CLASS TEST

Sl.: 03 SK_EC+EE_BCD_310823

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SIGNALS AND SYSTEMS

EC + EE

Date of Test : 31/08/2023

ANSWER KEY

1.	(b)	7.	(c)	13.	(c)	19.	(a)	25.	(c)
2.	(c)	8.	(c)	14.	(d)	20.	(d)	26.	(a)
3.	(d)	9.	(d)	15.	(a)	21.	(d)	27.	(c)
4.	(b)	10.	(a)	16.	(d)	22.	(b)	28.	(b)
5.	(b)	11.	(b)	17.	(d)	23.	(b)	29.	(a)
6.	(b)	12.	(a)	18.	(b)	24.	(c)	30.	(d)

DETAILED EXPLANATIONS

1. (b)

Given,
$$x(t) \xleftrightarrow{FS} c_k$$
$$x(t+a) \xleftrightarrow{FS} e^{jak\omega_0} c_k$$

Put $a = 2$

$$x(t+2) \xleftrightarrow{FS} e^{j2k\omega_0} c_k$$
$$x(-t) \xleftrightarrow{FS} c_{-k}$$
$$x(-t+2) \xleftrightarrow{FS} e^{j2k\omega_0} c_{-k}$$

2. (c)

Given,
$$\int_{-2}^{2} (t-3)\,\delta(2t+2)\,dt$$

From the property of impulse $\delta(at+b) = \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$

$$= \frac{1}{2}\int_{-2}^{2} (t-3)\,\delta(t+1)\,dt$$

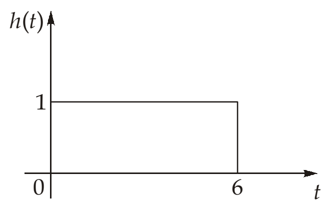
from
$$\int_{t_1}^{t_2} x(t)\delta(t-t_0)dt = x(t_0)\ ; \quad t_1 < t_0 < t_2$$

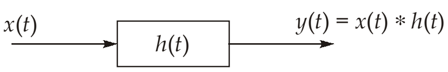
Clearly we can write,

$$= \frac{1}{2}(-1-3) = -2$$

3. (d)

Given,
$$h(t) = u(t) - u(t-6)$$





$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$y(2) = \int_{\tau=0}^{2} x(\tau)h(2-\tau)\,d\tau$$

$$= \int_{\tau=0}^{2} \left(\frac{1}{2}\tau\right) \cdot 1\, d\tau \qquad \left\{\because x(\tau) = \frac{1}{2}\tau \text{ from given diagram}\right\}$$

$$= \int_{\tau=0}^{2} \frac{1}{2}\tau\, d\tau = \left.\frac{\tau^2}{4}\right|_0^2 = 1$$

4. **(b)**

5. **(b)**

For DFT, the total number of multiplications = $N^2 = 64$

(Given, $N = 8$)

For FFT, the total number of multiplications = $\frac{N}{2}\log_2 N = \frac{8}{2}\log_2 8 = 12$

6. **(b)**

$\because x[n]$ is real and odd, the Fourier transform $X(e^{j\omega})$ will be purely Imaginary and odd function. Thus $\text{Re}\{X(e^{j\omega})\} = 0$ and the discrete time sequence corresponding to $\text{Re}\{X(e^{j\omega})\} = 0$.

7. **(c)**

Given, $\quad H(\omega) = -2j\omega$

From the definition of inverse fourier transform,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$

differentiate both sides,

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(\omega)\, e^{j\omega t} d\omega$$

$$-2\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{-2j\omega}_{H(\omega)}\ X(\omega)e^{j\omega t} d\omega$$

$\therefore$ Passing $x(t)$ through $H(w)$ is equivalent to perform $-2\frac{dx(t)}{dt}$

$\therefore \quad y(t) = -2\frac{dx(t)}{dt}$

given, $\quad x(t) = e^{jt}$

$\therefore \quad y(t) = -2\frac{d}{dt}[e^{jt}]$

$y(t) = -2je^{jt}$

8. **(c)**

From the pole-zero plot, it is shown that $r < 1$, so that the signal is a decaying signal.

