

Q. No. 1 to Q. No. 10 carry 1 mark each

- **Q.1** Given a max-heap tree of size \sqrt{n} , what is the worst case time complexity to delete the root element of heap tree?
 - (a) O(1)
 - (b) *O*(log n)
 - (c) *O*(*n*)
 - (d) None of these
- **Q.2** Consider a weighted complete graph on vertex set $\{V_1, V_2, \ldots, V_n\}$ such that the weight of the edge (V_i, V_j) is 2|i-j|. The weight of a minimum spanning tree using prim's algorithm is
 - (a) *n*−1
 - (b) 2*n*-2
 - (c) n/2
 - (d) *n*²
- **Q.3** Which of the following can be the best algorithm(s) for all pair shortest path problem?
 - I. 'V' invocations of Dijkshtra algorithm $\Rightarrow O(VE \log V)$
 - II. V' invocations of Bellman-ford algorithm \Rightarrow O(V^2E)
 - III. '1' invocations of Floyd-Warshall algorithm $\Rightarrow O(V^{\beta})$

where V = Number of vertices and E = Number of edges.

- (a) I only
- (b) I, II and III only
- (c) III and II only
- (d) III only
- **Q.4** Consider X[1, ..., n] and Y[1, ..., n] be two arrays each containing n-numbers both of which are already sorted. What is the time complexity to find median by combining the two arrays?
 - (a) O(log n) (b) O(n)
 - (c) $O(n \log n)$ (d) $O(\log \log n)$
- Q.5 What is the worst case time complexity to construct an AVL tree from an array which satisfies the min heap property ?
 - (a) $O(n \log n)$ (b) $O(n^2)$
 - (c) $O(n^2 \log n)$ (d) O(n)
- **Q.6** Which of the following procedure results same output as Dijkstra's algorithm on unweighted graph with '*n*' vertices ?

- (a) Breadth first search(b) Depth first search
- (c) Kruskal algorithm (d) Prim's algorithm
- Q.7 What is worst case time complexity to find all the nodes at height "v/2" from a particular node in a graph having 'v' vertices and 'e' edges ?
 (a) O(v²)
 (b) O(v log e)
 - (c) O(v + e) (d) $O(v^2 \log e)$
- Q.8 Which of the following is true ?
 - (a) The running time of Radix Sort is effectively independent of whether the input is already sorted.
 - (b) Changing the RELAX function to update is d[v] ≥ d[u] + w (u, v) (instead of strictly greater than) may produce shortest path, but will not effect the correctness of Bellman-Ford algorithms outputs.
 - (c) Both (a) and (b)
 - (d) None of these
- Q.9 Match List-I (Dynamic algorithm) with List-II (Average case running time) and select the correct answer using the codes given below the lists:
 - List-I
 - A. Matrix chain multiplication
 - **B**. Travelling salesman problem
 - C. 0/1 knapsack
 - D. Fibonacci series List-II
 - **1**. O(*mn*)
 - **2**. $O(n^3)$
 - **3**. O(*n*ⁿ)
 - **4**. O(*n*)
 - Codes:

	Α	В	С	D
(a)	1	3	2	4
(b)	1	3	3	2
(c)	2	3	3	2
(d)	2	3	1	4

- **Q.10** Let *G*(*V*, *E*) be an undirected graph with positive edge weights. What is the worst case time complexity to find minimum spanning tree using Kruskal algorithm is implemented using array data structure ?
 - (a) $O(|E| + |V| \log |V|)$ (b) $O(|V| \log |V|)$
 - (c) $O(|V|^2)$ (d) $O(|V| \log^2 |V|)$

Q. No. 11 to Q. No. 30 carry 2 marks each

- Q.11 Consider the following statements:
 - S1: An undirected graph is said to be Hamiltonian if it has a cycle containing all the vertices. Any DFS tree on a Hamiltonian graph must have depth V-1.
 - S2: For all weighted graphs, for any two vertices 's' and 't', Bellman-Ford at 's' will always return a shortest path to 't'.

