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Electromagnetic Theory

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CLASS TEST 2019-2020

ELECTRICAL ENGINEERING

Date of Test : 07/09/2019

ANSWER KEY ➤ Electromagnetic Theory

1.	(b)	7.	(c)	13.	(a)	19.	(b)	25.	(b)
2.	(d)	8.	(a)	14.	(b)	20.	(a)	26.	(a)
3.	(c)	9.	(c)	15.	(b)	21.	(c)	27.	(d)
4.	(b)	10.	(d)	16.	(a)	22.	(c)	28.	(b)
5.	(c)	11.	(a)	17.	(a)	23.	(a)	29.	(a)
6.	(b)	12.	(a)	18.	(d)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (b)

$\because$ $\quad \text{curl}\,[\vec{A}] = 0;$ irrotational or conservative

2. (d)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.5$$

$\therefore$ Reflected electric field = $E_r = \Gamma \times E_i = 50$ V/m

and Reflected magnetic field = $-50/100 = -\frac{1}{2}$ A/m

4. (b)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Net force on sides parallel to the wire = $\frac{\mu_0 I^2 a}{2\pi a} - \frac{\mu_0 I^2 a}{2\pi 2a}$

1st term attractive, 2nd term repulsive and repulsion less than attraction.

$\therefore$ $$F = \frac{\mu_0 I^2}{4\pi} \text{ towards the wire}$$

6. (b)

$$\nabla \times \vec{A} = \vec{B}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{A}\right) = \nabla \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$\Rightarrow$ $$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

7. (c)

A vector field $\bar{T}$ is said to be irrotational if $\nabla \times \bar{T} = 0$

$$\nabla \times \bar{T} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ T_x & T_y & T_z \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \alpha xy + \beta z^3 & 3x^2 - \gamma z & 3xz^2 - y \end{vmatrix} = 0$$

$\Rightarrow \hat{a}_x(-1+\gamma) - \hat{a}_y(3z^2 - 3\beta z^2) + \hat{a}_z(6x - \alpha x) = 0$

$\gamma = 1, \beta = 1, \alpha = 6$

8. (a)

For a sphere capacitance is given by

$$c = 4\pi\varepsilon_0 a = 4\pi \times \frac{10^{-9}}{30\pi} \times 6.37 \times 10^6 = 0.708 \text{ mF}$$

9. (c)

Since given function is a vector function and Stroke's theorem is valid for any vector function.

10. (d)

Total charges given by

$$Q = \int \rho_s ds = \int_0^1\int_0^1 xy(x^2+y^2+25)^{3/2} dx.dy\, nC$$

$$\Rightarrow \quad x\,dx = \frac{1}{2}dx^2$$

$$\therefore \quad Q = \frac{1}{2}\int_0^1 y\int_0^1 (x^2+y^2+25)^{3/2} d(x^2) dy\, nC = \frac{1}{2}\int_0^1 y\frac{2}{5}\Big[(x^2+y^2+25)^{5/2}\Big]_0^1 dy$$

$$= 5\int_0^1 y(y^2+26)^{5/2} - (y^2+25)^{5/2} dy = \frac{5}{2}(y^2+26)^{5/2} - (y^2+25)^{5/2} dy^2$$

$$= \frac{5}{2}\times\frac{2}{7}\Big[(y^2+26)^{7/2} - (y^2+25)^{7/2}\Big]_0^1 = \frac{5}{7}\Big[(27)^{7/2} - (26)^{7/2} - (26)^{7/2} + (25)^{7/2}\Big]$$

$$Q = 33.15 \text{ nC}$$

11. (a)

