



MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Bhubaneswar | Kolkata

Web: www.madeeasy.in | E-mail: info@madeeasy.in | Ph: 011-45124612

POWER SYSTEMS-2

ELECTRICAL ENGINEERING

Date of Test : 31/08/2023

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (c) | 7. (a) | 13. (b) | 19. (a) | 25. (c) |
| 2. (c) | 8. (d) | 14. (c) | 20. (c) | 26. (b) |
| 3. (b) | 9. (b) | 15. (a) | 21. (a) | 27. (d) |
| 4. (a) | 10. (b) | 16. (b) | 22. (b) | 28. (b) |
| 5. (d) | 11. (c) | 17. (c) | 23. (b) | 29. (d) |
| 6. (a) | 12. (c) | 18. (a) | 24. (b) | 30. (a) |

DETAILED EXPLANATIONS

1. (c)

The rating of the machine, $G = 100 \text{ MVA}$

Inertia constant, $H = 10 \text{ MJ/MVA}$

Accelerating power, $P_a = 90 \text{ MW} - 60 \text{ MW} = 30 \text{ MW}$

Angular momentum, $M = \frac{GH}{180f} = \frac{100 \times 10}{180 \times 50} = \frac{1}{9} \text{ MJ-s/electrical degree}$

Acceleration, $\alpha = \frac{P_a}{M} = \frac{30}{1/9} = 270 \text{ elec. degree/s}^2$

2. (c)

Assuming one-generator bus as slack bus

Number of slack bus-1

Number of PQ bus-2

Number of PV bus-1

Number of simultaneous equation to be solved is 2 (No. of PQ bus) + No. of PV bus
 $= 2 \times 2 + 1 = 5$

3. (b)

$$Y_{13} = j40$$

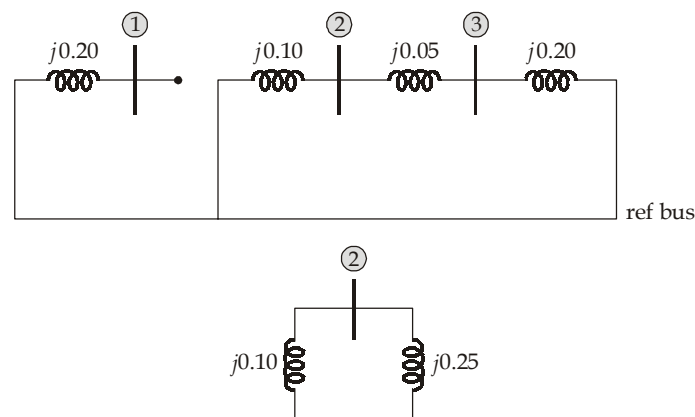
$$y_{13} = -j40$$

$$\frac{1}{Z_{13}} = -j40$$

$$Z_{13} = 0.025 j \text{ p.u.}$$

4. (a)

The zero sequence network of the above single line diagram is as shown below,



$$Z_0 = (j0.10) \parallel (j0.25)$$

$$= \frac{0.10j \times 0.25j}{0.10j + 0.25j} = 0.0714j$$

\therefore The zero sequence driving point reactance of bus-2 is 0.0714.

5. (d)

$$I_f = \frac{\sqrt{3}E_a}{X_1 + X_2} = \frac{\sqrt{3}}{0.2 + 0.2} = 4.33 \text{ p.u.}$$

$$I_f = 4.33 \times \frac{25 \times 10^3}{\sqrt{3} \times 11} = 5.68 \text{ kA}$$

6. (a)

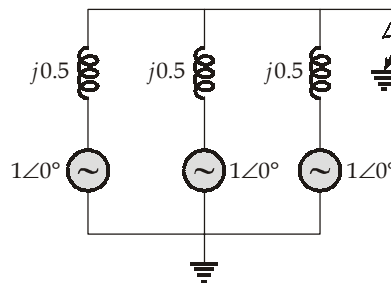
Fault current (kA) = Base current × Fault current (p.u.)

