## CLASS TEST



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## FLUID MECHANICS

## MECHANICAL ENGINEERING

Date of Test : 21/08/2023

## ANSWER KEY

1. (d)
2. (b)
3. (d)
4. (c)
5. (c)
6. (c)
7. (b)
8. (d)
9. (d)
10. (b)
11. (a)
12. (c)
13. (b)
14. (b)
15. (a)
16. (a)
17. (d)
18. (b)
19. (a)
20. (b)
21. (d)
22. (c)
23. (a)
24. (a)
25. (b)
26. (c)
27. (a)
28. (c)
29. (c)
30. (a)

## DETAILED EXPLANATIONS

1. (d)


From similar triangles, $\triangle \mathrm{ABC}$ and $\triangle \mathrm{CDE}$

$$
\begin{aligned}
\frac{4-x}{x} & =\frac{3}{0.75} \\
3 x & =(4-x)(0.75) \\
3 x & =3-0.75 x \\
x & =0.8 \mathrm{~mm} \\
y & =4-x=3.2 \mathrm{~mm} \\
\dot{V}_{\text {net }} & =\left(3.2 \times 10^{-3}\right)\left(5 \times 10^{-2}\right) \frac{3}{2}-\left(0.8 \times 10^{-3}\right)\left(5 \times 10^{-2}\right) \frac{0.75}{2} \\
\dot{V}_{\text {net }} & =24 \times 10^{-5}-1.5 \times 10^{-5}=225 \times 10^{-6} \mathrm{~m}^{3} / \mathrm{s}=225 \mathrm{~cm}^{3} / \mathrm{s}
\end{aligned}
$$

2. (c)

$$
\begin{aligned}
1 \text { Poise } & =0.1 \mathrm{~N}-\mathrm{s} / \mathrm{m}^{2} \\
\text { Shear stress, } \tau & =\mu \frac{d u}{d y} \\
\Rightarrow \quad \tau & =\left(0.1 \times 5 \frac{\mathrm{~N}-\mathrm{s}}{\mathrm{~m}^{2}}\right) \times\left(\frac{5 \mathrm{~m} / \mathrm{s}}{0.015 \mathrm{~m}}\right) \\
& =166.67 \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

3. (a)

Velocity component in $x$-direction, $u=\frac{\partial \psi}{\partial y}=\frac{\partial}{\partial y}\left(a x^{2}+b y^{2}+c y\right)$

$$
u=2 b y+c
$$

Note that you can use other sign convention also.
4. (a)


Total pressure on the triangle,

$$
\begin{aligned}
F & =p A=\gamma h_{c} A \\
& =(1000 \times 0.75 \times 9.81) \times\left(\frac{5}{3}\right) \times\left(\frac{1}{2} \times 5 \times 5\right) \\
& =750 \times 9.81 \times \frac{5}{3} \times \frac{25}{2}=1250 \times \frac{25}{2} \simeq 153.28 \mathrm{kN}
\end{aligned}
$$

5. (d)

Stable equilibrium condition

6. (c)

$$
\begin{aligned}
& \therefore \\
& \qquad \begin{array}{l}
\bar{V}=\frac{Q}{A}=\frac{Q}{\frac{\pi}{4} d^{2}} \Rightarrow \frac{4 Q}{\pi d^{2}}=\frac{4 \times 880 \times 10^{-9}}{\pi \times 0.50^{2} \times 10^{-6}}=4.48 \mathrm{~m} / \mathrm{s} \\
\text { We know, } \\
\qquad \quad Q=\frac{\pi \Delta P D^{4}}{128 \mu L} \\
\Rightarrow \quad \mu
\end{array} \\
& \qquad \begin{aligned}
128 Q L & \frac{\pi \Delta P D^{4}}{128 \times 880 \times 10^{-9} \times 1} \\
& \mu
\end{aligned} \\
&
\end{aligned}
$$

