S.No. : 05 CH_EE_F+T_060919

Signal and Systems

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CLASS TEST 2019-2020

ELECTRICAL ENGINEERING

Date of Test : 06/09/2019

ANSWER KEY ➤ Signal and Systems

1. (c)	7. (c)	13. (c)	19. (c)	25. (d)
2. (d)	8. (a)	14. (d)	20. (c)	26. (a)
3. (c)	9. (d)	15. (d)	21. (b)	27. (c)
4. (b)	10. (b)	16. (c)	22. (b)	28. (a)
5. (a)	11. (b)	17. (d)	23. (a)	29. (b)
6. (d)	12. (d)	18. (a)	24. (a)	30. (b)

DETAILED EXPLANATIONS

1. (c)

From the given figure of $x(t)$ and $y(t)$, we get conclude that,

$$x(0) = y(5)$$

at $t = 5$ sec, $\quad y(5) = x(-5a + 20) = x(0)$

$\therefore \quad -5a + 20 = 0$

$\therefore \quad a = 4$

2. (d)

Given, signal $x(t)$ has energy 'E'

for $\quad ax(t) \xleftrightarrow{E} a^2E$

$\therefore \quad ax(bt + c) \xleftrightarrow{E} \dfrac{a^2E}{b}$

From the given signal $a = 2$, $b = 5$ and hence

$\therefore$ The energy of signal $2x(5t - 6)$ is

$$E = \frac{(2)^2 \times 10}{5} = 8 \text{ J}$$

3. (c)

We know that, unit impulse let $x(t)$,

$$x(t) = \delta(t)$$

for $\quad \delta(t) \xleftrightarrow{LT} 1$

for $\quad \dfrac{d}{dt}x(t) \xleftrightarrow{LT} sX(s)$

$$\frac{d}{dt}\delta(t) \xleftrightarrow{LT} s$$

$$\frac{d^2}{dt^2}\delta(t) \xleftrightarrow{LT} s^2$$

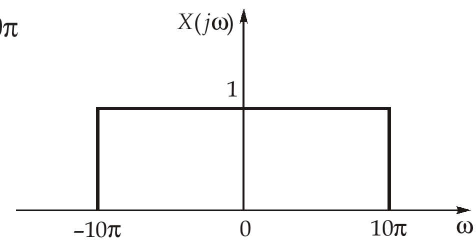
4. (b)

Given, $\quad x(t) = \dfrac{\sin(10\pi t)}{\pi t}$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or



$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$\therefore \quad T_s < \dfrac{1}{10}$

6. **(d)**

Given, $x(t) = \delta(t) * 2\delta(t-1) * 3\delta(t-2)$

From the convolution property of impulse,

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$\therefore \quad x(t) = \delta(t) * 6\delta(t-3)$$

$$x(t) = 6\delta(t-3)$$

7. **(c)**

Given, $H(z) = \dfrac{z}{z-0.2} = \dfrac{1}{1-0.2z^{-1}}$ ROC : $|z| > 0.2$

Since the ROC : $|z| > 0.2$, which includes unit circle.

$\therefore$ The impulse response will be stable.

9. **(d)**

$$\int_{-\infty}^{\infty} \delta(at)\,dt = \frac{1}{a}$$

Since $\delta(at) = \dfrac{1}{|a|}\delta(t)$

10. **(b)**

For sequence $X_1[n] \xleftrightarrow{Z} X_1[z]$; ROC = R

For sequence $X_2[n] = X_1[-n] \xleftrightarrow{Z} X_1[1/z]$; ROC = $1/R$

$\therefore$ ROC's are reciprocal of each other.

