CLASS TEST

Sl.: 01 SPME-EFGHIJK-07082023

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Bhubaneswar | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

STRENGTH OF MATERIALS

MECHANICAL ENGINEERING

Date of Test : 07/08/2023

ANSWER KEY

1.	(a)	7.	(d)	13.	(a)	19.	(c)	25.	(c)
2.	(c)	8.	(b)	14.	(a)	20.	(a)	26.	(c)
3.	(a)	9.	(d)	15.	(b)	21.	(a)	27.	(b)
4.	(a)	10.	(a)	16.	(c)	22.	(c)	28.	(b)
5.	(c)	11.	(c)	17.	(a)	23.	(c)	29.	(b)
6.	(a)	12.	(a)	18.	(b)	24.	(a)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

Elongation due to self weight, $$\Delta = \frac{\gamma L^2}{2E}$$

$$= \frac{\left(89.2\times10^{-6}\right)\times\left(15\times10^{3}\right)^2}{2\times\left(90\times10^{3}\right)} = 0.11 \text{ mm}$$

2. (c)

$$\text{Strain energy, } U = \frac{1}{2}\times P\times\Delta$$

$$= \frac{1}{2}\times P\times\frac{PL^3}{48EI}$$

$$\therefore \quad U = \frac{P^2L^3}{96EI}$$

For $P = 1$; $U = \dfrac{L^3}{96EI}$

3. (a)

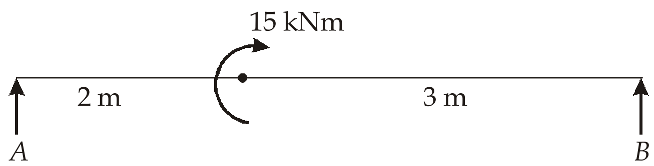
The length of column is very large as compared to its cross-sectional dimensions.

4. (a)

$$\text{Maximum shear stress, } \tau_{max} = \frac{16}{\pi D^3}\sqrt{M^2+T^2}$$

$$= \left[\frac{16}{\pi(100)^3}\sqrt{(8)^2+(6)^2}\right]\times10^6 = \frac{16}{\pi}\times\frac{10\times10^6}{10^6} = 50.93 \text{ MPa}$$

5. (c)



$\Sigma F_y = 0$

$$R_A + R_B = 0$$

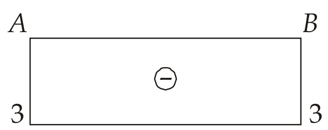
Also, $\Sigma M_A = 0$

$\Rightarrow$ $R_B \times 5 = 15$

$\Rightarrow$ $R_B = 3$ kN

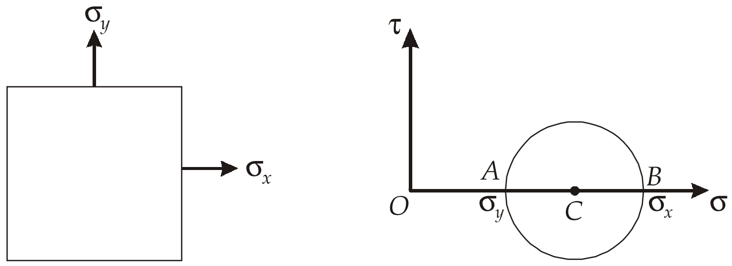
So, $R_A = -3$ kN

Now, the SFD for the beam will be as shown below:



6. (a)

Assuming ($\sigma_x > \sigma_y$)



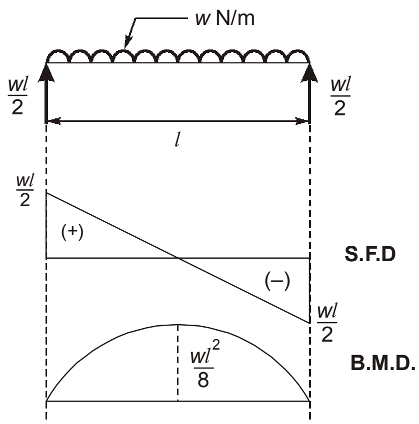
$$AB = \sigma_x - \sigma_y$$

$$AC = \frac{AB}{2} = \frac{\sigma_x - \sigma_y}{2}$$

$\therefore$
$$OC = OA + AC = \sigma_y + \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2}$$

7. (d)



8. (b)

$\frac{T}{J} = \frac{\tau}{R}$, Here T and τ are same, so $\frac{J}{R}$ should be same i.e. polar section modulus will be same.

9. (d)

Toughness is the ability of material to absorb the energy upto facture point i.e. toughness is the total area under stress-strain curve.

