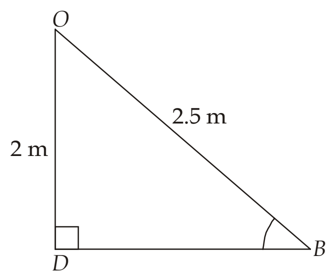
25. **(b)**

For a body under three forces to be in equilibrium, these forces must be coplanar and concurrent. So, force P, weight of cylinder W and reaction at $B(R_B)$ passes through the same point that's the upper most point of cylinder.



In ΔOBD,

$$OB^2 = OD^2 + DB^2$$
$$2.5^2 = (2.5 - 0.5)^2 + DB^2$$
$$DB^2 = 2.25$$
$$DB = 1.5 \text{ m}$$



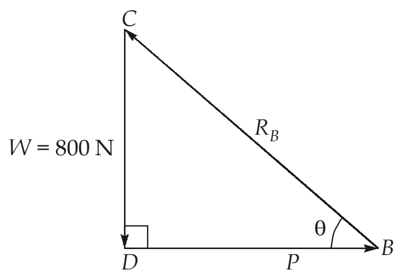
In ΔBCD, $$\tan\theta = \frac{2.5+2}{1.5} = \frac{4.5}{1.5} = 3$$

$$\theta = 71.565°$$

$$\frac{W}{P} = \tan\theta$$

$$\Rightarrow \quad \tan\ 71.565° = \frac{800}{P}$$

$$\Rightarrow \quad P = \frac{800}{3} = 266.67 \text{ N}$$



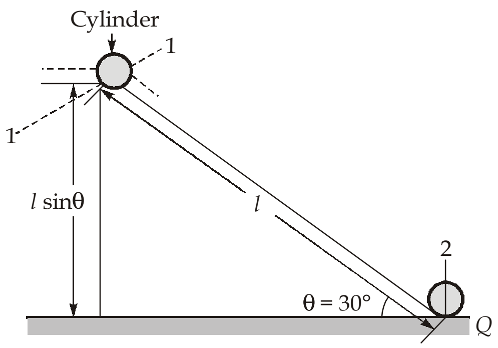
Alternate: Taking moment about B,

$$P \times CD = W \times DB$$

$$P = \frac{800\times 1.5}{4.5}$$

$$P = \frac{800}{3} = 266.67 \text{ N}$$

26. (d)



Cylinder rolls without slipping with angular speed $\omega = \frac{v}{r}$ about its axis.

By energy conservation,

$$(KE)_{1-1} + (PE)_{1-1} = (KE)_{2-2} + (PE)_{2-2}$$

$$0 + mgl\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0$$

$$mgl\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$mgl\sin\theta = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{4}{3}gl\sin\theta} = \sqrt{\frac{4}{3}\times 10\times 2\times \sin 30^\circ}$$

$$v = 3.65 \text{ m/s}$$

27. (c)

$$F \;\alpha\; v^3$$

$$F = kv^3$$

$$a = \frac{F}{m} = \frac{kv^3}{m}$$

We know that,

$$a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{kv^3}{m}$$

$$\frac{1}{v^3}dv = \left(\frac{k}{m}\right)dt$$

$$\left[\frac{v^{-2}}{-2}\right]_u^v = \left(\frac{k}{m}\right)\int_0^t dt$$

$$-\frac{1}{2}\left[\frac{1}{v^2}\right]_u^v = \left(\frac{k}{m}\right)[t]_0^t$$

$$-\frac{1}{2}\left[\frac{u^2 - v^2}{v^2u^2}\right] = \left(\frac{k}{m}\right)t$$

$$t \;\alpha\; \left(\frac{u^2 - v^2}{v^2u^2}\right)$$

28. (a)

Given data: m = 8.4 kg, ω = 6.9 rad/s, F = 6.6 N, M = 59 Nm, L = 4 m, $\omega_{\theta = 90°}$ = ?