9. (d)

Given impulse response,

$$h[n] = \left\{ p, \underset{\uparrow}{q}, p \right\} = pe^{j\omega} + q + pe^{-j\omega}$$

$$H(e^{j\omega}) = 2p\cos\omega + q$$

$$= q + 2p\cos(2\pi f) \quad \text{...(i)}$$

Given, $H(e^{j2\pi f}) = 0$ at $f = \frac{1}{4}$ Hz and $H(e^{j2\pi f}) = 1$ at $f = \frac{1}{8}$ Hz

From equation (i),

$$0 = q + 2p\cos\left(\frac{2\pi}{4}\right)$$

$$0 = q + 2p\cos\left(\frac{\pi}{2}\right)$$

$$\therefore \quad q = 0$$

$$1 = q + 2p\cos\left(\frac{2\pi}{8}\right)$$

$$1 = q + 2p\cos\left(\frac{\pi}{4}\right)$$

$$1 = q + 2p \cdot \frac{1}{\sqrt{2}}$$

$$\because \quad q = 0 \Rightarrow 1 = p \cdot \sqrt{2} \Rightarrow p = \frac{1}{\sqrt{2}}$$

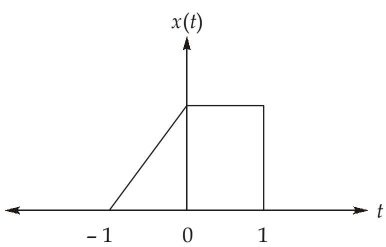
$$\therefore \quad h[n] = \left\{ 0.707, \underset{\uparrow}{0}, 0.707 \right\}$$

$$\therefore \quad \text{DC gain } H(e^{j0}) = \sum_{n=-1}^{1} h[n] = 0.707 + 0 + 0.707$$

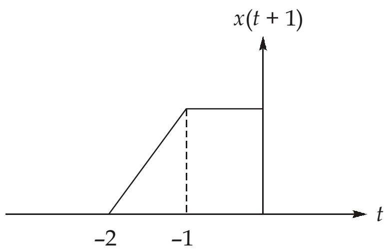
$$\therefore \quad H(e^{j0}) = 1.414$$

10. (a)

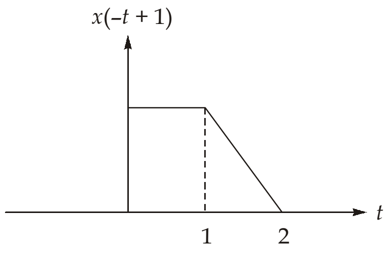
$x(t)$



By time shifting



By time reversal



11. (b)

$$X(s) - \frac{3H(s)}{s^2} = H(s)$$

$$\Rightarrow \quad X(s) = \left(1+\frac{3}{s^2}\right)H(s)$$

$$\bullet \quad 2H(s) + \frac{H(s)}{s} = Y(s)$$

$$\Rightarrow \quad \left(2+\frac{1}{s}\right)H(s) = Y(s)$$

$$\Rightarrow \quad \left(2+\frac{1}{s}\right)\frac{X(s)}{\left(1+\frac{3}{s^2}\right)} = Y(s)$$

$$\Rightarrow \quad \frac{Y(s)}{X(s)} = \frac{2+\frac{1}{s}}{\frac{3}{s^2}+1} = \frac{s+2s^2}{3+s^2}$$

$$\Rightarrow \quad \frac{d^2y(t)}{dt^2} + 3y(t) = \frac{dx(t)}{dt} + 2\frac{d^2x(t)}{dt^2}$$

12. (a)

$$x(t) = \sum_{m=-\infty}^{\infty} \delta(t-3m) + \delta(t-1-3m) - \delta(t-2-3m)$$

The period of $x(t)$ is $T = 3$. The fundamental frequency,

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

Let a_k represents the complex Fourier series coefficient for $x(t)$.

$$a_k = \frac{1}{3}\int_0^3 [\delta(t)+\delta(t-1)-\delta(t-2)]\cdot e^{\frac{-jk2\pi t}{3}}\cdot dt$$

$$a_k = \frac{1}{3}\left(1+e^{\frac{-jk2\pi}{3}} - e^{\frac{-jk4\pi}{3}}\right)$$