Which of the following is true?

- (a) S_1 is true, S_2 is false
- (b) S₁ is false, S₂ is true
 (d) Both the statements are false (c) Both the statements are correct
- Q.12 We are given two strings: String S of length n and string T of length m for the LCS problem, we have produced the following exponential time recursive program.

LCM (S, n, T, m)

if (n == 0 | m == 0) return 0 if (S[n] = T[m]) result t = 1 + LCS (S, n - 1, T, m - 1); else result = max (LCS (S, n - 1, T, m), LCS (S, n, T, m - 1)); return result;

}

{

Then the number of times that LCS (S, 1, T, 1) is recursively called equals _____

(a) $\binom{n+m-2}{m-1}$ (b) $\binom{n+m+1}{m-2}$ (d) $\binom{n+m}{m-1}$ (c) $\binom{n+m+2}{m}$

Q.13 Which of the following statement is false?

- (a) The depth of any DFS tree rooted at a vertex is at least as much as the depth of any BFS tree rooted at the same vertex.
- (b) If all the edges in a graph have distinct weight then the shortest path between two vertices is unique.
- (c) For a directed graph, the absence of back edges in a DFS tree means graph as no cycle.
- (d) BFS takes $O(V^2)$ time in a graph G(V, E) if graph is represented with an adjacency matrix.

Q.14 A m-ary tree has n leaf nodes. What is the expression for total number of nodes in the tree?

(a)
$$\frac{mn+1}{m-1}$$
 (b) $\frac{mn+2n-1}{m-1}$

(c)
$$\frac{mn-1}{m}$$
 (d) $\frac{mn-1}{m-1}$

- Q.15 Consider the following statements :
 - **S**₁: Given a minheap of height log*n*, it will take O(log*n*) time to delete the root element of the heap.
 - S_2 : Time complexity of fractional Knapsack using greedy algorithm is $O(n^2)$.
 - S_3 : Selection sort works on greedy approach.
 - Which of the following option is correct?
 - (a) S_1 and S_2 are correct (b) S_1 and S_3 are correct
 - (d) Only S_3 is correct (c) Only S_1 is correct
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Q.16 Consider the following statements

- Let *T* be a minimum spanning tree of a graph *G*. Then for any two vertices *u* and *v* the path from *u* to *v* in *T* is the shortest path from *u* to *v* in the graph *G*.
- II. Suppose that average edge weight for a graph *G* is A_{avg} . Then the minimum spanning tree of *G* will have weight at most $(n-1) A_{avg}$, where *n* is number of vertices in graph *G*.

Which of the above statements are true?

- (a) Only I (b) Only II
- (c) Both I and II (d) None of these
- Q.17 Consider the graph given below:



Suppose you are running Dijkstra's algorithm starting from source vertex 'D'. What will be the order of vertices which will be relaxed using Dijkastra's algorithm?

- (a) DFBHEGCA (b) DFBHGECA
- (c) DFBHEGAC (d) DFBEHGCA
- Q.18 Consider two vertices 'a' and 'b' that are simultaneously on the FIFO queue at same point during the execution of breadth first search from 's' in an undirected graph. Consider the following statements:
 - S₁: The number of edges on the shortest path between 's' and 'a' is atmost one more than the number of edges on the shortest path between 's' and 'b'.
 - S₂: The number of edges on the shortest path between 's' and 'a' is atleast one less than the number of edges on the shortest path between 's' and 'b'.

 S_3 : There is a path between 'a' and 'b'.

Which of the following is true?

- (a) S_1 only (b) S_1 and S_2 only
- (c) S_1 and S_3 only (d) S_1 , S_2 and S_3

Q.19 Consider the following function.



What is the worst case running time of the function *f* for any positive value of *n*?

(a)	O(1)	(b)	O(n)
(C)	$O(n^2)$	(d)	O(<i>n</i> ³)

Q.20 Consider the already built heap (Max-Heap).

24, 20, 20, 7, 8, 9, 19, 1, 2 After root deletion operation, which of the following Max Heaps are valid?