8. (b)

$$
\text { Sensitivity }=\frac{1}{\sin \theta}=\frac{1}{\sin 30^{\circ}}=2
$$

9. (c)

A dimension is a measure of a physical quantity (without numerical values), while a unit is a way to assign a number to that dimension.
10. (d)
$R e=\frac{\rho U D}{\mu}$, where symbols have the usual meaning.
Given, $\quad \rho=100 \mathrm{~kg} / \mathrm{m}^{3}, \quad U=40 \mathrm{~m} / \mathrm{s}, \quad D=0.1 \mathrm{~m}, \quad \mu=0.048$
$\therefore \quad R e=\frac{\rho U D}{\mu}=\frac{100 \times 40 \times 0.1}{0.048}$
$R e=8333.33$
As
$R e>2300$
$\therefore \quad$ Flow will be turbulent.
Nothing can be said about the path as there will be rapid mixing and eddies formations.
11. (c)

As per given data:

$$
\begin{aligned}
u^{*} & =\frac{u}{U} \text { and } y^{\star}=\frac{y}{\delta} \\
d y^{*} & =\delta^{-1} d y
\end{aligned}
$$

The given parabolic velocity distribution and the expression for the displacement thickness can then be expressed as

$$
u^{*}=2 y^{\star}-y^{* 2}, \text { and } \delta^{*}=\delta \int_{0}^{1}\left(1-u^{*}\right) d y^{*}
$$

Combining these equations gives,

$$
\begin{aligned}
\delta^{\star} & =\delta \int_{0}^{1}\left(1-2 y^{\star}+y^{\star 2}\right) d y^{\star} \\
\delta^{\star} & =\delta\left[y^{\star}-y^{\star 2}+\frac{1}{3} y^{\star 3}\right]_{0}^{1} \\
\delta^{\star} & =\frac{1}{3} \delta \\
\frac{\delta^{\star}}{\delta} & =\frac{1}{3}
\end{aligned}
$$

12. (a)

Applying Bernoulli's equation between the two reservoirs, we get

$$
\begin{aligned}
12.5 & =0.5 \frac{V^{2}}{2 g}+\frac{f L V^{2}}{2 g D}+\frac{V^{2}}{2 g} \\
\Rightarrow \quad 12.5 & =\frac{V^{2}}{2 g}\left[1.5+\frac{f L}{D}\right]
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow & 12.5 & =\frac{V^{2}}{2 \times 10}\left[1.5+\frac{0.04 \times 1000}{0.5}\right] \\
\Rightarrow & 12.5 & =\frac{V^{2}}{20} \times 81.5 \\
\Rightarrow & V & =1.75 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

13. (d)

Let the parabolic velocity distribution is

$$
V=A+B y+C y^{2}
$$

where, constants, $A, B$ and $C$ are to be determined from boundary conditions.

$$
\begin{aligned}
& V=0, \quad \text { at } y=0(\text { No slip at the plate surface }) \\
& V=1.125 \text { at } y=0.075 \mathrm{~m}
\end{aligned}
$$

$\frac{d V}{d y}=0$, at $y=0.075$ (condition of vertex of parabola)
Substituting the boundary conditions, we have

$$
\begin{array}{rlrl}
A & =0 \\
& & & \\
\Rightarrow & 1.125 & =0.075 B+(0.075)^{2} C \\
\therefore & 0 & =B+0.15 C \\
\therefore & B & =30, \quad C=-200 \\
\therefore & V & =30 y-200 y^{2} \\
& \frac{d V}{d y} & =30-400 y
\end{array}
$$

at $y=0.05 \mathrm{~m}, \quad \quad \frac{d V}{d y}=30-400 \times 0.05, \frac{d V}{d y}=10$

$$
\therefore \quad \tau=\frac{\mu d V}{d y}=0.05 \times 10=0.5 \mathrm{~N} / \mathrm{m}^{2}
$$

14. (d)

All the four are correct.
15. (b)