The given vector is in cylindrical coordinates,

$$\bar{A} = \rho z \sin\phi\, \hat{a}_\rho + 3\rho z^2 \cos\phi\, \hat{a}_\phi$$

$$\bar{A} = A_\rho \cdot \hat{a}_\rho + A_\phi \cdot \hat{a}_\phi + A_z \cdot \hat{a}_z$$

$$div \cdot \vec{A} = \nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho \cdot A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho \times \rho z \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(3\rho z^2\cos\phi) + 0$$

$$= \frac{1}{\rho}(2\rho z\sin\phi) - \frac{1}{\rho}\cdot(3\rho z^2 \cdot \sin\phi)$$

$$= 2\,z\sin\phi - 3z^2\sin\phi = (2-3z)\,z\sin\phi$$

At point $\left(5, \frac{\pi}{2}, 1\right)$

$$div \cdot \vec{A} = (2 - (3\times 1)) \times 1 \sin\frac{\pi}{2} = -1$$

12. (a)

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\cdot\varepsilon_0}$$

So, $q = EA\,\varepsilon_0 = 10^3 \times 100 \times 10^{-4} \times 8.854 \times 10^{-12} = 8.854 \times 10^{-11}$ C

13. (a)

$$\nabla \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Bx & By & Bz \end{vmatrix} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y + z\cos xz & x & x\cos xz \end{vmatrix}$$

$$= \hat{a}_x(0) - \hat{a}_y(-x\sin xz - z + \cos xz + xz\sin xz - \cos xz) + \hat{a}_z(1-1)$$

$$= 0$$

As $\nabla \times B = 0$, the field is conservative.

$$\nabla.\bar{B} = \frac{\partial}{\partial x}Bx + \frac{\partial}{\partial y}By + \frac{\partial}{\partial z}Bz = -z^2 \sin xz + 0 - x^2 \sin xz \neq 0$$

$$\nabla.\bar{B} \neq 0 \text{ (Not solenoidal)}$$

14. (b)

An emf is not induced if the plane of the coil is parallel to the lines of the magnetic field, since the magnetic flux through the coil does not change when the coil shrinks.

15. (b)

The electric field is given by E = VB

$$= (0.01) \times (37.5) = 0.375 \text{ N/C} \approx 0.38 \text{ N/C}$$

16. (a)

Net flux spreading out of any closed surface is equal to the total charge enclosed

$$\Psi = Q_{enclosed}$$

$$\Psi = \int \rho_v dv = \int_{r=0}^{1} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{r^2} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_{r=0}^{1} dr \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta \, d\theta = \int_{r=0}^{1} dr \int_{\phi=0}^{2\pi} d\phi \, [-\cos\theta]\Big|_0^{\pi}$$

$$= \int_{r=0}^{1} dr \int_{\phi=0}^{2\pi} d\phi \, 2 = 2(2\pi)1 = 4\pi \text{ Coulombs}$$

17. (a)

We know,
$$\nabla \times D = \rho_v$$

$\therefore$
$$\rho_v = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z = 0 + 4x - 0 = 4x$$

$$\text{Charge stored, } Q = \int \rho_v dv = 4\int_1^2 x\,dx \int_1^2 dy \int_{-1}^4 dz$$

$$= 4\left[\frac{x^2}{2}\right]_1^2 [y]_1^2 [z]_{-1}^4 = 4 \times \frac{3}{2} \times 1 \times 5 = 30 \text{ C}$$

18. (d)

We know that if both $\vec{B}$ and $\vec{E}$ are present then, force on an electron is :

$$\vec{F} = q\left[\vec{E} + V \times \vec{B}\right]$$

Since $\vec{F} = 0$

therefore,

$$0 = q\left[\vec{E} + V \times \vec{B}\right]$$

or, $$\vec{E} = -V \times \vec{B} = \vec{B} \times \vec{V} = \begin{vmatrix} 10 & 20 & 30 \\ 3 & 12 & -4 \end{vmatrix} \times 10^5 \times 10^{-3}$$

or, $\vec{E} = \left(-4.4\vec{a}_x + 1.3\vec{a}_y + 11.4\vec{a}_z\right) kV/m$

19. (b)

For spherical co-ordinate, $\frac{1}{r}\frac{\delta}{\delta r}\left[r^2 \frac{dV}{dr}\right] = 0$

$\Rightarrow$ $$V = \frac{-c_1}{r} + c_2$$

(c_1 and c_2 are constants)

Given, $V = 0$

at $r = \infty$

$\Rightarrow$ $c_2 = 0$

$\Rightarrow$ $$V = -\frac{c_1}{r}$$

When $V = V_a$

at $r = a$

$\Rightarrow$ $$V_a = -\frac{c_1}{a}$$

And $c_1 = -a\,V_a$

$\therefore$ $V = \frac{aV_a}{r}$ ..... Laplace equation in spherical co-ordinates.