11. **(b)**

By the differentiation property of Fourier transform,

$$\frac{dx(t)}{dt} \xleftrightarrow{\text{F.T}} j\omega X(\omega)$$

$$\frac{d^2x(t)}{dt^2} \xleftrightarrow{\text{F.T}} -\omega^2 X(\omega)$$

By the time shifting property,

$$-\frac{d^2x(t+2)}{dt^2} \xleftrightarrow{\text{F.T}} \omega^2 e^{j2\omega} X(\omega)$$

12. **(d)**

Let us consider two signals,

$$x_1(t) = 1, \quad \forall\, t$$

$$x_2(t) = -1, \quad \forall\, t$$

Clearly $x_1(t) \neq x_2(t)$ but $(x_1(t))^2 = (x_2(t))^2$

Therefore different inputs gives the same output hence the system is non invertible.

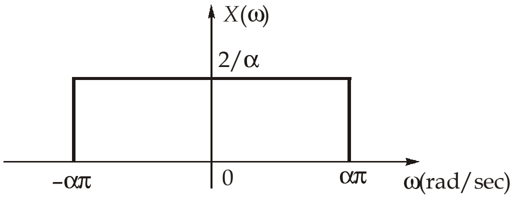
And also it is non linear system.

13. (c)

We know that,

Energy, $E = \int_{-\infty}^{\infty} x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$

Now, $X(\omega) =$



$$E = \frac{1}{2\pi}\int_{-\alpha\pi}^{\alpha\pi}\left|\frac{2}{\alpha}\right|^2 d\omega$$

$$0.5 = \frac{1}{2\pi}\times\frac{4}{\alpha^2}[2\alpha\pi]$$

$$0.5 = \frac{4}{\alpha}$$

$\therefore$ $\alpha = 8$

14. (d)

Given, $x[n] = \cos\frac{\pi}{4}n + \sin\left(\frac{\pi}{3}n + \frac{1}{2}\right)$

Let $x[n] = x_1[n] + x_2[n]$

Let N_1 be the period of $x_1[n]$.

$$\frac{\pi/4}{2\pi} = \frac{m}{N_1}$$

$\Rightarrow$ $N_1 = 8$

Let N_2 be the period of $x_2[n]$.

$$\frac{\pi/3}{2\pi} = \frac{m}{N_2}$$

$\Rightarrow$ $N_2 = 6$

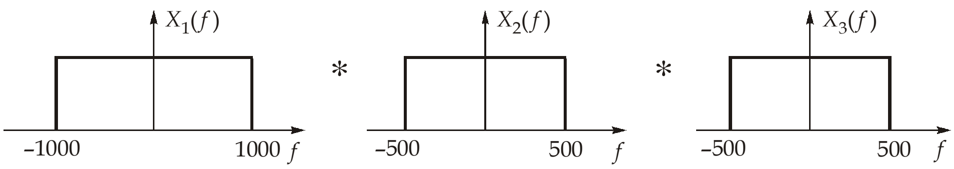
Overall time period, $N = \text{LCM}(N_1, N_2) = \text{LCM}(6, 8)$

$\therefore$ $N = 24$

15. (d)

We know that,

For $X(f) = X_1(f) * X_2(f) * X_3(f)$



Sampling frequency, $f_s = 2(1000 + 500 + 500)$

$f_s = 4000$ samples/sec

16. (c)

Consider a right sided signal $x(t)$

So, that $x(t) = 0$ for $t < t_0$, and $X(s)$ converges for Re{s} = σ_0, then

$$|X(s)| \leq \int_{-\infty}^{\infty} \left|x(t)e^{-st}\right| dt = \int_{-\infty}^{\infty} |x(t)|\, e^{-\sigma_0 t}\, dt = \int_{t_1}^{\infty} |x(t)|\, e^{-\sigma_0 t}\, dt < \infty$$

Let Re{s} = $\sigma_1 > \sigma_0$. Then

$$\int_{t_1}^{\infty} |x(t)|\, e^{-\sigma_1 t}\, dt = \int_{t_1}^{\infty} |x(t)|\, e^{-\sigma_0 t}\, e^{-(\sigma_1 - \sigma_0)t}\, dt$$

$$< e^{-(\sigma_1 - \sigma_0)t_1} \int_{t_1}^{\infty} |x(t)|\, e^{-\sigma_0 t}\, dt < \infty$$

Thus, $X(s)$ converges for Re{s} = σ_1 and the ROC of $X(s)$ is of the form Re{s} > σ_0. Since the ROC of $X(s)$ cannot include any poles of $X(s)$, we conclude that it is of the form Re{s} > σ_{max}.