The frontal area of a sphere is $A=\frac{\pi D^{2}}{4}$
The drag force acting on the balloon is

$$
F_{D}=C_{D} A \frac{\rho V^{2}}{2}=(0.2)\left[\frac{\pi(7)^{2}}{4}\right] \frac{(1.20)\left(\frac{40 \times 5}{18}\right)^{2}}{2}=570.14 \mathrm{~N}
$$

Acceleration in the direction of the winds

$$
a=\frac{F_{D}}{m}=\frac{570.14}{350}=1.63 \mathrm{~m} / \mathrm{s}^{2}
$$

16. (b)


By symmetry, the net horizontal force is zero.
Vertical force, $F_{V}=$ Weight of fluid above the hemisphere MPN

$$
\begin{aligned}
& =\gamma\left[\pi R^{2} H-\frac{1}{2} \cdot \frac{4}{3} \pi R^{3}\right] \\
& =9.81 \times\left[\pi(0.75)^{2} \times 5-\frac{2}{3} \times \pi \times(0.75)^{3}\right] \\
& =78.01 \mathrm{kN}
\end{aligned}
$$

Resultant force is same as the vertical force $F_{V}=78.01 \mathrm{kN}$ acting vertically at the center of the hemisphere.
17. (a)

The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

$$
\begin{aligned}
p_{\text {bottom }} & =(1000 \mathrm{~g})(1.5)+1.60(1000 \mathrm{~g})(1.0)=1.60(1000 \mathrm{~g}) h+(2.5-h) \times 667 \mathrm{~g} \\
h & =1.535 \mathrm{~m}
\end{aligned}
$$

Solve for
18. (c)
$x$-component acceleration,

$$
\begin{aligned}
a_{x} & =u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z} \\
a_{x} & =x^{2} y(2 x y)+y^{2} z\left(x^{2}\right)-\left(2 x y z+y z^{2}\right)(0) \\
& =2 x^{3} y^{2}+x^{2} y^{2} z \\
& =2(3)^{3}(2)^{2}+(3)^{2}(2)^{2}(1) \\
& =2 \times 27 \times 4+9 \times 4 \times 1 \\
& =54 \times 4+36 \\
& =216+36 \\
& =252 \text { units }
\end{aligned}
$$

19. (c)

$$
\begin{aligned}
u & =\frac{\partial \psi}{\partial y} \\
\therefore \text { for face } O B, \quad & =2 x^{2}+(x+t) 2 y \\
x & \Rightarrow 0 \\
u_{O B} & =2 t y
\end{aligned}
$$

Discharge through $O B$

$$
\left.\begin{array}{ll}
\therefore & \\
\text { At } & Q_{O B}
\end{array}=\int_{0}^{2} u_{O B} \cdot 5 d y=\int_{0}^{2} 2 t y \cdot 5 d y\right]
$$

20. (d)

$$
\begin{aligned}
V_{2} & =\frac{Q}{A_{2}}=\frac{1.13 \times 10^{-6}}{\frac{\pi}{4} \times(0.0012)^{2}} \simeq 1 \mathrm{~m} / \mathrm{s} \\
\frac{P_{1}}{\rho g}+\frac{\alpha_{1} V_{1}^{2}}{2 g}+z_{1} & =\frac{P_{2}}{\rho g}+\alpha_{2} \frac{V_{2}^{2}}{2 g}+z_{2}+h_{f} \\
h_{f} & =z_{1}-z_{2}-\frac{\alpha_{2} V_{2}^{2}}{2 g} \\
\Rightarrow \quad h_{f} & =0.6-0-\frac{(2)(1)^{2}}{2 \times 9.81}=0.5 \mathrm{~m} \\
h_{f} & =\frac{32 \mu V L}{\rho g D^{2}} \\
\Rightarrow \quad 0.5 & =\frac{32 \times \mu \times 0.3 \times 1}{9000 \times 0.0012^{2}} \\
\Rightarrow \quad \mu & =6.75 \times 10^{-4} \mathrm{~Pa}-\mathrm{s}
\end{aligned}
$$