20. (a)

We know $E = -\nabla N$

$$E = -\left[\frac{\partial v}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial v}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial v}{\partial \phi}\hat{a}_\phi\right]$$

$$= -\left[\frac{-10}{r^2}\sin\theta\cos\phi\,\hat{a}_r + \frac{10}{r^2}\cos\theta\cos\phi\,\hat{a}_r\,\frac{10}{l^2}\sin\phi\right]$$

$$= \frac{10}{r^2}\sin\theta\cos\phi\,\hat{a}_r - \frac{10}{r^2}\cos\theta\cos\phi\,\hat{a}_r + \frac{10}{l^2}\sin\phi\,\hat{a}_\phi$$

$D = \varepsilon_0 E$, at $(2, \pi/2, 0)$

So, $$D = \varepsilon_0\left[\frac{10}{4}\hat{a}_r - 0.+0\right] = 2.5\varepsilon_0\hat{a}_r \text{ C/m}^2 = 22.1\hat{a}_r \text{ pC/m}^2$$

21. (c)

$$\nabla\cdot\vec{D} = \rho$$

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\frac{r^2\theta}{\pi r^2}(1-\cos 3r)\right) = \rho$$

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\frac{\theta}{\pi}(1-\cos 3r)\right) = \rho$$

$$\frac{\theta}{r^2\pi}\left(\frac{\partial}{\partial r}(-\cos 3r)\right) = \rho$$

$$\rho = \frac{3\theta}{r^2\pi}\sin 3r$$

22. (c)

$$V = \frac{kq_1}{L_1}+\frac{kq_2}{L_2}+\frac{kq_3}{L_3}+\frac{kq_4}{L_4}$$

$$q_1 = q \qquad L_1 = L$$
$$q_2 = q \qquad L_2 = L$$
$$q_3 = -q \qquad L_3 = \sqrt{5}L$$
$$q_4 = -q \qquad L_4 = \sqrt{5}L$$

$$V = \frac{1q}{4\pi\epsilon_0}\left(\frac{1}{L}+\frac{1}{L}-\frac{1}{\sqrt{5}L}-\frac{1}{\sqrt{5}L}\right) = \frac{1}{4\pi\epsilon_0}\cdot\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$$

23. (a)

The magnetic field intensity produced due to a small current element $I\,dl$ is defined as

$$d\vec{H} = \frac{I\,dl\times\hat{a}_R}{4\pi R^2}$$

where dl is the differential line vector and $\hat{a}_R$ is the unit vector directed towards the point where field is to be determined. So for the circular current carrying loop, we have

$$dl = a\,d\phi\,\hat{a}_\phi$$

$$\hat{a}_R = -\hat{a}_\rho$$

Therefore the magnitude field intensity produced at the center of the circular loop is

$$\vec{H} = \int_{\phi=0}^{2\pi}\frac{Iad\phi\hat{a}_\phi\times(-\hat{a}_P)}{4\pi a^2} = \frac{I_a}{4\pi a^2}[\phi]_0^{2\pi}\,\hat{a}_z = \frac{I}{2a}\hat{a}_z \text{ A/m}$$

24. (c)

Given, Magnetization, $M = 2.8$ A/m,

$$\chi_m = 0.0025$$

The magnetic field intensity in a material,

$$H = \frac{M}{\chi_m} = \frac{2.8}{0.0025} = 1120 \text{ A/m}$$

The flux density, $B = (1+\chi_m)\mu_0 H = (1+0.0025)\times 4\pi\times 10^{-7}\times 1120$

$$= 1.411 \text{ mT}$$

25. (b)