For $k = 3$,
$$a_3 = \frac{1}{3}\left(1+e^{-j2\pi} - e^{-j4\pi}\right) = \frac{1}{3}$$

The frequency response of the LTI system is given by,

$$H(j\omega) = e^{j\omega/4} - e^{-j\omega/4} = 2j\sin\left(\frac{\omega}{4}\right)$$

If C_k is the complex Fourier series coefficient of $y(t)$, then

$$C_k = H\left(\frac{j2\pi k}{3}\right)a_k$$

$$C_k = \left(j2\sin\frac{2\pi k}{12}\right)a_k$$

Hence,
$$C_3 = \left(j2\sin\frac{\pi}{2}\right)\times\frac{1}{3} = \frac{j2}{3}$$

13. (c)

The fourier transform can be written as:

$$X(j\omega) = |X(j\omega)|\angle X(j\omega)$$

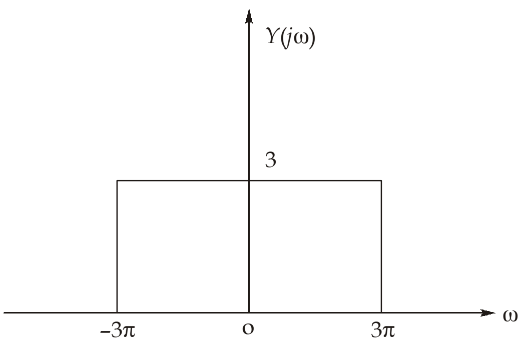
$$X(j\omega) = \begin{cases} j3\omega, & |\omega| < 3\pi \\ 0, & \text{otherwise} \end{cases}$$

Let
$$Y(j\omega) = \begin{cases} 3, & |\omega| < 3\pi \\ 0, & \text{otherwise} \end{cases}$$

$X(j\omega)$ is $j\omega$ times $Y(j\omega)$. Hence,

$$x(t) = \frac{dy(t)}{dt}$$

We have,



$$A\,\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{FT} \frac{AT\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}$$

Using duality property,
$$\frac{AT\sin\left(\frac{tT}{2}\right)}{\left(\frac{tT}{2}\right)} \xleftrightarrow{FT} 2\pi A\,\text{rect}\left(\frac{\omega}{T}\right)$$

$$\frac{A}{\pi t}\sin\left(\frac{tT}{2}\right) \xleftrightarrow{FT} A\,\text{rect}\left(\frac{\omega}{T}\right)$$

For $T = 6\pi$ and $A = 3$,

$$\frac{3}{\pi t}\sin(3\pi t) \xleftrightarrow{FT} 3\,\text{rect}\left(\frac{\omega}{6\pi}\right)$$

Hence, $$y(t) = \frac{3\sin(3\pi t)}{\pi t}$$

$\therefore$ $$x(t) = \frac{d}{dt}y(t) = \frac{3}{\pi t^2}(3\pi t\cos 3\pi t - \sin 3\pi t).$$

14. (d)

From the given data, we can write

$$X(z) = \frac{kz^2}{(z - e^{j\pi/2})(z - e^{-j\pi/2})}$$

$$= \frac{kz^2}{\left[z - \left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)\right]\left[z - \left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right)\right]}$$

$$X(z) = \frac{kz^2}{(z - j1)(z + j1)}$$

It is given, $X(1) = 1$

i.e., $$X(1) = \frac{k}{(1 - j1)(1 + j1)}$$

$$\frac{k}{1+1} = 1 \quad \Rightarrow k = 2$$

$\therefore$ $$X(z) = \frac{2z^2}{(z^2 + 1)} \quad \text{ROC is } |z| > 1$$

15. (a)

$$x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t - n)$$

Let $x(t)$ be periodic with period T.