21. (b)

$$
\begin{aligned}
R e_{L} & =\frac{U L}{V}=\frac{1.75 \times 5}{1.475 \times 10^{5}} \\
R e_{L} & =5.932 \times 10^{5} \\
C_{f} & =\frac{0.074}{\operatorname{Re}_{L}^{1 / 5}}=\frac{0.074}{\left(5.932 \times 10^{5}\right)^{1 / 5}}=5.183 \times 10^{-3}
\end{aligned}
$$

Drag force on one side of the plate,

$$
\begin{aligned}
F_{d} & =C_{f} \times \operatorname{area} \times \frac{1}{2} \rho U^{2} \\
& =5.183 \times 10^{-3} \times(1.8 \times 5) \times 1.22 \times \frac{(1.75)^{2}}{2} \\
F_{d} & =0.0871 \mathrm{~N}
\end{aligned}
$$

22. (a)

$$
\begin{aligned}
& a_{x}=\frac{d u}{d t}+u \frac{\partial u}{d x}+v \frac{\partial u}{\partial y} \Rightarrow 2 t+\left(t^{2}+3 y\right) 0+(4 t+5 x)(3) \\
& a_{y}=\frac{d v}{d t}+u \frac{\partial v}{d x}+v \frac{\partial v}{\partial y} \Rightarrow 4+\left(t^{2}+3 y\right) 5+(4 t+5 x) 0 \\
&=4+5 t^{2}+15 y \\
& \text { At point }(5,3), \quad \begin{aligned}
a_{x} & =(14 \times 2)+(15 \times 5)=103 \\
a_{y} & =4+\left(5 \times 2^{2}\right)+(15 \times 3)=69 \\
a & =\sqrt{103^{2}+69^{2}}=123.97 \text { units }
\end{aligned}
\end{aligned}
$$

23. (a)


Original volume of

$$
\begin{aligned}
\text { Cylinder } & =\pi r^{2} h \\
V_{1} & =\pi r^{2} \times 1.2
\end{aligned}
$$

Volume of liquid spilled out

$$
=\frac{1}{2} \pi r^{2} \times h
$$

$$
\begin{aligned}
V_{2} & =\frac{1}{2} \pi r^{2} \times 0.6 \\
\therefore \quad & \frac{V_{2}}{V_{1}}
\end{aligned}=\frac{\frac{1}{2} \times 0.6 \pi r^{2}}{\pi r^{2} \times 1.2}=\frac{1}{4}
$$

24. (c)

From force balance at point of contact,


$$
\sigma_{1} \cos (180-\theta)+\sigma_{3}=\sigma_{2}
$$

or

$$
\cos (180-\theta)=\frac{\sigma_{2}-\sigma_{3}}{\sigma_{1}}=-\cos \theta
$$

$$
\therefore \quad \sigma_{1}=0.0720 \mathrm{~N} / \mathrm{m} \quad \text { (liquid and air) }
$$

$$
\sigma_{2}=0.0418 \mathrm{~N} / \mathrm{m} \quad \text { (liquid and solid) }
$$

$$
\sigma_{3}=0.0008 \mathrm{~N} / \mathrm{m} \quad \text { (air and solid) }
$$

$$
\cos \theta=\frac{0.0008-0.0418}{0.072}=-0.56944
$$

$$
\theta=124.7^{\circ}
$$

25. (c)

$$
\begin{aligned}
f & =\frac{64}{R e} \\
R e & =\frac{U D}{v}=\frac{0.1 \times 0.1}{10^{-5}}=1000 \\
f & =\frac{64}{1000}=0.064
\end{aligned}
$$

26. (b)

At stagnation point, $u=0, \quad v=0$
$\Rightarrow \quad x+2 y+2=0,2 x-y=3.5$
On solving above equations,

$$
\begin{aligned}
x & =1, \quad y=-1.5 \\
D & =\sqrt{(x-0)^{2}+(y-0)^{2}}=1.8027 m
\end{aligned}
$$