17. (d)

Given signal, Let $x(t) = \dfrac{1}{\pi(1+t^2)}$

We know that,

$$e^{-a|t|} \overset{FT}{\longleftrightarrow} \frac{2a}{a^2 + \omega^2}$$

Put $a = 1$

$$e^{-|t|} \overset{FT}{\longleftrightarrow} \frac{2}{1+\omega^2}$$

By using duality property,

$$\frac{2}{1+t^2} \overset{FT}{\longleftrightarrow} 2\pi e^{-|-\omega|}$$

$$\frac{2}{1+t^2} \overset{FT}{\longleftrightarrow} 2\pi e^{-|\omega|}$$

$$\frac{1}{\pi(1+t^2)} \overset{FT}{\longleftrightarrow} e^{-|\omega|}$$

18. (a)

Given, $y(t) = \dfrac{1}{2}x\left(\dfrac{t}{2}\right) * \delta(t-4) = \dfrac{1}{2}x\left(\dfrac{t-4}{2}\right) = \dfrac{1}{2}x\left(\dfrac{1}{2}t - 2\right)$

at $t = 2$; $y(2) = \dfrac{1}{2}x\left(\dfrac{1}{2}(2) - 2\right) = \dfrac{1}{2}x(-1)$

Slope of given signal $x(t)$ is 2.

$$x(-1) = -2$$

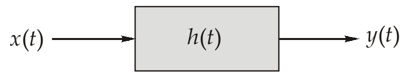
$$y(2) = \frac{1}{2}(-2) = -1$$

19. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3 + j\omega}$$

and output, $y(t) = e^{-3t}\,u(t) - e^{-4t}\,u(t)$



We know that, $$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$$\therefore \quad X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t}\,u(t)$$

20. **(c)**

Given, sinusoidal pulse

$$z(t) = \begin{cases} e^{j10t}\ ; & |t| < \pi \\ 0\ ; & |t| > \pi \end{cases}$$

We may express $z(t)$ as the product of a complex sinusoid e^{j10t} and a rectangular pulse $x(t)$.

Let, $$x(t) = \begin{cases} 1\ ; & |t| < \pi \\ 0\ ; & |t| > \pi \end{cases}$$

Fourier transform of $x(t)$ is $X(j\omega)$

$$\therefore \quad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{-\pi}^{\pi} 1\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\pi}^{\pi}$$

$$= -\frac{1}{j\omega}\left[e^{-j\pi\omega} - e^{+j\omega\pi}\right] = \frac{e^{j\omega\pi} - e^{-j\omega\pi}}{j\omega} = \frac{2}{\omega}\left[\frac{e^{j\omega\pi} - e^{-j\omega\pi}}{2j}\right]$$

$$\therefore \quad X(j\omega) = \frac{2}{\omega}\sin(\omega\pi)$$

By using frequency shift property of Fourier transform, we get,

$$z(t) = e^{j10t}\cdot x(t) \xleftrightarrow{\text{FT}} X(j(\omega - 10))$$

$$\therefore \quad z(t) \xleftrightarrow{\text{FT}} \frac{2}{\omega-10}\sin\left((\omega-10)\pi\right)$$

21. **(b)**

Given, $X(s) = \log(s+2) - \log(s+3)$

Differentiating both the sides with respect to s

$$\frac{d}{ds}X(s) = \frac{1}{s+2} - \frac{1}{s+3} \qquad \ldots\text{(i)}$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds}X(s)$$