$$x(t + T) = \sum_{n=-\infty}^{\infty} e^{-(2(t+T)-n)}u(2(t + T) - n)$$

$$= \sum_{n=-\infty}^{\infty} e^{-(2t+2T-n)}u(2t + 2T - n)$$

$$= x(t)$$

i.e. $$2T - n = -m$$

Thus, $x(t)$ is periodic if,

$$2T - n = -m$$

$$T = \frac{n-m}{2}, \text{ Thus } T_{\min} = \frac{1}{2}$$

$$x(t) = \begin{cases} \sum_{n=-\infty}^{\infty} e^{-(2t-n)}, & t > n/2 \\ 0, & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{0} \exp(-(2t-n)),\ 0 < t < \frac{1}{2}$$

$$= e^{-2t}\sum_{n=-\infty}^{0} e^{n} = e^{-2t}\sum_{n=0}^{\infty}(e^{-1})^{n}$$

$$x(t) = \frac{e^{-2t}}{1-e^{-1}}, 0 < t < \frac{1}{2}$$

$$P_X = \frac{1}{T}\int_0^T |x(t)|^2\, dt$$

$$= 2\int_0^{1/2}\left[\frac{e^{-2t}}{1-e^{-1}}\right]^2 dt = \frac{2}{(1-e^{-1})^2}\int_0^{1/2} e^{-4t}dt = \frac{1}{2(1-e^{-1})^2}\cdot(1-e^{-2})$$

$$= 1.082 \text{ W}$$

16. (d)

By using convolution property,

$$y_1(t) = \int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau$$

From the given first fact,

at $t = 5$

$$y_1(5) = \int_{-\infty}^{\infty} x_1(\tau)h(5-\tau)d\tau$$

$$y_1(5) = A\int_{5-T}^{5} x_1(\tau)d\tau = 0$$

if the lower limit is equal to 1, then the area of the triangle between $\tau = 1$ and $\tau = 3$ is 2 and cancels the area of the rectangle between $\tau = 4$ and $\tau = 5$.

Hence, the value for T should be 4.

$$y_2(t) = \int_{-\infty}^{\infty} x_2(\tau)h(t-\tau)d\tau$$

$$= A\int_{t-T}^{t} x_2(\tau)d\tau$$

$$y_2(t)\big|_{t=9} = A\int_5^9 x_2(\tau)d\tau \quad (\text{given } t = 9)$$

from the second fact, we have

$$y_2(t)\big|_{t=9} = 9 = A\int_5^9 x_2(\tau)d\tau$$

$$= A\int_5^9 \sin\left(\frac{\pi\tau}{3}\right)d\tau$$

$$= -\frac{A}{\pi/3}\cos\left(\frac{\pi\tau}{3}\right)\Bigg]_5^9$$

$$9 = \frac{9A}{2\pi}$$

$\therefore$ $A = 2\pi$

$\therefore$ The value of $A \times T = 2\pi \times 4 = 8\pi = 25.13$

17. (d)

We can rewrite the given $x(n)$ signal

$$x(n) = a^{|n|},\ 0 < a < 1$$

The z-transform of the $x(n)$ is $X(z)$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{n=-\infty}^{-1} a^{-n} z^{-n}$$

(or) $x[n] = a^n\, u(n) + a^{-n}\, u[-n-1]$

$$X(z) = \frac{1}{1-az^{-1}} - \frac{1}{1-a^{-1}z^{-1}};\ a < |z| < \frac{1}{a}$$

(or)

$$\left(\because a^{-n}u[-n-1] \overset{z}{\longleftrightarrow} \frac{-1}{1-a^{-1}z^{-1}};\ |z| < \frac{1}{a}\right)$$

$$= \frac{z}{z-a} - \frac{z}{z-\frac{1}{a}};\ a < |z| < \frac{1}{a} = \frac{z^2 - \frac{z}{a} - z^2 + az}{(z-a)\left(z-\frac{1}{a}\right)}$$

$$X(z) = \frac{\left[a-\frac{1}{a}\right]z}{(z-a)\left(z-\frac{1}{a}\right)};\ a < |z| < \frac{1}{a}$$

This z-transform has poles at $z = a$, $z = \frac{1}{a}$, and a zero at $z = 0$.

$\therefore$ None of the given pole-zero diagram represents $x(n)$.

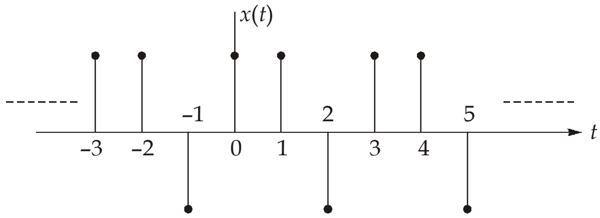
Hence option (d) is correct.

