CLASS TEST

S.No. : 02 SK_ABCDFGHI_3923

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ENGINEERING MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 03/09/2023

ANSWER KEY

1.	(b)	7.	(b)	13.	(d)	19.	(a)	25.	(c)
2.	(c)	8.	(b)	14.	(d)	20.	(a)	26.	(c)
3.	(b)	9.	(d)	15.	(a)	21.	(a)	27.	(b)
4.	(b)	10.	(b)	16.	(b)	22.	(b)	28.	(b)
5.	(a)	11.	(a)	17.	(b)	23.	(d)	29.	(b)
6.	(d)	12.	(c)	18.	(d)	24.	(d)	30.	(a)

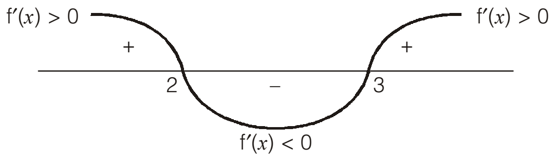
DETAILED EXPLANATIONS

1. (b)

$$f'(x) = 6x^2 - 30x + 36$$
$$= 6(x-2)(x-3)$$

So, $f'(x) > 0$ when $x < 2$ and also when $x > 3$. $f(x)$ is increasing in $]-\infty, 2[\; \cup \;]3, \infty[$.

OR, by Wavy-Curve Method



2. (c)

$$A + A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ $$2\cos\alpha = 1$$

$\Rightarrow$ $$\alpha = \frac{\pi}{3}$$

3. (b)

A is skew-symmetric,

$\Rightarrow$ $$A = -A^T$$

Now, $$(A \cdot A)^T = A^T \cdot A^T = (-A) \cdot (-A) = A \cdot A$$

$\therefore$ $A \cdot A$ is a symmetric matrix.

4. (b)

$$\lambda = np = \frac{1}{100} \times 100 = 1$$

$$P(X > 2) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X = 0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = e^{-\lambda}$$

$$P(X = 1) = \frac{e^{-\lambda}\lambda'}{1!} = e^{-\lambda} \cdot \lambda$$

$$P(X > 2) = 1 - e^{-1}(2) = \frac{1-2}{e} = \frac{e-2}{e}$$

5. (a)

Given
$$P = \int_0^1 x e^x dx$$
$$= \left[x\int e^x\,dx\right]_0^1 - \int_0^1\left[\frac{d}{dx}(x)\int e^x\,dx\right]dx = \left[xe^x\right]_0^1 - \int_0^1 (1)e^x\,dx$$
$$= (e^1 - 0) - \left[e^x\right]_0^1 = e^1 - [e^1 - e^0] = e - e + 1 = 1$$

6. (d)

$$I = \int_0^{\pi/4} \log\left(\frac{\sin x}{\cos x}\right) dx = \int_0^{\pi/4} [\log(\sin x)dx - \log(\cos x)dx]$$
$$= \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right) dx - \int_0^{\pi/2} \log(\cos x)dx$$
$$I = 0$$

7. (b)

$$|A - \lambda I| = 0$$
$$\left|\begin{bmatrix} 1-\lambda & \sin x \\ \sin x & 1-\lambda \end{bmatrix}\right| = 0$$
$$(1-\lambda)^2 - \sin^2 x = 0$$
$$1 + \lambda^2 - 2\lambda - \sin^2 x = 0$$
$$\lambda^2 - 2\lambda + \cos^2 x = 0$$
$$\lambda = \frac{2 \pm \sqrt{4 - 4\cos^2 x}}{2}$$
$$\lambda = 1 \pm \sin x$$

8. (b)

$$\frac{\partial f}{\partial x} = 2 - 2x \qquad \frac{\partial f}{\partial y} = 2 - 2y$$
$$r = \frac{\partial^2 f}{\partial x^2} = -2 \qquad t = \frac{\partial^2 f}{\partial y^2} = -2, \qquad s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

finding stationary points,
$$\frac{\partial f}{\partial x} = 2 - 2x = 0$$
$\Rightarrow x = 1$
$$\frac{\partial f}{\partial y} = 2 - 2y = 0$$
$\Rightarrow y = 1$

at the stationary point (1, 1)
$$rt - s^2 = (-2)(-2) - 0 = 4 > 0$$
So, $f(x, y)$ is maxima at (1, 1)

Maximum value of $f(x, y) = 2 + 2 + 2 - 1 - 1$
$$= 4$$

9. (d)

The constant term in any characteristic polynomial is always $|A|$.