$$\text{Base current} = \frac{\text{MVA}}{\sqrt{3} \times \text{KV}} \times 1000 \text{ A} = \frac{100}{\sqrt{3} \times 132} \times 1000 = 437.4 \text{ A}$$

$$\text{Fault current} = 437.4 \times 5 = 2.187 \text{ KA}$$

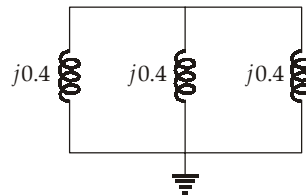
7. (a)

Positive sequence reactance diagram:



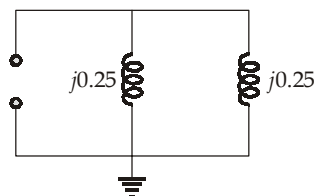
$$\Rightarrow Z_{01} = j\frac{0.5}{3}$$

Negative sequence reactance diagram:



$$\Rightarrow Z_{02} = j\frac{0.4}{3}$$

Zero sequence reactance diagram:



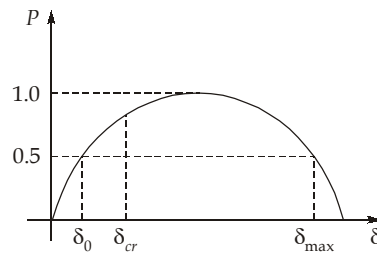
$$\Rightarrow Z_{00} = j\frac{0.25}{2}$$

$$I_f = 3 \left(\frac{E}{Z_{01} + Z_{02} + Z_{00}} \right) = 3 \left(\frac{1}{j0.425} \right)$$

$$I_f = -j7.058 \text{ p.u.}$$

$$\Rightarrow |I_f| = 7.058 \text{ p.u.}$$

8. (d)



$$P_m = P_{\max} \sin \delta_0$$

$$\sin \delta_0 = 0.5$$

$$\delta_0 = 30^\circ \text{ or } \frac{\pi}{6} \text{ rad}$$

and

$$\delta_{\max} = \pi - \delta_0 = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ rad or } 150^\circ$$

$$\cos \delta_{cr} = \frac{P_m}{P_{\max}} (\delta_{\max} - \delta_0) + \cos \delta_{\max} = \frac{0.5}{1} \left(\frac{5\pi}{6} - \frac{\pi}{6} \right) + \cos(150^\circ)$$

The critical clearing angle,

$$\delta_{cr} = \cos^{-1} (0.18117) = 79.562^\circ$$

9. (b)

Sum of the line currents in a Δ is always zero

$$I_a + I_b + I_c = 0$$

$$I_b = -I_a$$

$$I_{a1} = \frac{1}{3} [I_a + \alpha I_b + \alpha^2 I_c] = \frac{1}{3} [I_a - \alpha I_a]$$

$$= \frac{I_a(1 - 1 \angle 120^\circ)}{3} = \frac{(10 \angle 0^\circ)(1 - 1 \angle 120^\circ)}{3}$$

$$I_{a1} = 5.77 \angle -30^\circ \text{ A}$$

10. (b)

This method is not directly applicable to multi-machine system.

11. (c)

Only Y_{22} , Y_{24} , Y_{42} , Y_{44} will change because transmission line is connected between 2nd and 4th buses.

$$Y_{22} = -j60 + \frac{1}{Z_{se}} + \frac{Y_{sh}}{2}$$

$$= -j60 + \frac{1}{j0.1} + j20 = -j60 - j10 + j20 = -j50$$

$$Y_{24} = Y_{42} = 0 - \frac{Y_{sh}}{2} = -j20$$

$$Y_{44} = -j25 + \frac{1}{Z_{se}} + \frac{Y_{sh}}{2} = -j25 + \frac{1}{j0.1} + j20 = -j25 - j10 + j20$$

$$Y_{44} = -j15$$

12. (c)