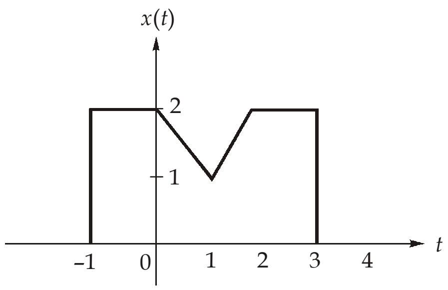
Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or,
$$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t}\right] u(t)$$

22. **(b)**

Given,



By the definition of Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot e^{j\omega t}\,d\omega$$

at $t = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega$$

$$\therefore \quad \int_{-\infty}^{\infty} X(j\omega)\,d\omega = 2\pi\, x(0) = 2\pi(2) = 4\pi \approx 12.57$$

23. **(a)**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!} z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

$$X(z) = 1 + \frac{\frac{1}{2z}}{1!} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \ldots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

24. (a)

By Parsevals theorem,

Energy of a signal $x[n]$ is, $E = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X(e^{j\omega})^2\right|d\omega$

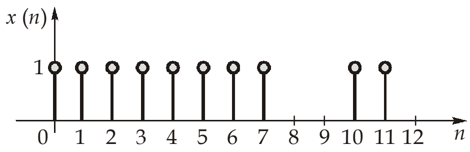
where $X(e^{j\omega})$ is discrete time Fourier transform of $x[n]$,

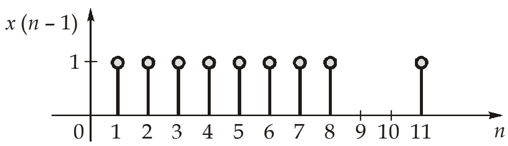
So, $$\frac{\sin(2n)}{\pi n} \xleftrightarrow{\text{DTFT}} X(e^{j\omega}) = \begin{cases} 1; & |\omega| \le 2 \\ 0; & \text{otherwise} \end{cases}$$

$\therefore$ $$\text{Energy, } E = \frac{1}{2\pi}\int_{-2}^{2} 1 \cdot d\omega = \frac{1}{2\pi}[4]$$

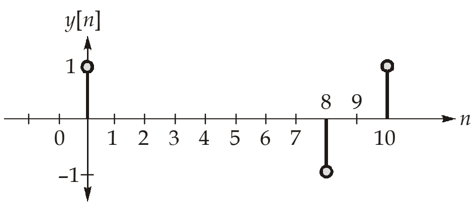
$\therefore$ $$E = \frac{2}{\pi} = 0.64 \text{ J}$$

25. (d)





Subtracting the two signal, we get



$\Rightarrow N = 10$

26. (a)

Given, $$x[n] = [1, \underset{\uparrow}{2}, 3, 4]$$

$$h[n] = \delta[n-1]$$

$$y[n] = x[n] * h[n] \qquad [* \text{ denotes convolution}]$$

$$= x[n] * \delta[n-1] = x[n-1]$$

$\therefore$ $$y[0] = x(-1) = 1 \qquad (\because n = 0)$$

27. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$\therefore$ $$\cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

$$= \cos\pi t[u(t) - u(t-1)]$$
$$= \cos(\pi t)u(t) - \cos\pi t\, u(t-1)$$
$$= \cos\pi t\, u(t) - \cos\pi(t - 1 + 1)u(t-1)$$
$$= \cos\pi t\, u(t) - \cos[\pi(t-1) + \pi]\, u(t-1)$$
$$x(t) = \cos(\pi t)u(t) + \cos[\pi(t-1)]\, u(t-1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2+\pi^2} + \frac{s e^{-s}}{s^2+\pi^2} \quad [\because x(t-t_0) = X(s)\cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1+e^{-s}]}{s^2+\pi^2}$$

28. (a)