- **1.** 20, 8, 20, 7, 2, 9, 19, 1
- **2.** 20, 20, 19, 7, 8, 9, 2, 1
- **3.** 20, 20, 9, 7, 8, 2, 19, 1
- **4.** 20, 7, 20, 2, 8, 9, 19, 1
- (a) Only 3 and 4 (b) Only 2 and 3
- (c) Only 1 and 2 (d) Only 1 and 4

Q.21 Consider the following graph G.



Modified BFS traversal on G applied as follows

- Assume starting vertex is 'a'.
- At any level (breadth), vertices are visited in alphabetic order.
- In above graph G, after visiting vertex 'a', the order of vertices visited in G are *c*, *d* and *e*.
- It works same as BFS except the restriction on order of vertices visited.

Which of the following is not a cross edge during given BFS traversal on G?

(a) { <i>d</i> , <i>c</i> }	(b)	{ <i>d</i> , <i>b</i>
-----------------------------	-----	-----------------------

(C)	{ <i>e</i> , <i>b</i> }	(d)	{ <i>e</i> , <i>f</i> }

Q.22 Consider the following array: 27, 12, 45, 31, 23, 26 (in the given order) Find number of inversions in the array.

(a) 6 (b) 8

(d) None of these (c) 10

Q.23 Consider the following "Max-heapify" algorithm. Array has size atleast n and $1 \le i \le n$. After applying the Max-heapify rooted at A[i], the result will be the subtree of A[1, ...,n] rooted at A[i] is max heap. [Assume that except root A[i], all its children satisfies heap property] Max-heapify (int A[], int n, int i)

```
{
     int p, m;
     p = i;
     hile (X)
     ł
           if (Y && Z)
           m = 2p + 1;
           else m = 2p;
           if (A[p] < A[m])
                 Swap (A[p], A[m]);
                 p = m;
           else
     return;
     {
}
```

Find missing statements at X, Y and Zrespectively to apply the heapify for subtree rooted at A[i].

- (a) $p \le n$, $(2p + 1) \ge n$, A[2p + 1] > A[2p]
- (b) $2p \le n$, $(2p + 1) \le n$, A[2p + 1] > A[2p]
- (c) $2p \le n$, $(2p+1) \ge n$, A[2p+1] < A[2p]
- (d) $p \le n$, $(2p + 1) \le n$, A[2p + 1] < A[2p]
- Q.24 Consider the following knapsack problem with capacity W = 8

Item	Profit	Weight
I_1	13	1
I ₂	8	5
I ₃	7	3
I_4	3	4

Which of the following item is not selected in the optimal solution of 0/1 knapsack problem?

- (a) I_1 only (b) I_2 only (d) None of these (c) I_3 only
- Q.25 Consider the following graph



Which of the following is NOT a depth first search traversal of the above graph?

(a) ACFHEBDG (b) ACFHGEBD

(c) ABDHFCGE (d) ABDEHFCG

Q.26 Consider the following direct graph



What is the adjacently matrix after floyd's algorithm applied on the above graph to find all pairs shortest paths?

		Α	В	С			Α	В	С	?
	Α	2	8	5]		Α	2	8	5	
(a)	В	3	0	8	(b)	В	3	11	8	
	С	_5	2	0]		С	_5	2	0	
		Α	В	С			Α	В	С	
	Α	0	8	5]		Α	0	8	5	
(C)	В	3	0	8	(d)	В	3	11	8	
	С	5	2	0		С	5	2	0	

Q.27 Consider the following problems.

- Longest common subsequence 1.
- Optimal Binary search tree 2.
- 0/1 knapsack problem З.

Matrix chain multiplication 4.

Which of the above problems can be solved using dynamic programming?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1, 3 and 4 only
- (d) 1, 2, 3 and 4

Q.28 The pseudo code for insertion sort is presented as a procedure called **Insertion-sort**, which takes array A[1...*n*] as parameter containing sequence of n elements to be sorted. Find the missing statements at X and Y respectively.