Inertia constant, $H = 4 \text{ MW-sec/MVA}$
 $= 4 \text{ MJ/MVA}$

No load voltage, $V_1 = 1.2 \text{ p.u.}$

Infinite bus voltage, $V_2 = 1 \text{ p.u.}$

Total reactance, $X = X_G + X_L$
 $= 0.25 + 0.15$
 $= 0.4 \text{ p.u.}$

Angular momentum, $M = \frac{GH}{\pi f} = \frac{1 \times 4}{\pi \times 50} = 0.0254$

For 80% loading,

$$\sin \delta_0 = \frac{80}{100} = 0.8$$

$$\cos \delta_0 = \sqrt{1 - 0.8^2} = 0.6$$

$$\frac{dP_c}{d\delta} = \frac{V_1 V_2}{X} \cos \delta_0 = \frac{1.2 \times 1}{0.4} \times 0.6 = 1.8$$

$$f_n = \sqrt{\frac{\left. \frac{dP_e}{d\delta} \right|_{\delta_0}}{M}} = \sqrt{\frac{1.8}{0.0254}} = 8.41 \text{ rad/sec} = 1.34 \text{ Hz}$$

13. (b)

Full load current of each alternator,

$$= \frac{20 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 1.05 \text{ kA}$$

Since the two identical alternators are operating in parallel

$$Z_1 = \frac{j0.18}{2} = j0.09 \text{ p.u.}$$

$$Z_2 = \frac{j0.15}{2} = j0.075 \text{ p.u.}$$

$$Z_0 = j0.10 + 3R_n$$

$$= j0.1 + 3 \times \frac{2 \times 20}{11^2} = j0.1 + 0.992$$

For an L-G fault,

fault current, $I_f = 3I_{a1} = \frac{3E_a}{Z_1 + Z_2 + Z_0} = \frac{3}{j0.09 + j0.075 + j0.1 + 0.992}$

$$= 2.92 \angle -14.96^\circ \text{ p.u.}$$

fault current, $I_f = 2.92 \times 1.05 = 3.066 \text{ kA}$

Voltage drop across grounding resistor

$$= 3.066 \times 2 = 6.132 \text{ kV}$$

14. (c)

Voltage magnitude at bus-2,

$$V_2 = 1 - \frac{Z_{12}}{Z_{11}} = 1 - Z_{12} I_{f1} \quad \left(\because I_{f1} = \frac{1}{Z_{11}} \right)$$

$$0.9 = 1 - Z_{12} \times 12.5$$

$$Z_{12} = \frac{0.1}{12.5} = 0.008 \text{ p.u.}$$

and voltage magnitude at bus-1,

$$V_1 = 1 - \frac{Z_{12}}{Z_{22}} = 1 - Z_{12} \cdot I_{f2}$$

$$= 1 - 0.008 \times 10$$

$$= 1 - 0.08$$

$$V_1 = 0.92 \text{ p.u.}$$

15. (a)

Base MVA = 2 MVA

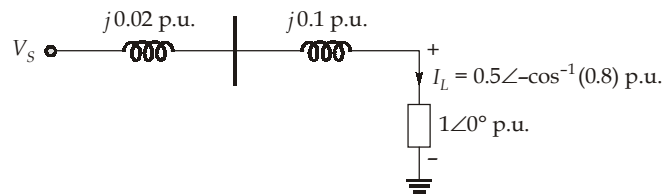
The p.u. reactance of 2 MVA transformer is $j0.1$ p.u.