10. (a)

$$\text{Circumferential stress} = \frac{Pd_i}{2t}$$

$$P = 6 \text{ MPa}$$
$$d_i = 600 \text{ mm}$$
$$t = 10 \text{ mm}$$

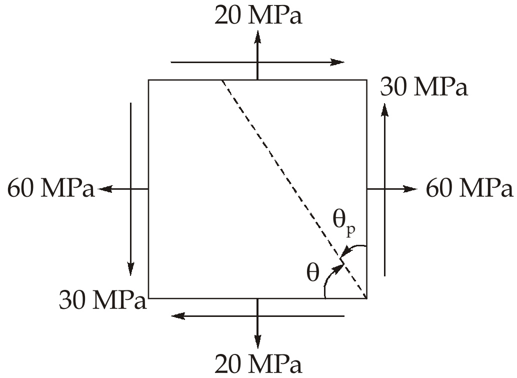
$$\sigma_c = \frac{6\times600}{2\times10} = 180 \text{ MPa}$$

$$= 180 \times 1000 \text{ kPa}$$

$$= 18 \times 10^4 \text{ kPa}$$

11. (c)

Plane having zero shear stress is called principal planes.



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2\times30}{60-20}$$

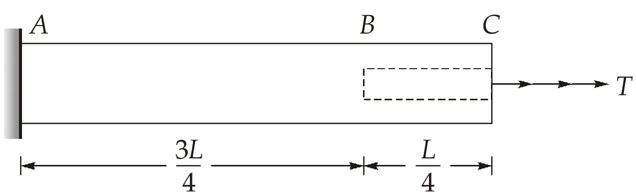
$$\tan 2\theta_p = 1.5$$

$$2\theta_p = \tan^{-1}(1.5)$$

$$\theta_p = 28.15°$$

Required angle from plane B, $\theta = 90° - 28.15° = 61.85°$ (Clockwise)

12. (a)



$$\theta_{AC} = \theta_{AB} + \theta_{BC}$$

$$\theta_C = \frac{T_{AB}\times L_{AB}}{GJ_{AB}} + \frac{T_{BC}\times L_{BC}}{GJ_{BC}} \quad (\because \theta_A = 0)$$

$$J_{BC} = \frac{\pi}{32}\left[D^4 - \left(\frac{D}{2}\right)^4\right]$$

$$J_{BC} = \frac{\pi}{32}D^4\left[1-\frac{1}{16}\right]$$

$$J_{BC} = \frac{15}{16}J_{AB} = \frac{15}{16}J$$

$$\therefore \qquad \theta_C = \frac{T\times\frac{3L}{4}}{GJ} + \frac{T\times\frac{L}{4}}{G\times\frac{15}{16}J}$$

$$= \frac{3}{4}\frac{TL}{GJ} + \frac{4}{15}\frac{TL}{GJ}$$

$$= \frac{45TL+16TL}{60GJ} = \frac{61}{60}\frac{TL}{GJ}$$

13. (a)

Given, M = 54.0 kNm and T_z = 72.0 kNm

Consider external and internal diameter as D and d (= 0.5 D) respectively,

Now, section modulus, $Z = \frac{\pi D^3}{32}\left(1-\frac{d^4}{D^4}\right) = \frac{\pi D^3}{32}\left(1-\left(\frac{1}{2}\right)^4\right) = \frac{15\pi D^3}{512}$

Polar section modulus,

$$Z_p = \frac{\pi D^3}{16}\left(1-\frac{d^4}{D^4}\right) = \frac{15\pi D^3}{256}$$

Maximum shear stress is given by,

$$\tau_{max} = \frac{T}{Z_p} = \left(\frac{256}{15\pi D^3}\right)\times T_e$$

$$T_e = \sqrt{M^2+T^2} = \sqrt{54^2+72^2} = 90 \text{ kNm}$$

Now, $$96 = \left(\frac{256}{15\pi D^3}\right)\times 90\times 10^6 \qquad (\because \tau_{permissible} = 96 \text{ MPa})$$

$$D^3 = \frac{256}{15\pi}\times\frac{90\times 10^6}{96} = 5092958$$

$$D = 172.05 \text{ mm}$$

14. (a)

Principal strains, $$\varepsilon_{1/2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= \left[\frac{800+200}{2} \pm \sqrt{\left(\frac{800-200}{2}\right)^2+\left(\frac{-600}{2}\right)^2}\right]\times 10^{-6}$$

$$\varepsilon_1 = 924.264\times 10^{-6}$$

$$\varepsilon_2 = 75.74\times 10^{-6}$$

Thus major principal stress is,

$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2) = \frac{200\times10^3}{1-0.3^2}(924.264 + 0.3\times75.74)\times10^{-6}$$

$$= 208.13 \text{ MPa}$$

15. (b)