Given,
$$\vec{A} = yz\hat{a}_x + xy\hat{a}_y + xz\hat{a}_z$$

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xy & xz \end{vmatrix}$$

$$\nabla\times\vec{A} = \hat{a}_x(0-0) - \hat{a}_y(z-y) + \hat{a}_z(y-z)$$

$$= -\hat{a}_y(z-y) + \hat{a}_z(y-z)$$

At point (0, 1, 2)

$$\nabla\times\vec{A} = -\hat{a}_y(2-1) + \hat{a}_z(1-2)$$

$$\nabla\times\vec{A} = -\hat{a}_y - \hat{a}_z$$

$$|\nabla\times\vec{A}| = \sqrt{1^2+1^2} = \sqrt{2} = 1.41$$

26. (a)

$$\frac{C}{L} = \frac{2\pi\in}{ln\frac{b}{a}} = \frac{2\pi\times 4\times 8.854\times 10^{-12}}{ln 5} = 138.3 \text{ pF}$$

27. (d)

According to Gauss Law, the volume charge density in a certain region is equal to the divergence of electric flux density in that region.

i.e.
$$\rho_v = \nabla.\vec{D}$$

$$= \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right).\left[(3y^2+4z)\hat{a}_x + 2xy\,\hat{a}_y + 4x\,\hat{a}_z\right]$$

$$\rho_v = 2x$$

So, total charge enclosed by the cube is:

$$Q = \int \rho_v dV = \int_0^2\int_0^2\int_{-1}^1 (2x)(dx\, dy\, dz) = \left[x^2\right]_0^2 \times 2\times 2$$

Hence,
$$Q = 16 \text{ C}$$

28. (b)

According to Ampere's circuital law the contour integral of magnetic field intensity in a closed path is equal to the current enclosed by the path.

i.e. $$\oint \vec{H}.d\vec{l} = I_{enclosed}$$

Now using right hand rule, we obtain the direction of the magnetic field intensity in the loop which will be opposite to the direction of L.

So, $$\oint \vec{H}.d\vec{l} = -I_{enclosed} = -20 \text{ A}$$

(10 A is not inside the loop so it will not be considered)

29. (a)

Electric field at any point due to infinite surface charge distribution is defined as

$$\vec{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_n$$

Where $\rho_s \rightarrow$ surface charge density,

$\hat{a}_n \rightarrow$ unit vector normal to the sheet directed towards the point where field is to be determined.

At origin electric field intensity due to sheet at $y = 1$ is

$$\vec{E}_1 = \frac{\rho_s}{2\epsilon_0}(-\hat{a}_y) = -\frac{5}{2\epsilon_0}\hat{a}_y$$

$$(\hat{a}_n = -\hat{a}_y)$$

and electric field intensity at origin due to sheet at $y = -1$ is

$$\vec{E}_{-1} = \frac{\rho_s}{2\epsilon_0}(\hat{a}_y) = \frac{5}{2\epsilon_0}\hat{a}_y$$

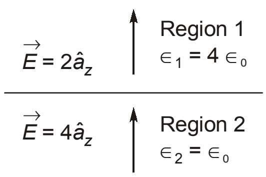
$$(\hat{a}_n = \hat{a}_y)$$

So, net field intensity at origin is

$$\vec{E} = \vec{E}_1 + \vec{E}_{-1} = -\frac{5}{2\epsilon_0}\hat{a}_y + \frac{5}{2\epsilon_0}\hat{a}_y = 0 \text{ V/m}$$

30. (a)

Consider the two dielectric as shown below,



Since the field is normal to the interface so, the fields are

$$E_{1n} = 2$$

and $$E_{2n} = 4$$

From boundary condition, we have

$$D_{1n} - D_{2n} = \rho_s$$

$$\epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \rho_s$$

(where ρ_s is the surface charge density on the interface)

$$4(\epsilon_0)(2) - (\epsilon_0)(4) = \rho_s$$

$$8\epsilon_0 - 4\epsilon_0 = \rho_s$$

$$4\epsilon_0 = \rho_s$$

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