So, $|A| = -\frac{1}{4}$ since constant term of $p(\lambda)$ is $-\frac{1}{4}$.

10. (b)

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + z = 0$$

$$[A : B] = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & -11 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$,

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

Rank of $[A : B] = 3$

Rank of $[A] = 3 =$ Rank of $[A : B] =$ number of unknowns

So, unique soluton exists

11. (a)

Total possible outcomes = $^{52}C_2 = 1326$

Favourable outcomes = Drawing any spade apart from king of spades along with any king left in pack + Drawing king of spades with any three kings left in pack

Note: It is necessary that spade and king's card should be different. So in 2nd case, when king of spade's is drawn it is considered as a spade.

$\therefore$ Favourable outcomes = $^{12}C_1 \times {}^4C_1 + {}^1C_1 \times {}^3C_1 = 51$

$$\text{Probability} = \frac{51}{1326} = \frac{1}{26}$$

12. (c)

$$\lim_{x\to\infty}\left(\frac{x}{2+x}\right)^{2x} = \lim_{x\to\infty}\left(\frac{2+x}{x}\right)^{-2x}$$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{-2x} = \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}(-4)} \qquad \because 2x = \frac{x}{2}(-4)$$

$$= e^{-4} \left(\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right)$$

13. (d)

$$a_{ij} = \begin{cases} i; & i = j \\ 0; & i \neq j \end{cases}$$

$$\Rightarrow \quad [A] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & n \end{bmatrix}$$

Determinant of all n eigen value of A

$= $ Product of diagonal elements $= 1 \times 2 \times \dots \times n = n!$

14. (d)

15. (a)

$[A : B]$

$$\begin{bmatrix} 1 & 2 & -2 & 0 & : & 0 \\ 2 & -1 & -1 & 0 & : & 0 \\ 1 & 2 & -1 & 0 & : & 0 \\ 4 & -1 & -1 & 3 & : & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_2, \quad R_3 \to (R_3 - R_1)$

$$= \begin{bmatrix} 1 & 2 & -2 & 0 & : & 0 \\ 2 & -1 & -1 & 0 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 \\ 2 & 0 & 0 & 3 & : & 0 \end{bmatrix}$$

$R_1 \to \frac{1}{3}(R_1 + 2R_2 + 4R_3)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 \\ 2 & -1 & -1 & 0 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 \\ 2 & 0 & 0 & 3 & : & 0 \end{bmatrix}$$

$R_2 \to (2R_1 - R_2 - R_3), \quad R_4 \to \frac{1}{3}(R_4 - 2R_1)$

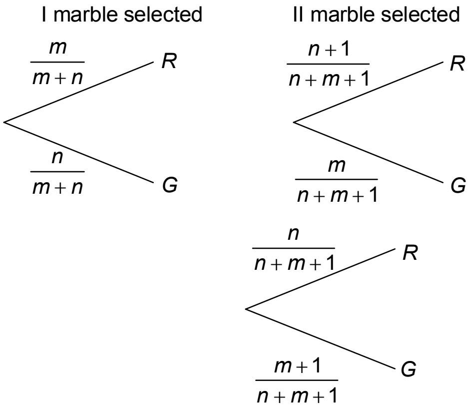
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 \\ 0 & 1 & 0 & 0 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 \\ 0 & 0 & 0 & 1 & : & 0 \end{bmatrix}$$

$\rho(A : B) = \rho(A) = 4 =$ number of variables

$\Rightarrow$ System is consistent with trivial solution.