Moment of Inertia of rod about hinge O,

$$I_O = \frac{mL^2}{12} + m\times\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3} = \frac{8.4\times4\times4}{3} = 44.8 \text{ kg m}^2.$$

By conservation of energy:

$$mgh_{cm} + (M + F \times L)\Delta\theta = \frac{1}{2}I_0\left(\omega_1^2 - \omega_0^2\right)$$

$$\text{For } \theta = 90°,\ h_{cm} = 2 \text{ m}$$

$$(8.4\times9.81\times2) + (59 + 6.6\times4)\times\frac{\pi}{2} = \frac{1}{2}\times(44.8)\left[\omega_1^2 - 6.9^2\right]$$

$$\frac{298.954\times2}{44.8} = \omega_1^2 - 6.9^2$$

$$\omega_1^2 = 60.9562$$

$$\omega_1 = 7.807 \text{ rad/s}$$

29. (c)

$$\text{Normal reaction, } N = \text{mg}\cos30°$$

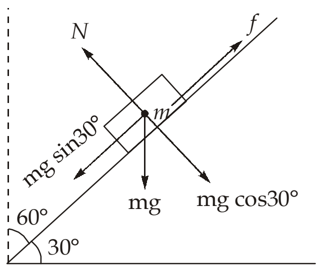
$$\text{friction force, } f = \mu N$$

$$f = \mu \text{mg}\cos30°$$

Now, resultant force in downward direction will be = (mg sin30° – μmg cos30°)

We known that,

$$a = \frac{F}{m} = \frac{m(g\sin30° - \mu g\cos30°)}{m}$$



$$\frac{dv}{dt} = g(0.5 - 0.866 \times 3x)$$

$$v\frac{dv}{dx} = g(0.5 - 0.866 \times 3x)$$

Given, $v_i = 0,\ v_f = 0$

Integrating, $$\int_0^0 v\,dv = g\int_0^x (0.5 - 0.866\times3x)\,dx$$

$$\left[\frac{v^2}{2}\right]_0^0 = g\left[0.5x - \frac{0.866}{2}\times3x^2\right]_0^x$$

$$0 = 0.5x - 0.433 \times 3x^2$$

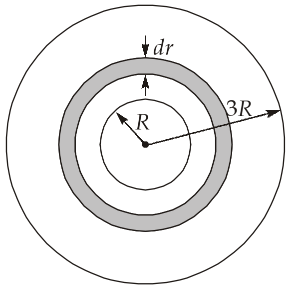
$$0.433 \times 3x^2 = 0.5x$$

$$x = \frac{0.5}{0.433\times3}$$

$$x = 0.3849 \text{ m}$$

30. (a)

Given that, $m = 20$ kg, $r_i = 0.1$ m $= R$, $r_0 = 0.3$ m $= 3R$



Cross-section area $= \pi[(3R)^2 - R^2] = 8\pi R^2$

Mass of small strip of thickness 'dr' at a distance 'r' from centre.

$$dm = \frac{m \times 2\pi r dr}{8\pi R^2}$$

We know that,
$$I = \int_R^{3R} dm \cdot r^2$$

$$I = \int_R^{3R} \frac{m \times 2\pi r dr \times r^2}{8\pi R^2} = \frac{m}{4R^2} \times \left[\frac{r^4}{4}\right]_R^{3R}$$

$$= \frac{m}{4 \times 4R^2} \times \left(81R^4 - R^4\right) = \frac{20}{4} mR^2 = 5\, mR^2$$

$$(\text{K.E.})_{total} = (\text{K.E.})_{rotational} + (\text{K.E.})_{translation}$$

$$\text{K.E.} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} m \times 9R^2\omega^2 + \frac{1}{2} I\omega^2 = \frac{9}{2} mR^2\omega^2 + \frac{5}{2} mR^2\omega^2$$

$$\text{K.E.} = 7\, mR^2\omega^2$$

$$62.5 = 7 \times 20 \times (0.1)^2 \times \omega^2$$

$$\omega = 6.6815 \text{ rad/s}$$

$$v = 3R\omega = 3 \times 0.1 \times 6.6815$$

Velocity of centre of mass, $v =$ 2 m/s

Note: for rolling without slipping, $v = r\omega$, $v = 3R\omega$

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