The p.u. reactance of 10 MVA transformer,

$$X_{\text{p.u. (new)}} = X_{\text{p.u. (old)}} \times \frac{2}{10}$$

$$= 0.1 \times \frac{2}{10} = 0.02 \text{ p.u.}$$

The load current is 0.5 p.u. for 2 MVA base



KVL in the loop:

$$V_S = I_L Z + V \angle 0^\circ$$

$$= [(0.5 \angle -36.87^\circ)(j0.12)] + (1 \angle 0^\circ)$$

$$= 1.037 \angle 2.65^\circ \text{ p.u.}$$

$$V_S = 1.037 \times 33 = 34.22 \text{ kV}$$

16. (b)

Natural frequency of oscillations is,

$$f_n = \sqrt{\frac{\left(\frac{\partial P_e}{\partial \delta} \right)_{\delta_0}}{M}}$$

$$GH = \frac{1}{2} M \omega$$

$$M = \frac{GH}{\pi f} = \frac{1 \times 3}{\pi \times 50} = 0.019$$

$$P_e = \frac{|E||V|}{X} \sin \delta_0$$

$$0.6 P_{\max} = P_{\max} \sin \delta_0$$

$$\delta_0 = \sin^{-1}(0.6) = 36.87^\circ$$

$$\left. \frac{\partial P_e}{\partial \delta} \right|_{\delta_0} = P_e \cos \delta_0 = \frac{1.1}{0.5} \cos(36.87^\circ) = 1.76 \text{ p.u.}$$

$$f_n = \sqrt{\frac{1.76}{0.019}} = 9.62 \text{ rad/sec}$$

$$f_n = \frac{9.62}{2\pi} = 1.53 \text{ Hz}$$

17. (c)

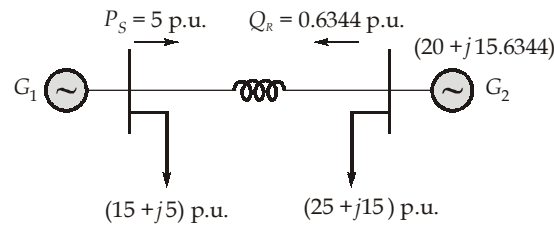
By equalizing the station,

$$P_{G_1} = P_{G_2} = 20 \text{ p.u.}$$

Now,

$$5 = \frac{|E||V|}{|X|} \sin \delta = \frac{1 \times 1}{0.05} \sin \delta$$

$$\delta = 14.47^\circ$$



$$Q_R = \frac{|V_1||V_2|}{X} \cos \delta - \frac{|V_1|^2}{X} = -0.6344 \text{ p.u.}$$

$$\begin{aligned} \text{Total load on station 2} &= (25 + j15) + (-5 + j0.6344) \\ &= (20 + j15.6344) \end{aligned}$$

$$\begin{aligned} \text{Power factor of station 2} &= \cos \left(\tan^{-1} \left(\frac{15.6344}{20} \right) \right) \\ &= 0.78 \text{ lagging} \end{aligned}$$

18. (a)

$$I_{f3-\phi} = \frac{E_f}{X_1}$$

$$I_{fL-G} = \frac{3E_f}{X_1 + X_2 + X_0}$$

$$\frac{I_{f3-\phi}}{I_{fL-G}} = \frac{X_1 + X_2 + X_0}{3X_1}$$

$$= \frac{0.30 + 0.30 + 0.10}{3 \times 0.30} = 0.778$$

19. (a)

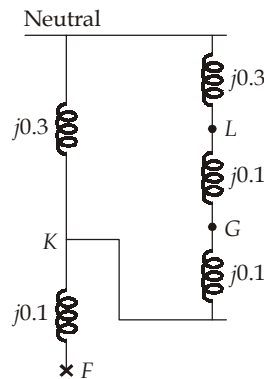
Let base MVA = 10 MVA

Per unit reactance of each generator is 0.3 p.u.

Per unit reactance of each reactor is 0.1 p.u.