16. (b)

The tree diagram for problem is



$$p(R) = \frac{m}{m+n}\times\frac{n\times 1}{n+m+1}+\frac{n}{m+n}\times\frac{n}{n+m+1}$$

$$= \frac{m(n+1)+n^2}{(m+n)(m+n+1)}$$

17. (b)

$$\lim_{x\to 0}\frac{e^{ax}-e^{-ax}}{\log(1+bx)} = \lim_{x\to 0}\frac{(e^{ax}-e^{-ax})\times 2ax\times b}{2ax\times bx\times\log(1+bx)}$$

$$= \lim_{x\to 0}\left(\frac{e^{ax}-e^{-ax}}{2ax}\right)\times\lim_{x\to 0}\frac{bx}{\log(1+bx)}\left(\frac{2a}{b}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sinh ax}{ax}\right)\lim_{x\to 0}\frac{bx}{\log(1+bx)}\left(\frac{2a}{b}\right)$$

$$= 1\times 1\times\frac{2a}{b} = \frac{2a}{b}$$

18. (d)

$$y = -\int\frac{1-\sin x-1}{1-\sin x}dx$$

$$= -\int 1\cdot dx+\int\frac{1}{1-\sin x}dx$$

$$y = -x+\int\frac{dx}{1-\sin x}$$

$$\int\frac{dx}{1-\sin x} = \int\frac{1+\sin x\,dx}{(1-\sin x)(1+\sin x)} = \int\frac{(1+\sin x)}{(1-\sin^2 x)}dx$$

$$= \int\frac{1+\sin x}{\cos^2 x}dx$$

$$= \int\sec^2 x\,dx+\int\sec x\tan x\,dx$$

$$= \tan x+\sec x+C$$

$$y = -x+\tan x+\sec x+C$$

19. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx+6k)dx = 1$$

$$\left.\frac{kx^2}{2}\right|_0^2 + 2kx|_2^4 + \left.\left(\frac{-kx^2}{2}+6kx\right)\right|_4^6 = 1$$

$$\frac{k}{2}(2^2-0)+2k(4-2)-\frac{k}{2}(6^2-4^2)+6k(6-4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2dx + \int_2^4 \frac{1}{4}xdx + \int_4^6\left(-\frac{1}{8}x^2+\frac{3}{4}x\right)dx$$

$$= \left.\frac{1}{8}\frac{x^3}{3}\right|_0^2 + \left.\frac{1}{4}\frac{x^2}{2}\right|_2^4 - \left.\frac{1}{8}\frac{x^3}{3}\right|_4^6 + \left.\frac{3}{4}\frac{x^2}{2}\right|_4^6$$

$$= \frac{1}{3}+\frac{3}{2}-\frac{19}{3}+\frac{15}{2} = 3$$

20. (a)

$$|x-2| = \begin{cases} -(x-2)\,; & x<2 \\ (x-2)\,; & x>2 \end{cases}$$

$$\int_1^3 \frac{|x-2|}{x}dx = \int_1^2 \frac{-(x-2)}{x}dx + \int_2^3 \frac{x-2}{x}dx$$

$$= \int_1^2\left(-1+\frac{2}{x}\right)dx + \int_2^3\left(1-\frac{2}{x}\right)dx = -(2-1)+(2\,ln x)_1^2+(x)_2^3-2(ln x)_2^3$$

$$= 2ln2 - 2ln\frac{3}{2} = 2ln\frac{2}{\frac{3}{2}} = 2ln\frac{4}{3}$$

$$= 0.575$$

21. (a)

To obtain maximum value of $f(x)$, first $f'(x)$ should be equated to zero.

$\Rightarrow \quad f'(x) = 6x^2 - 6x - 36 = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad f'(x) = 0 \quad$ at $x = 3$ and -2

Now, $\quad f''(x) = 12x - 6$

$f''(3) = 30 > 0$

at $x = 3$, there is local minima

and $\quad f''(2) = -30 < 0$

$\therefore$ at $x = -2$, a local maxima is observed.