Insertion-sort (A)

```
{
      for j \leftarrow 2 to length [A]
      {
             key \leftarrow A[/]
             i = X
             While (i > 0 \&\& A[i] > \text{key})
                   A[i + 1] \leftarrow A[i]
                   i = Y
             A [i + 1] \leftarrow key.
      }
 }
                                                γ
                               (b)
(a)
               i+1
                                        +1
                                               i + 1
```

Q.29 Consider the following functions.

i-1

f(n) = 3 n + 100, n > 0g(n) = n + log n, n > 0

Which of the following is correct for larger values of n?

(d)

- (a) $f(n) = \Omega(g(n))$ and $f(n) \neq O(g(n))$
- (b) g(n) = O(f(n)) and $f(n) \neq \Omega(g(n))$

(c)
$$f(n) = \theta(g(n))$$

(C)

(d) None of these

Q.30 Suppose that you are running Dijkstra's algorithm on the edge-weighted digraph below, starting from vertex A.



The table gives 'Distance' and 'Parent' entry of each vertex after vertex E has been deleted from the priority queue and relaxed.

Vertex	Distance	Parent
A	0	NULL
В	2	A
С	13	F
D	23	A
Ε	11	F
F	7	В
G	36	F
Н	19	Ε

What could be the possible values of *x* and y?

(a) x = 11, y = 7
(b) x = 8, y= 12

- (c) x = 6, y = 10
- (d) x = 10, y = 14

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1.	(b)	7.	(c)	13.	(b)	19.	(c)	25.	(d)
_									
2.	(b)	8.	(a)	14.	(d)	20.	(c)	26.	(c)
2. 3.	(b) (d)	8. 9.	(a) (d)	14. 15.	(d) (b)	20. 21.	(c) (b)	26. 27.	(c) (d)
2. 3. 4.	(b) (d) (a)	8. 9. 10.	(a) (d) (c)	14. 15. 16.	(d) (b) (d)	20. 21. 22.	(c) (b) (b)	26. 27. 28.	(c) (d) (c)
2. 3. 4. 5.	(b) (d) (a) (a)	8. 9. 10. 11.	(a) (d) (c) (d)	14. 15. 16. 17.	(d) (b) (d) (a)	 20. 21. 22. 23. 	(c) (b) (b) (b)	26. 27. 28. 29.	(c) (d) (c) (c)

DETAILED EXPLANATIONS

1. (b)

Size $m \Rightarrow$ To delete root: $O(\log n)$

Size
$$\sqrt{n} \Rightarrow$$
 To delete root: $O(\log \sqrt{n}) = O(\log n^{1/2}) = O(\frac{1}{2}\log n) = O(\log n)$

2. (b)

Prim's algorithm will pick up the edge with least weight for a particular node, [provided it does not form a cycle] weight of edge (V_{i-1}, V_i) or $(V_i, V_{i-1}) = 1$

: MST will be

$$\underbrace{V_1}_{1} \underbrace{V_2}_{1} \underbrace{V_3}_{1} \underbrace{V_4} \cdots \underbrace{V_n}_{n}$$

 \therefore Total edge weight = 2 × (*n* - 1) = 2*n* - 2.

3. (d)

In worst case, i.e., dense graph, $E = O(V^2)$

- :. Time complexities of
- **1.** '*V* invocations of Dijkshtra algo = $O(V^3 \log V)$
- **2.** '*V* invocations of Bellman-ford algo = $O(V^4)$
- **3.** '1' invocation of Floyd-warshall algo = $O(V^3)$
- : Floyd-warshall is best.

4. (a)

Since two arrays are already in sorted order. We first select median of X and Y array in O(1) time lets say 'a' and 'b' respectively. Compare X[a] and Y[b] if X[a] < = Y(b) then we apply above procedure to X[a,...n], Y[b,...,n] till their are ≤ 2 elements left in each array.

So, total time = log(n) + O(1) = log(n).