Per unit reactance of each transformer on the base MVA

$$= \frac{5}{100} \times \frac{10}{5} = 0.1 \text{ p.u.}$$



Total per unit impedance from generator neutral upto fault point F

$$= 0.1 + [(0.3) \parallel (0.5)]$$

$$= 0.1 + \frac{(0.3)(0.5)}{(0.3) + (0.5)} = 0.2875 \text{ p.u.}$$

$$\text{Short circuit MVA} = \frac{\text{Base MVA}}{\text{Per unit fault reactance}} = \frac{10}{0.2875} = 34.78 \text{ MVA}$$

20. (c)

The rating of the machine, $G = 100 \text{ MVA}$ Inertia constant, $H = 5 \text{ kW-s/kVA}$

$$= 5 \text{ KJ/KVA} = 5 \text{ MJ/MVA}$$

Kinetic energy stored in the rotating parts of generator and turbine at synchronous speed ($f = 50 \text{ Hz}$)

$$= HG = 5 \times 100 = 500 \text{ MJ}$$

Excess power input to the generator shaft before the steam valve begins to close,

$$= 100 - 60 = 40 \text{ MW}$$

Excess energy transferred to rotating parts in 0.5 sec

$$= 40 \times 0.5 = 20 \text{ MJ}$$

Since, Kinetic energy, $K.E. \propto (\text{speed})^2 \propto f^2$

So, frequency at the end of 0.5 sec

$$f_2 = f_1 \sqrt{\frac{\text{Total energy stored in 0.5 sec}}{\text{Energy stored at synchronous speed}}}$$

$$f_2 = 50 \sqrt{\frac{500 + 20}{500}} = 50 \times 1.02 \approx 51 \text{ Hz}$$

$$\text{Change in frequency} = f_2 - f_1$$

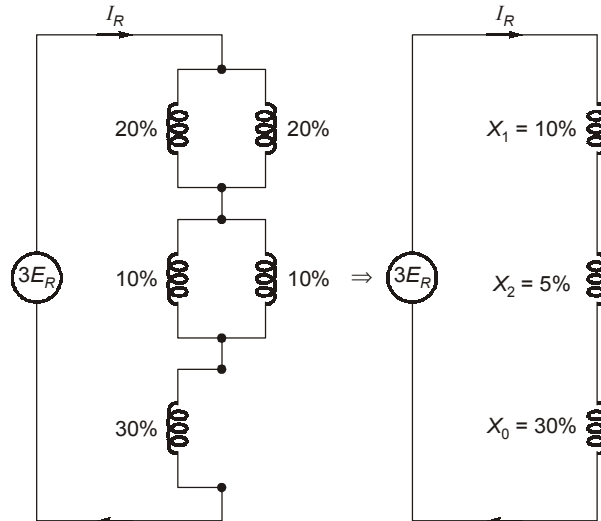
$$= 51 - 50 \approx 1 \text{ Hz}$$

21. (a)

The earth fault is assumed to occur on the red phase. Taking red phase as the reference, its phase e.m.f.

$$E_R = \frac{11 \times 1000}{\sqrt{3}} = 6351 \text{ V}$$

For line to ground fault the equivalent circuit will be



The percentage reactances can be converted into ohmic values as under :

$$\% X = \frac{Z (MVA_b)}{(KV)^2} \times 100$$

$$Z = \frac{\% X \times (KV)^2}{(MVA_b) \times 100}$$

$$X_1 = \frac{10 \times (11)^2}{20 \times 100} = 0.605 \Omega$$

$$X_2 = \frac{5 \times 11^2}{20 \times 100} = 0.3025 \Omega$$

$$X_0 = \frac{30 \times 11^2}{20 \times 100} = 1.815 \Omega$$

$$\text{Fault current } \vec{I}_R = \frac{3 \vec{E}_R}{X_1 + X_2 + X_0} = \frac{3 \times 6351}{j0.605 + j0.3025 + j1.815}$$

$$\vec{I}_R = -j 6998 \text{ A}$$

$$|I_R| = 6998 \text{ A}$$

22. (b)