27. (a)

As per given information,
1.5 m wide, $\rho_{\text {water }}=1000 \mathrm{~kg} / \mathrm{m}^{3}$


The resultant hydrostatic force acting on the dam becomes,

$$
F_{R}=\rho g \bar{x} A=1000 \times 9.81 \times \frac{3.6}{2} \times 3.6 \times 1.5 \mathrm{~N}=95353.2 \mathrm{~N}
$$

The line of action of the force passes through the pressure centre which is $\frac{2 h}{3}$ from the free surface.

$$
\bar{h}=\frac{2 h}{3}=\frac{2 \times 3.6}{3}=2.4 \mathrm{~m}
$$

Taking the moment about point $A$ and setting it equal to zero gives,

$$
\begin{aligned}
\Sigma M_{A} & =0 \\
F_{R}(0.9+\bar{h}) & =W \times 2.4 \\
W & =131110.65 \\
\text { Mass } & =\frac{W}{9.81}=\frac{131110.65}{9.81}=13365 \mathrm{~kg}=13.36 \times 10^{3} \mathrm{~kg}
\end{aligned}
$$

28. (b)

$$
\begin{aligned}
h_{1} & =-\frac{10}{100} \times 13.6=-1.36 \mathrm{~m} \text { of water } \\
h_{2} & =\frac{1 \times 10^{4}}{9810}=1.019 \mathrm{~m} \text { of water } \\
\Delta h & =\left(h_{2}-h_{1}\right)=1.019-(-1.36)=2.379 \mathrm{~m} \\
V & =C \sqrt{2 g \Delta h} \\
& =0.98 \sqrt{2 \times 9.81 \times 2.379}=6.698 \mathrm{~m} / \mathrm{s} \\
\text { Mean velocity in pipe } & =0.85 \mathrm{~V} \\
& =0.85 \times 6.698=5.691 \mathrm{~m} / \mathrm{s} \\
Q & =A_{p} V_{m} \\
& =\frac{\pi}{4} \times(0.3)^{2} \times 5.691 \\
& =0.402 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

29. (b)

$$
\begin{aligned}
& D_{i}=6 \times 10^{-2} \mathrm{~m} \\
& D_{f}=6.9 \times 10^{-2} \mathrm{~m}
\end{aligned}
$$

As soap bubble has two surfaces,
Therefore total change in surface area $=2\left[4 \pi\left(R_{f}^{2}-R_{i}^{2}\right)\right]=2\left[\pi\left(D_{f}^{2}-D_{i}^{2}\right)\right]$

$$
=2(0.003647)=7.294 \times 10^{-3} \mathrm{~m}^{2}
$$

Work input required, $W=\sigma \times \Delta A=0.039 \times 7.294 \times 10^{-3}$

$$
=2.845 \times 10^{-4} \text { Joule }
$$

30. (a)

Given,

$$
\rho=981 \mathrm{~kg} / \mathrm{m}^{3}
$$

and $\quad \tau=0.2452 \mathrm{~N} / \mathrm{m}^{2}$
Velocity gradient, $\quad \frac{d u}{d y}=0.2 \mathrm{~s}^{-1}$
Now, using the equation

$$
\begin{aligned}
\tau & =\mu \frac{d u}{d y} \\
\Rightarrow \quad 0.2452 & =\mu \times 0.2 \\
\Rightarrow \quad \mu & =\frac{0.2452}{0.2}=1.226 \mathrm{Ns} / \mathrm{m}^{2}
\end{aligned}
$$

Kinematic viscosity is given by

$$
\begin{aligned}
v & =\frac{\mu}{\rho}=\frac{1.226}{981}=0.125 \times 10^{-2} \mathrm{~m}^{2} / \mathrm{sec} \\
& =12.5 \mathrm{~cm}^{2} / \mathrm{sec} \\
& =12.5 \text { stokes }
\end{aligned}
$$