We know that,

$$x[n] \xleftrightarrow{\text{DTFT}} X(\omega)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = \sum_{n=-2}^{3} x[n]e^{-j\omega n}$$

$$X(\omega) = e^{j2\omega} + 2e^{j\omega} + 3 + 2e^{-j\omega} + e^{-j2\omega}$$

$$X(\omega) = 3 + 4\cos\omega + 2\cos 2\omega \qquad \text{...(i)}$$

$$H(k) = \sum_{n=0}^{5} h[n]e^{-j\frac{2\pi}{6}nk}$$

$$H(k) = \sum_{n=0}^{5} h[n]e^{-j\frac{\pi}{3}nk} = 3 + 2e^{-j\frac{\pi}{3}k} + e^{-j\frac{\pi}{3}2k} + 0 + e^{-j\frac{\pi}{3}4k} + 2e^{-j\frac{\pi}{3}5k}$$

$$= 3 + 2e^{-j\frac{\pi}{3}k} + 2e^{+j\frac{\pi}{3}k} + e^{-j\frac{2\pi}{3}k} + e^{j\frac{2\pi}{3}k}$$

$$= 3 + 4\cos\frac{\pi k}{3} + 2\cos\frac{2\pi k}{3} \qquad \text{...(ii)}$$

By comparing equation (i) and equation (ii), we get,

$$\omega = \frac{\pi k}{3}$$

29. (b)

Given signals, $x(t) = \sin\omega_0 t$

$h(t) = \text{sgn}t$

from the multiplication property of Fourier transform,

$$x(t)h(t) = \frac{1}{2\pi}[X(\omega) * H(\omega)]$$

Fourier transform of $x(t)$ is $X(\omega)$,

$$X(\omega) = \frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$$

Fourier transform of $h(t)$ is $H(\omega)$,

$$H(\omega) = \frac{2}{j\omega}$$

$$\therefore \quad x(t)\ h(t) \xleftrightarrow{FT} \frac{1}{2\pi}\left[\frac{\pi}{j}(\delta(\omega-\omega_0)-\delta(\omega-\omega_0)) * \frac{2}{j\omega}\right]$$

$$\xleftrightarrow{FT} \frac{1}{2\pi}\left[\left[\frac{\pi}{j}\times\frac{2}{j(\omega-\omega_0)}\right]-\left[\frac{\pi}{j}\times\frac{2}{j(\omega+\omega_0)}\right]\right]$$

$$(\because X(\omega)*\delta(\omega-\omega_0)=X(\omega-\omega_0))$$

$$\xleftrightarrow{FT} \left[\frac{-1}{\omega-\omega_0}+\frac{1}{\omega+\omega_0}\right] = \frac{-\omega-\omega_0+\omega-\omega_0}{\omega^2-\omega_0^2}$$

$$\therefore \quad x(t)h(t) \xleftrightarrow{FT} \frac{-2\omega_0}{\omega^2-\omega_0^2}$$

30. (b)

Given,
$$X(z) = \frac{1}{1-2.5z^{-1}+z^{-2}} = \frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)}$$

$$\frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)} = \frac{A}{z^{-1}-2}+\frac{B}{z^{-1}-\frac{1}{2}}$$

$$\therefore \quad A = \frac{1}{2-\frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

$$B = \frac{1}{\frac{1}{2}-2} = -\frac{2}{3}$$

$$\therefore \quad X(z) = \frac{\frac{2}{3}}{z^{-1}-2}+\frac{-\frac{2}{3}}{z^{-1}-\frac{1}{2}}$$

$$= \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}}+\frac{\frac{4}{3}}{1-2z^{-1}}$$

Given $X(z)$ is a causal system, the ROC is right of the right most pole.

$\therefore\ |z|>2$

hence,
$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n]+\frac{4}{3}(2)^n u[n]$$

$$\therefore \quad x(0) = -\frac{1}{3}+\frac{4}{3} = \frac{3}{3} = 1$$

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