For 3- ϕ transmission line we can use relation,

$$X_1 = X_s - X_m \quad \dots(1)$$

$$X_0 = X_s + 2X_m \quad \dots(2)$$

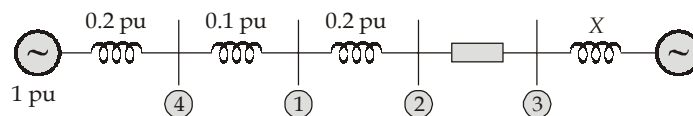
Also given, $X_0 = 32 \Omega$ and $X_1 = 16 \Omega$

∴ Solving (1) and (2) simultaneously,

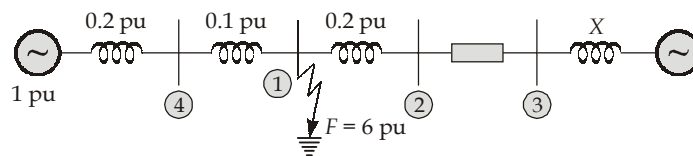
$$\begin{aligned} X_s - X_m &= 16 \\ X_s + 2X_m &= 32 \\ (-) \quad (-) \quad (-) \\ \hline -3X_m &= -16 \\ X_m &= \frac{16}{3} \approx 5.33 \Omega \end{aligned}$$

23. (b)

Drawing single line diagram, assuming system B reactance be X ,



If fault occurs at bus 1 after reactance,



$$\begin{aligned} \text{Thevenin impedance, } Z_{th} &= (0.2 + 0.1) \parallel (0.2 + X) \\ &= \frac{(0.3)(0.2 + X)}{0.5 + X} \end{aligned}$$

$$\text{Also, per unit fault current} = \frac{1}{Z_{th}} = 6 \text{ pu}$$

$$\therefore \frac{1}{6} = \frac{(0.3)(0.2 + X)}{0.5 + X}$$

$$0.5 + X = (1.8)(0.2 + X)$$

$$0.8X = 0.5 - 0.36 \text{ or } X = 0.175 \text{ pu}$$

Now using $X = 0.175$ pu, finding fault level at bus 3 after interconnection

$$\text{New, } Z_{th \text{ pu}} = \frac{(0.5)(0.175) \text{ pu}}{0.675} = 0.1296 \text{ pu}$$

$$I'_f = \frac{1}{Z_{th \text{ pu}}} = 7.71 \text{ pu}$$

24. (b)

For the power system finding admittance value,

$$y_{10} = \frac{1}{Z_{10}} = \frac{1}{0.3} = \frac{10}{3}$$

$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.5} = 2$$

$$\therefore y_{20} = \frac{1}{Z_{20}} = \frac{1}{0.5} = 2$$

$$\text{Admittance matrix, } Y_{\text{bus}} = \begin{bmatrix} 10/3+2 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 5.33 & -2 \\ -2 & 4 \end{bmatrix}$$

∴ Impedance matrix,

$$\begin{aligned} Z_{\text{bus}} &= [Y_{\text{bus}}]^{-1} = \frac{1}{(5.33 \times 4) - (2 \times 2)} \times \begin{bmatrix} 4 & 2 \\ 2 & 5.33 \end{bmatrix} \\ &= 0.0577 \times \begin{bmatrix} 4 & 2 \\ 2 & 5.33 \end{bmatrix} = \begin{bmatrix} 0.230 & 0.115 \\ 0.115 & 0.308 \end{bmatrix} \end{aligned}$$

25. (c)

$$v = 100 \sin(100\pi t + 15^\circ)$$

It will be maximum when,

$$(100\pi t + 15^\circ) = 90^\circ$$

$$100\pi t = \frac{75 \times \pi}{180}$$

$$t = \frac{75}{100 \times 180} = 4.16 \text{ ms}$$

Now short circuit current is given by

$$i = \frac{V_{\text{max}}}{Z} \sin(\omega t + \alpha - \phi) + \frac{V_{\text{max}}}{Z} \sin(\phi - \alpha) e^{-t/\tau}$$