When both ends are clamped, $(l_{\text{eff}})_1 = \frac{l}{2}$

When one end is free, $(l_{\text{eff}})_2 = 2l$

Buckling load, $P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}$

So, $(P_{cr})_1 = \frac{4\pi^2 EI}{l^2}$

Similarly, $(P_{cr})_2 = \frac{\pi^2 EI}{4l^2}$

So,
$$\% \text{ change} = \frac{\frac{4\pi^2EI}{l^2} - \frac{\pi^2EI}{4l^2}}{\frac{4\pi^2EI}{l^2}} \times 100 = \frac{4-(1/4)}{4}\times 100$$

$$= \left(1-\frac{1}{16}\right)\times 100 = 93.75\%$$

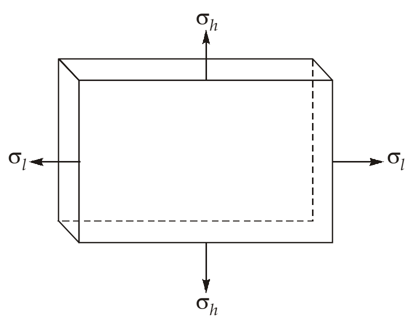
16. (c)

For a closed cylinder (thin), the two stress components induced due to internal pressure are,

$$\sigma_h = \frac{pd}{2t} \qquad \text{(Hoop stress)}$$

$$\sigma_l = \frac{pd}{4t} \qquad \text{(Longitudinal stress)}$$

If we neglect the pressure in radial direction, this becomes a plane stress condition.



For this condition, $\tau_{\max} = \max.\left\{\frac{\sigma_h}{2}, \frac{\sigma_l}{2}, \frac{\sigma_h - \sigma_l}{2}\right\} = \frac{pd}{4t}$

For safety, $$\tau_{max} \le \frac{(f_y/2)}{\text{FOS}}$$

$$\Rightarrow \quad \frac{p \times 2 \times 100}{4 \times 5} = \frac{100/2}{2}$$

$$\Rightarrow \quad p = 2.5 \text{ MPa}$$

17. (a)

$$\frac{1}{2} \times P \times \delta l = 50 \qquad \in = \frac{\delta l}{l}$$

or $$\frac{1}{2} P \times \in \times l = 50$$

or $$\in l = \frac{100}{10} = 10 \text{ m}$$

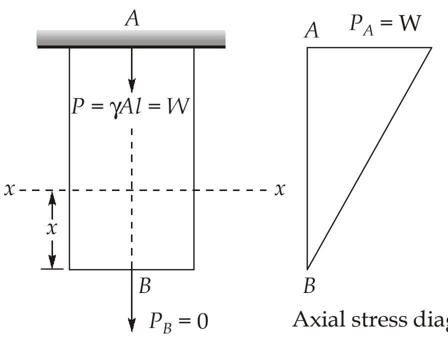
18. (b)

$$M = 80 \cdot x - 64(x - 1) \ \forall \ x \in (1, 4)$$

At centre $$x = 4 \text{ m}$$

$$M = (80 \times 4) - 64(3) = 128 \text{ kNm}$$

19. (c)



Axial stress diagram

$$P_{x-x} = \gamma \times A \times x$$

where γ is load density

$$(\sigma_{axial})_{x-x} = \frac{P_{x-x}}{A_{x-x}}$$

$$(\sigma_{axial})_{x-x} = \gamma \times x$$

Axial stress diagram is triangle and independent of area.

20. (a)

$$K = \frac{E}{3(1-2\mu)} = \frac{100}{3(1-2\times0.2)} = \frac{100}{3\times0.6} = 55.555 \text{ GPa}$$

$$\sigma_x = \sigma_y = \sigma_z = \frac{P}{A} = \frac{250\times10^3}{40\times40\times10^{-6}} = 156.25 \text{ MPa}$$

$$K = \frac{\sigma}{\epsilon_v} = \frac{\sigma\times V}{(\Delta V)}$$

$$\Delta V = \frac{156.25\times(40)^3}{55.555\times10^3}\text{mm}^3$$

Change in volume, $\Delta V = 180 \text{ mm}^3$

21. (a)

$$\sigma_h = \frac{pd}{2t\times\eta_{LJ}} = \frac{6\times150}{2\times12.5\times0.8} = 45 \text{ MPa}$$

$$\sigma_l = \frac{pd}{4t\times\eta_{CJ}} = \frac{6\times150}{4\times12.5\times0.9} = 20 \text{ MPa}$$

$$\frac{\delta d}{d} = \frac{1}{E}(\sigma_h - \mu\sigma_L) = \frac{1}{200\times10^3}(45-0.25\times20)$$

$$\frac{\delta d}{d} = 0.2\times10^{-3}$$

$$\delta d = 0.2\times150\times10^{-3} \text{ mm}$$

$$\delta d = 0.03 \text{ mm}$$

22. (c)