18. (b)

Given, periodic signal

$$x(t) = \sum_{m=-\infty}^{\infty} \delta(t-3m) + \delta(t-1-3m) - \delta(t-2-3m)$$

redrawing the periodic signal



Clearly the period of $x(t)$ is $T = 3$

$\therefore$ $y(t)$ is also periodic with period $T = 3$.

The fundamental frequency of $x(t)$ is,

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

Let a_k represents the fourier series coefficient of $x(t)$. Then

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-j\omega_0 kt}dt$$

$$= \frac{1}{3}\int_0^3 (\delta(t) + \delta(t-1) - \delta(t-2)\mathrm{e}^{-j\frac{2\pi}{T}kt}\,dt$$

$$a_k = \frac{1}{3}\left[1 + e^{-j\frac{2\pi}{3}k} - e^{-j\frac{2\pi}{3}k\times 2}\right]$$

at $k = 3$; $$a_3 = \frac{1}{3}\left(1 + e^{-j2\pi} - e^{-j4\pi}\right)$$

$$a_3 = \frac{1}{3}$$

The frequency response of the system is given,

$$H(j\omega) = e^{j\omega/4} - e^{-j\omega/4}$$

$$= 2j\sin\frac{\omega}{4}$$

$\therefore$ $$b_k = H\left(j\frac{2\pi}{3}k\right)a_k = \left(j2\sin\frac{2\pi}{12}k\right)a_k$$

at $k = 3$ $$b_3 = \left(j2\sin\frac{\pi}{2}\right)\frac{1}{3} = j\frac{2}{3}$$

$\therefore$ $$|b_3| = 0.66$$

19. (a)

Given, $y[n] \xleftrightarrow{DTFT} y(e^{j\omega})$

also, $$Im[y(e^{j\omega})] = 3\sin\omega + \sin 3\omega$$

We know that, $\text{Even}\{y[n]\} \xleftrightarrow{DTFT} \text{Re}\{y(e^{j\omega})\}$

$$\text{Odd}\{y[n]\} \xleftrightarrow{DTFT} jIm\{y(e^{j\omega})\}$$

$\therefore$ the inverse DTFT of $jIm\{y(e^{j\omega})\}$ is the odd part of $y[n]$.

Let $y_0[n]$

$$y_0[n] = \text{inverse DTFT}\{j3 \sin \omega + j\sin 3\omega\}$$

$$= DTFT^{-1}\left[\frac{1}{2}\left[3e^{j\omega} - 3e^{-j\omega} + e^{j3\omega} - e^{-j3\omega}\right]\right]$$

$$y_0[n] = \frac{1}{2}\{3\delta[n+1] - 3\delta[n-1] + \delta[n+3] - \delta[n-3]\}$$

Since $y[n]$ is real and causal (given)

$$y[n] = 2y_0[n]u[n] + y[0]\delta[n]$$

$$= y[0]\delta[n] - 3\delta[n-1] - \delta[n-3]$$

also given, $$y(e^{j\omega})\Big|_{\omega=\pi} = \sum_{n=-\infty}^{\infty} y[n](-1)^n$$ (by definition of DTFT)

$$3 = y(0) + 3 + 1$$

$\therefore$ $y(0) = -1$

Hence, $y[n] = -\delta[n] - 3\delta[n-1] - \delta[n-3]$

at $n = 3$

$$y[3] = -1$$

20. (d)

Given $x(t)$ is real,

i.e., $$\text{Even}\{x(t)\} = \frac{x(t)+x(-t)}{2} \xleftrightarrow{\text{FT}} \text{Re}\{X(j\omega)\}$$

given inverse Fourier Transform,

$$\text{IFT}\{\text{Re}(X(j\omega)\} = |t|e^{-|t|}$$

$\therefore$ $$\text{Even}\{x(t)\} = \frac{x(t)+x(-t)}{2} = |t|e^{-|t|}$$

also it is known that $x(t) = 0$ for $t \le 0$

This implies that $x(-t)$ is zero for $t > 0$

$\therefore$ We conclude that, $x(t) = 2|t|e^{-|t|}$ for $t \ge 0$

$\therefore$ at $t = 1$, $x(1) = 2e^{-1} = 0.736$

21. (d)