Where τ is time constant,

$$\text{Now, } Z = \sqrt{R^2 + X^2} = \sqrt{5^2 + (100\pi \times 0.1)^2} = 31.81 \Omega$$

$$V_{\text{max}} = 100 \text{ Volts}$$

$$\alpha = 15^\circ$$

$$\phi = \tan^{-1}\left(\frac{100\pi \times 0.1}{5}\right) = 80.96^\circ$$

$$\tau = \frac{L}{R} = \frac{0.1}{5} = \frac{1}{50}$$

So,

$$\begin{aligned} i &= \frac{100}{31.81} \sin(100\pi t + 15 - 80.96^\circ) + \frac{100}{31.81} \sin(80.96 - 15) e^{-50t} \\ &= \frac{3.1435 \sin(100\pi t - 65.96^\circ)}{\text{Particular}} + \frac{2.871 e^{-50t}}{\text{Complementary}} \end{aligned}$$

(d) at $t = 4.16 \text{ ms}$

$$\begin{aligned} i &= 3.1435 \sin\left(\left(\frac{100\pi}{\pi} \times 180 \times 4.16 \times 10^{-3}\right) - 65.96^\circ\right) + 2.871 e^{-50 \times 4.16 \times 10^{-3}} \\ &= 0.4874 + 2.3318 = 2.819 \text{ Amp} \end{aligned}$$

26. (b)

$$X_{1 \text{ eq}} = \frac{j0.18}{2} = j0.09 \text{ p.u.}$$

$$X_{2 \text{ eq}} = \frac{j0.15}{2} = j0.075 \text{ p.u.}$$

$$Z_{0 \text{ eq}} = j0.1 + \frac{3 \times 2}{11^2} \times 20 = (0.9917 + j0.1) \text{ p.u.}$$

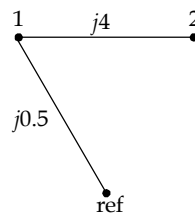
$$\begin{aligned} \text{Fault current, } I_f &= \frac{3E_f}{j(X_1) + jX_2 + Z_{0 \text{ eq}}} \\ &= \frac{3 \times 1 \angle 0^\circ}{1.0265 \angle 14.96^\circ} = 2.922 \angle -14.96^\circ \text{ A} \end{aligned}$$

Current in grounding resistor,

$$I_f = 2.922 \times \frac{20}{11\sqrt{3}} = 3.07 \text{ kA}$$

27. (d)

Existing system and bus matrix is



$$Z_{\text{Bus}} = \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j4.5 \end{bmatrix}$$

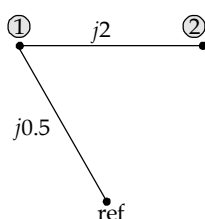
Modifying line with reactance $j2$ is equivalent to adding a line in parallel with impedance $j4$. Thus it is type-4 modification.

$$[Z_{\text{new}}] = [Z_{\text{old}}] - \frac{1}{Z_{11} + Z_{22} - 2Z_{12} + Z_s} \begin{bmatrix} \text{subtract} \\ 2^{\text{old}} \text{ column} \\ \text{to first column} \end{bmatrix} [\text{Transpose}]$$

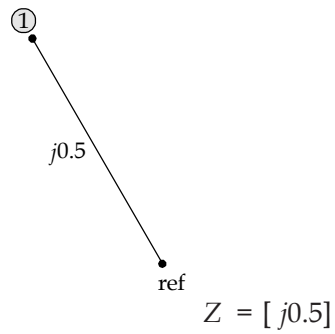
$$\begin{aligned} [Z_{\text{Bus}}]_{\text{new}} &= \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j4.5 \end{bmatrix} - \frac{1}{j0.5 + j4.5 - 2(j0.5) + j4} \begin{bmatrix} j0 \\ -j4 \end{bmatrix} \begin{bmatrix} j0 & -j4 \end{bmatrix} \\ &= \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j4.5 \end{bmatrix} - \frac{1}{j8} \begin{bmatrix} 0 & 0 \\ 0 & -16 \end{bmatrix} \\ &= \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j4.5 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & -\frac{16}{j8} \end{bmatrix} \\ &= \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j4.5 - \frac{j16}{8} \end{bmatrix} = \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j2.5 \end{bmatrix} \end{aligned}$$