22. (b)

$$\lambda_1^2+\lambda_2^2+\lambda_1\lambda_2 = \lambda_1^2+\lambda_2^2+2\lambda_1\lambda_2-\lambda_1\lambda_2$$

$$= (\lambda_1+\lambda_2)^2-\lambda_1\lambda_2$$

Sum of eigen values, $\lambda_1 + \lambda_2$ = trace of matrix

= sum of diagonal elements

$$= 1-\frac{1}{3}=\frac{2}{3}$$

Products of eigen values, $\lambda_1\lambda_2$ = determinant of matrix

$$= 1\left(-\frac{1}{3}\right)-(-1)\left(\frac{4}{9}\right)$$

$$= \frac{1}{9}$$

$$\therefore \quad (\lambda_1+\lambda_2)^2-\lambda_1\lambda_2 = \left(\frac{2}{3}\right)^2-\frac{1}{9}$$

$$= \frac{4}{9}-\frac{1}{9}=\frac{3}{9}=0.33$$

23. (d)

$6(13 \times 11 - 4 \times 37) - 3(32 \times 11 - 10 \times 37) + 7(32 \times 4 - 10 \times 13)$

$= -30 + 54 - 14$

$= 10$

24. (d)

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 7\lambda^2 + 36 = 0$$

since $\lambda = 3$ is root of the equation

$$(\lambda-3)(\lambda^2-4\lambda-12) = 0$$

$$(\lambda-3)(\lambda+2)(\lambda-6) = 0$$

highest eigen value = 6

$$(A-\lambda I)X = 0$$

for $\lambda = 6$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-5x_1 + x_2 + 3x_3 = 0, \quad x_1 - x_2 + x_3 = 0, \quad 3x_1 + x_2 - 5x_3 = 0$

$$\frac{x_1}{4}=\frac{x_2}{8}=\frac{x_3}{4} \text{ or } \frac{x_1}{1}=\frac{x_2}{2}=\frac{x_3}{1}$$

so eigen vector is $[1, 2, 1]^T$

25. (c)

Using Crout's method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$l_{11} = 2 \qquad l_{11} u_{12} = 4$$

$$u_{12} = \frac{4}{2} = 2$$

$$l_{21} = 6 \qquad l_{21} u_{12} + l_{22} = 3$$

$$6 \times 2 + l_{22} = 3$$

$$l_{22} = 3 - 12$$

$$l_{22} = -9$$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

26. (c)

$$A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$

$$\alpha = a^2 + b^2, \beta = 2ab$$

27. (b)

Consider $n = 3$

Then $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

and $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix}$ $\begin{matrix} R_3 \leftarrow 3R_1 - R_3 \\ R_2 \leftarrow 2R_1 - R_2 \end{matrix}$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

This if $n = 3$ then Rank (A) = 1.

28. (b)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x-2) = (2-2) = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3-8) = (2^3-8) = 0$$

Also $f(2) = 2-2 = 0$

Thus $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\therefore$ f is continuous at $x = 2$

$$f'(x) = \begin{cases} 3x^2 & 2 < x < \infty \\ 1 & -\infty < x \le 2 \end{cases}$$

and $Lf'(2) = 1$ and $Rf'(2) = 12$

$\therefore$ f is not differentiable at $x = 2$.

29. (b)

$$|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$$

$$|\text{adj}(\text{adj } A^2)| = |A^2|^{(n-1)^2}$$

$$= |A^2|^{(3-1)^2} = |A|^{2\times(4)}$$

$$= |A|^8$$

30. (a)

Let $A = \lim_{x\to\pi/2} \dfrac{\cos x}{\left(x-\dfrac{\pi}{2}\right)^3}$

By putting $\left(x-\dfrac{\pi}{2}\right) = t$

when $x \to \dfrac{\pi}{2}$, $t \to 0$

then, $A = \lim_{t\to 0} \dfrac{\cos\left(\dfrac{\pi}{2}+t\right)}{t^3}$

$$= \lim_{t\to 0} \frac{-\sin t}{t^3} = \lim_{t\to 0}(-1)\frac{\sin t}{t}\cdot\frac{1}{t^2}$$

$$= (-1).1.\frac{1}{0} = -\infty$$

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