5. (a)

We need to delete element from the array and insert into the AVL tree. Deletion from the array satisfying heap property will take $O(\log n)$ time for each element.

Total time = $O(n \log n) + O(n \log n)$

 $= O(n \log n)$

Note: $O(n \log n)$ time to satisfy the heap property of the array for every deletion and $O(n \log n)$ time to construct the AVL tree.

6. (a)

Dijkstra's algorithm will output same as breadth first search on graph and will take O(m + n) time.

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7. (c)

Apply BFS algorithm on graph will give number of nodes at height "v/2" from particular node it will take O(v + e) time.

8. (a)

- In Radix Sort, all input orderings give the worst-case running time, the running time does not depend on the order of the inputs in any significant way.
- The parent pointers may not lead back to the source node if a zero length cycle exists.
 In the example below, relaxing the (s, a) edge will set d[a] = 1 and π[a] = s. Then, relaxing the (a, a) edge will set d[a] = 1 and π[a] = a

Following the π pointers from *t* will no longer give a depth to *s*, so the algorithm is incorrect.

9. (d)

A. Matrix chain multiplication : (n^3)

- **B.** Travelling salesman problem : (*nⁿ*)
- **C.** 0/1 knapsack : (*mn*)
- **D.** Fibonacci series : O(*n*)

10. (c)

Finding minimum edge each time will take (v) time, this will done for (V-1) time. Will take worst case time

=
$$O(|V|) \times (V-1)$$

= $O(|V|^2)$

11. (d)

- 1. If the graph has a Hamiltonian cycle, then it is possible depending on the graph ordering of the graph, that DFS will find that cycle and that the DFS tree will have depth V-1. However, DFS is not guaranteed to find that cycle. If DFS does not find that cycle then the depth of the DFS tree will be less than V-1.
- 2. False because, if the graph contains a negative-weight cycle, then no shortest path exist.

13. (b)

Even if no two edges have the same weight, there could be two paths having the same weight.



To find the shortest path between s and t.

$$s \rightarrow p \rightarrow t \Rightarrow 8$$

$$s \rightarrow q \rightarrow l \Rightarrow s$$

So, there are two distinct paths.

14. (d)

Number of leaf nodes = n

Let internal nodes be K

 \therefore Total nodes = K + n

For m-ary tree, number of leaf nodes with K internal nodes = (m - 1) K + 1

∴ (m – 1) K + 1 = n

$$\therefore \qquad \mathsf{K} = \frac{n-1}{m-1}$$

:. Total number of nodes

$$= \frac{n-1}{m-1} + n = \frac{n-1+n(m-1)}{m-1} = \frac{n-1+nm-n}{m-1} = \frac{nm-1}{m-1}$$

15. (b)

Considering each statement :

- S_1 : It takes O(1) time to delete the node but, the heap property gets destroyed, so to resume the property of a heap, time taken will be O(log *n*).
- S_2 : Fractional Knapsack using greedy approach takes $O(n \log n)$ time complexity.
- $\overline{S_3}$:Selection sort algorithm is designed using Greddy approach.
- 16. (d)
 - I. MST contain *ac* and *bc* but not contain *ab*, which is the shortest path between *a* and *b*



II. We may be forced to select the edges with weight much higher than average



Average weight = $\frac{50+6}{7} = 8$

Expected MST weight = $4 \times 8 = 32$ Actual MST weight = 50 + 6 = 56

17. (a)

Starting vertex is D

	Α	В	С	D	Ε	F	G	Н
D	∞	8	∞	0 Nill	8	8	8	8
F	16	17 D	13 D	-	8	3 D	8	8 D
В	16	7 E	13 D	_	8	_	8	8 D
Н	16	_	13 D	_	∞	_	12 B	8 D
E	16	-	13 D	_	10 H	_	12 B	
G	16	-	13 D	-	—	_	12 B	
С	16	_	_	_	-	_	_	_
A	_	_	_	-	_	_	_	_

So the order of relaxed the vertices by using Dijkastra's algorithm is DFBHEGCA.