We know that, $X(K)$ is DFT of $x(n)$

$$X(K) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi nK}{N}}$$

$$= \sum_{n=0}^{5} x(n)\cdot e^{-\frac{j2\pi nK}{6}}$$

$$X(K) = \sum_{n=0}^{5} x(n)e^{-j\frac{\pi nK}{3}}$$

$$= x(0)e^{-j0} + x(1)e^{-j\pi K/3} + x(2)e^{-j2\pi K/3} + x(3)e^{-j\pi K} + x(4)e^{-j\frac{4\pi K}{3}} + x(5)e^{-j\frac{5\pi K}{3}}$$

$$= 3e^{-j0} + 2e^{-j\pi K/3} + 1\cdot e^{-j\frac{2\pi K}{3}} + 0 + 1\cdot e^{-j\frac{4\pi K}{3}} + 2e^{-j\frac{5\pi K}{3}}$$

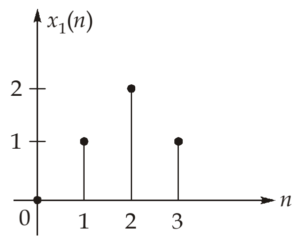
$$= 3 + 2e^{-j\pi K/3} + e^{-j2\pi K/3} + e^{j2\pi K/3} + 2e^{j\pi K/3} \qquad \left[\because e^{-j\frac{5\pi K}{3}} = e^{j\frac{\pi K}{3}}\right]$$

$$X(K) = 3 + 4\cos\frac{\pi K}{3} + 2\cos\frac{2\pi K}{3}$$

22. **(b)**

The given signal $x(n)$ can be expressed as follows:

where, $N_0 = 4$, $$x(n) = \sum_{K=0}^{\infty} x_1(n - KN_0) \qquad ...(1)$$



$x_1(n)$ can be expressed as

$$x_1(n) = \delta(n-1) + 2\delta(n-2) + \delta(n-3)$$

by taking z-transform,

$$X_1(z) = z^{-1} + 2z^{-2} + z^{-3}$$

$$X_1(z) = z^{-1}[1 + 2z^{-1} + z^{-2}]$$

$\therefore$ $$X(z) = X_1(z)\left[1 + z^{-N_0} + z^{-2N_0} + z^{-3N_0} + ...\right] \qquad \text{[from equation (1)]}$$

$$= X_1(z)\sum_{m=0}^{\infty}(z^{-N_0})^m$$

$\therefore$ $$X(z) = X_1(z)\cdot\frac{1}{1 - z^{-N_0}}; |z^{-N_0}| < 1 \longrightarrow |z| > 1$$

where, $N_0 = 4$

$\therefore$ $$X(z) = \frac{X_1(z)}{1 - z^{-4}};\quad |z| > 1$$

(or) $$X(z) = \frac{z^{-1}\left[1 + 2z^{-1} + z^{-2}\right]}{1 - z^{-4}}; |z| > 1$$

23. **(b)**

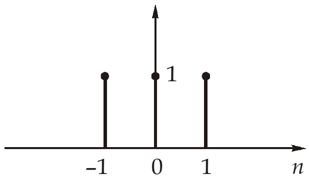
The given waveform has half wave symmetry

i.e., $$x(t) = -x\left(t \pm \frac{T}{2}\right)$$

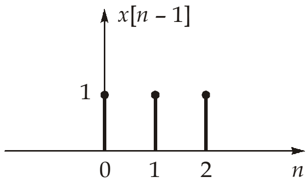
$\therefore$, a_k will be zero for even integer values of k.