Alternative :

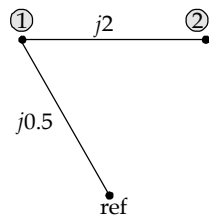
New system will



First branch :



Type - 2 modification



$$Z = \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j0.5 + j2 \end{bmatrix} = \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j2.5 \end{bmatrix}$$

28. (b)

For single line to ground fault , fault current is

$$I_f = 3I_a^{(0)}$$

and

$$I_a^{(0)} = \frac{-V_a^{(0)}}{Z_{g0}}$$

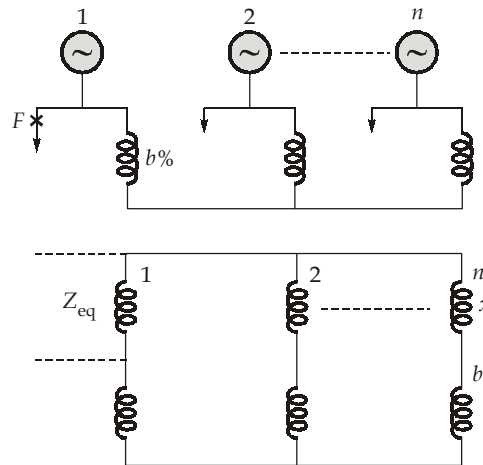
$$\begin{aligned} V_a^{(0)} &= \frac{1}{3}[V_a + V_b + V_c] = \frac{1}{3}[0 + 1.013\angle -102.25^\circ + 1.013\angle 102.25^\circ] \\ &= -0.1433 \text{ p.u.} \end{aligned}$$

$$I_a^{(0)} = -\left(\frac{-0.1433}{j0.1}\right) = -j1.43 \text{ p.u.}$$

$$I_f = 3 \times (-j1.43) = -j4.29 \text{ p.u.}$$

$$I_f = -j4.29 \times \left(\frac{20000}{\sqrt{3} \times 13.8}\right) = -j3.59 \text{ kA}$$

29. (d)



Equivalent impedance Z_{eq} between the zero potential bus and the fault point is

$$\left(\frac{b+x}{n-1} + b\right) \parallel x = \left(\frac{bn+x}{n-1}\right) \parallel x$$

$$\frac{1}{Z_{eq}} = \frac{1}{x} + \frac{n-1}{bn+x}$$

$$\begin{aligned} \text{SC kVA} &= \frac{8}{Z_{eq}} \times 100 \\ &= 8 \left[\frac{1}{x} + \frac{n-1}{bn+x} \right] \times 100 \end{aligned}$$

If n is very large.

$$\text{Short circuit kVA} = 8 \left[\frac{1}{x} + \frac{1}{b} \right]$$

30. (a)

$$\begin{aligned} \delta_0 &= \sin^{-1}(0.25) \\ &= 14.48^\circ \end{aligned}$$

$$\begin{aligned} \delta_c &= \sin^{-1}(0.5) \\ &= 30^\circ \end{aligned}$$

$$\int_{\delta_0}^{\delta_c} (0.5 - \sin \delta) d\delta = \int_{\delta_c}^{\delta_m} (\sin \delta - 0.5) d\delta$$

$$\Rightarrow 0.5(\delta_c - \delta_0) + \cos \delta_c - \cos \delta_0 = \cos \delta_c - \cos \delta_m - 0.5(\delta_m - \delta_c)$$

$$0.5(\delta_m - \delta_0) = \cos 14.48^\circ - \cos \delta_m$$

$$0.5 \delta_m + \cos \delta_m = 1.0945$$

$$\delta_m = 46.41^\circ$$