18. (c)

Consider a graph



During the breadth first traversal of the graph. The status of the queue will be as follows :



19. (c)

$$f(n) = \sum_{i=0}^{99} \sum_{j=0}^{n-1} \left(\sum_{k=0}^{j-1} 1 + \sum_{m=0}^{i-1} 1 \right) = \sum_{i=0}^{99} \sum_{j=0}^{n-1} \sum_{k=0}^{j-1} 1 + \sum_{i=0}^{99} \sum_{j=0}^{n-1} \sum_{m=0}^{i-1} 1 = O(n^2) + O(n) = O(n^2)$$

20. (c)

The given heap is



When we delete root element in max heap we replace root with last node in the tree. Now, while Max-Heapify down, we select the maximum of the two children and swap it with root.

Here the two children are both 20 and 20 and we can swap with either of them and the only two possible heaps after deletion and Heapifying are

- **1.** 20, 8, 20, 7, 2, 9, 19, 1
- **2.** 20, 20, 19, 7, 8, 9, 2, 1

21. (b)

Given BFS traversal on G is :

Tree edges are: $\{a, d\}, \{a, c\}, \{a, e\}, \{d, b\}, \{c, f\}$ **Cross edges are:** $\{d, c\}, \{e, b\}, \{e, f\}, \{f, b\}, \{f, e\}$ $\Rightarrow \{d, b\}$ is not a cross edge.



22. (b)



A has following inversions:

 $(i, j) \Rightarrow (1, 2), (1, 5), (1, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6)$

If i < j and A[i] > A[j] then (i, j) is inversion pair [for $1 \le \{i, j\} \le n$]

23. (b)



{If
$$A[2p + 1] > A[2p]$$
 then $m = 2p + 1$
else $m = 2p$ }
X is : $2p \le n$
Y is : $(2p + 1) \le n$
Z is : $A[2p + 1] > A[2p]$

24. (b)

W = 8 (capacity)

Feasible solutions:

(i) $\{I_1, I_3, I_4\},\$ (ii) $\{I_2, I_3\}$ Profit of $\{I_1, I_3, I_4\}$ is 23 profit of $\{I_2, I_3\}$ is 15 Optimal solution is $\{I_1, I_3, I_4\}$ with capacity of 8 and maximum profit 23 produced. $\therefore I_2$ is not selected in the solution.

25. (d)

AB<u>DE H</u> FGH is not DFS traversal

Traversal from node D to E fail the DFS algorithm.

26. (c)



 $\begin{array}{cccc} A & B & C \\ A \begin{bmatrix} 2 & 8 & 5 \\ 3 & \infty & \infty \\ C \begin{bmatrix} \infty & 2 & \infty \end{bmatrix} \end{array}$ is adjacency matrix for above graph.

27. (d)

Longest common subsequence, longest increasing subsequence, sum of subsets, optimal BST, 0/1 knapsack, matrix chain multiplication, Travelling salesperson, Balanced partition, Fibonacci sequence, Multistage graph problems are solved by using dynamic programming.

28. (c)

Only j-1 at X and i-1 at Y gives the correct implementation of insertion sort.

29. (c)

 $f(n) = 3n + 100 = \theta(n)$ $g(n) = n + \log n = \theta(n)$ $f(n) = \theta(g(n)) \text{ is correct}$ 30. (b)

. ,

 \Rightarrow

	Α	В	С	D	Ε	F	G	Н
	0 Nill	8	8	8	8	8	8	~
A	-	2 A	15 A	23 A	~	~	~	~
В	-	-	15 A	23 A	17 B	7 B	8	~
F	_	_	13 F	23 A	11 F	_	36 F	y + 7 F
Given $\rightarrow E$	-	-	13 F	23 A	-	-	36 F	<i>x</i> +11 <i>E</i>

Compare above table with given table

<i>x</i> + 1	1	=	19
	x	=	8
<i>y</i> +	7	\geq	x + 11
<i>y</i> +	7	\geq	8 + 11
	y	\geq	12