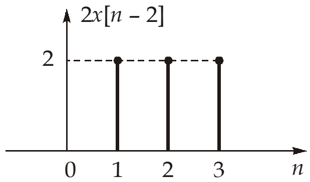
24. (c)

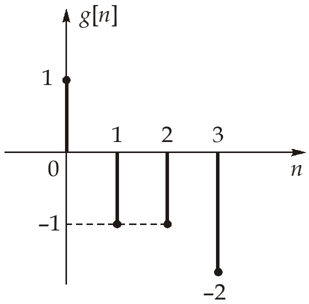
$$x[n] = \text{rect}_2[2n]$$



$\therefore$
$$g[n] = x[n] \otimes \{\delta[n-1] - 2\delta[n-2]\}$$
$$= x[n-1] - 2x[n-2]$$



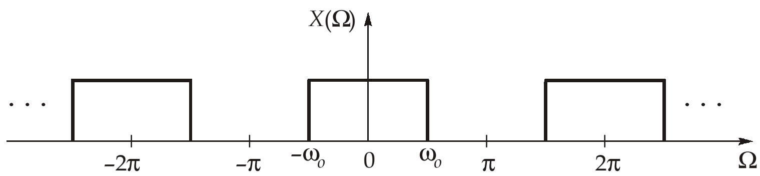




$\Rightarrow$
$$g[2] = -1 \text{ and } g[3] = -2$$

25. (c)

$$\frac{\sin n\omega_o}{\pi n} \longleftrightarrow$$



Given :
$$h(n) = \frac{\sin\left(\frac{n\pi}{4}\right)}{\pi n}$$

$\therefore$
$$H(\Omega) = \begin{cases} 1 \;; & |\Omega| < \frac{\pi}{4} \\ 0 \;; & \frac{\pi}{4} < |\Omega| < \pi \end{cases}$$

Using DTFS,
$$x(n) = \sum_{k=0}^{4} C_k e^{jk\frac{2\pi}{5}n}$$

Since, $\Omega_0 = \frac{2\pi}{N_o} = \frac{2\pi}{5}$, and the filter passes only frequencies in the range $|\Omega| \le \frac{\pi}{4}$, hence only the DC term is passed.

$$C_o = \frac{1}{N}\sum_{n=0}^{N-1} x(n) = \frac{1}{5}\sum_{n=0}^{4} x(n) = \frac{3}{5}$$

Thus, output $\quad y(n) = \frac{3}{5}$ for all n.

26. (a)

Since, $\quad e^{-j\frac{\pi}{2}} = -j$ and $e^{j\frac{\pi}{2}} = j$

$\therefore \quad H(\omega) = -j\,\text{sgn}(\omega)$

$$\text{sgn}(t) \xleftrightarrow{\text{FT}} \frac{2}{j\omega}$$

Using duality property, $\quad \frac{2}{jt} \xleftrightarrow{\text{FT}} 2\pi\,\text{sgn}(-\omega) = -2\pi\,\text{sgn}(\omega)$

or $\quad \frac{1}{\pi t} \xleftrightarrow{\text{FT}} -j\,sgn(\omega)$

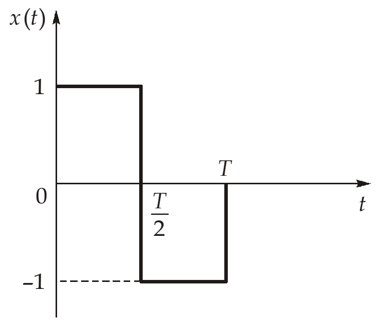
$\therefore \quad h(t) = \frac{1}{\pi t}$

27. (c)

For a periodic wave,

$$F(s) = \frac{X(s)}{1 - e^{-sT}}$$

where $X(s)$ is the Laplace transform of signal for one time period.



$$X(s) = \int_0^T x(t)\,e^{-st}dt = \int_0^{T/2} e^{-st}dt + \int_{T/2}^{T} -1e^{-st}dt$$

$$X(s) = \left.\frac{e^{-st}}{-s}\right|_0^{T/2} - \left.\frac{e^{-st}}{-s}\right|_{T/2}^{T} = -\frac{1}{s}\left[e^{-\frac{sT}{2}} - 1\right] + \frac{1}{s}\left[e^{-sT} - e^{-\frac{sT}{2}}\right]$$

$$X(s) = \frac{e^{-sT} - 2e^{-\frac{sT}{2}} + 1}{s} = \frac{\left[1 - e^{-\frac{sT}{2}}\right]^2}{s}$$

$$\therefore \quad F(s) = \frac{\left[1 - e^{-\frac{sT}{2}}\right]^2}{s(1 - e^{-sT})} = \frac{\left[1 - e^{-\frac{sT}{2}}\right]^2}{s\left[1 - e^{-\frac{sT}{2}}\right]\left[1 + e^{-\frac{sT}{2}}\right]}$$

$$F(s) = \frac{1 - e^{-sT/2}}{s(1 + e^{-sT/2})}$$

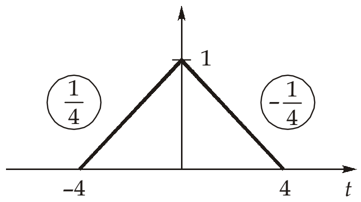
28. (b)

$$X(\omega) = \frac{\sin^2 2\omega}{\omega^2}\cos 2\omega = \left(\frac{\sin 2\omega}{\omega}\right)^2 \cos 2\omega$$

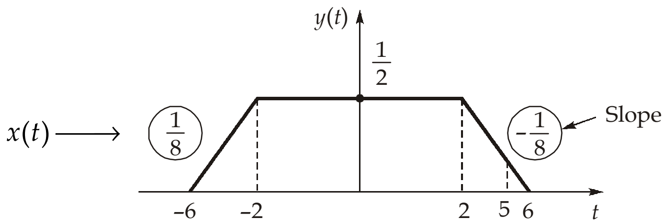
$$X(\omega) = 4S_a^2(2\omega)\cos 2\omega$$

Let $$G(\omega) = 4S_a^2(2\omega)$$

$\therefore$ $g(t) =$



$\therefore$ $$x(t) = \frac{g(t-2)+g(t+2)}{2}$$

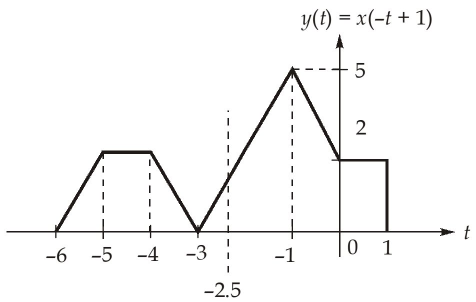


$\therefore$ $$x(t)\big|_{t=5} = \frac{1}{8} = 0.125$$

29. (a)

Let, $y(t) = x(-t+1)$.

$$\int_{-\infty}^{\infty} x(-t+1)\delta'(t+2.5)dt = \int_{-\infty}^{\infty} y(t)\delta'(t+2.5)dt = -y'(t)\big|_{t=-2.5}$$



$$\int_{-\infty}^{\infty} x(-t+1)\delta'(t+2.5)dt = -y'(t)\big|_{t=-2.5} = -(\text{slope of } y(t) \text{ at } t=-2.5)$$

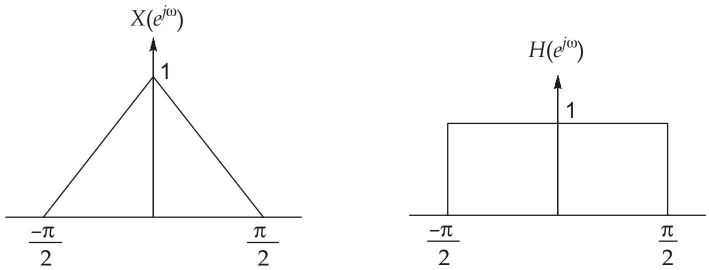
$$= -\left(\frac{5}{2}\right) = -2.50$$

30. (d)

$$y[n] = x[n] * h[n]$$

$$= \left(\frac{\sin\frac{\pi}{4}n}{\pi n}\right)^2 * \left(\frac{\sin\omega_c n}{\pi n}\right)$$

Convolution in time domain specifies multiplication in frequency domain thus, $x[n]$ has a Fourier transform equal to a triangular pulse with width $\frac{\pi}{2}$ and the width of the filter $H(e^{j\omega})$ should also have a limit of $\frac{\pi}{2}$ to have $x[n] = y[n]$.



$$\therefore \quad \omega_c = \frac{\pi}{2}$$

$$2\pi f_c = \frac{\pi}{2}$$

$$f_c = \frac{1}{4} = 0.25 \text{ Hz}$$

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