In case of composite bar,

$$(\delta_{th})_{Cu} - (\delta_{axial})_{Cu} = (\delta_{th})_{steel} + (\delta_{axial})_{steel}$$

$$(\delta_{axial})_{Cu} + (\delta_{axial})_{steel} = (\delta_{th})_{Cu} - (\delta_{th})_{steel}$$

$$\frac{\sigma_1 L}{E_1} + \frac{\sigma_2 L}{E_2} = \alpha_1 TL - \alpha_2 TL$$

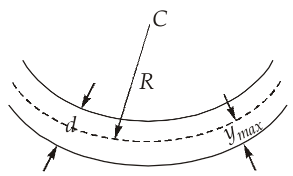
$$\left(\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2}\right)L = (\alpha_1 - \alpha_2)TL$$

$$\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} = (\alpha_1 - \alpha_2)T$$

23. (c)

Diameter of wire, $d = 20$ mm

So, $\qquad y_{max} = \frac{d}{2} = \frac{20}{2} = 10 \text{ mm}$



Radius of curvature, $R = 10$ m

Bending equation

$$\frac{\sigma_{max}}{y_{max}} = \frac{E}{R}$$

$$\Rightarrow \quad \frac{\sigma_{max}}{10\times10^{-3}} = \frac{200\times10^3}{10}$$

$$\Rightarrow \quad \sigma_{max} = 200 \text{ MPa}$$

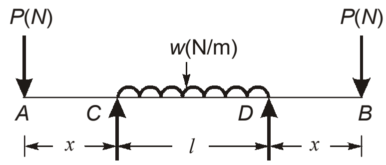
24. (a)

$$\Delta L_s = \Delta L_A$$

$$\Rightarrow \quad \left(\frac{PL}{AE}\right)_S = \left(\frac{PL}{AE}\right)_A$$

$$\frac{P_s}{P_A} = \frac{A_S E_S/L_S}{A_A E_A/L_A} = \frac{0.5\times200/2}{2\times100/1} = 0.25$$

25. (c)



From the symmetry of the figure,

$$R_C = R_D = P + \frac{wl}{2}$$

Bending moment at mid point,

$$= -\frac{wl}{2}\times\frac{l}{4} + R_C\times\frac{l}{2} - P\left(x+\frac{l}{2}\right) = 0$$

gives $$x = \frac{wl^2}{8P}$$

26. (c)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \quad \text{or} \quad \tau = \frac{GR\theta}{L}$$

$$\tau \propto \frac{1}{L}$$

27. (b)

$$\text{Area} = 50\times110 = 5500 \text{ mm}^2$$

$$P = 30 \text{ kN} = 30\times10^3 \text{ N}$$

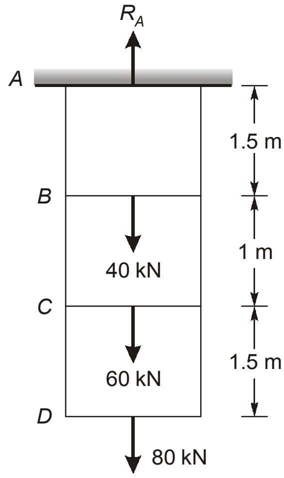
$$\text{Young's modulus} = \frac{\text{Longitudnal stress}}{\text{Longitudnal strain}}$$

$$\text{Stress} = \frac{F}{A} = \frac{30\times10^3}{5500} = 5.4545 \text{ N/mm}^2$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{0.625}{1500} = 4.16\times10^{-4}$$

$$E = \frac{5.4545}{4.16\times10^{-4}} = 13090.90 \text{ N/mm}^2$$

28. (b)



$$R_A = 40 + 60 + 80 = 180 \text{ kN}$$

Force P_1 on portion AB = 180 kN (tensile)

Force P_2 on portion BC = 180 – 40 = 140 kN (tensile)

Force P_3 on portion CD = 80 kN (tensile)

$$\Delta = \frac{1}{AE}(P_1L_1 + P_2L_2 + P_3L_3)$$

$$\Delta = \frac{1}{1200 \times 2.05 \times 10^5} \times [180 \times 1500 + 140 \times 1000 + 80 \times 1500] \times 10^3$$

$$= \frac{530000 \times 10^3}{1200 \times 2.05 \times 10^5} = 2.15 \text{mm}$$

29. (b)

$$J_s = \frac{\pi}{32} \times (50)^4 = 613592 \text{ mm}^4$$

$$J_b = \frac{\pi}{32} \times \left(75^4 - 50^4\right) = 2492719 \text{ mm}^4$$

$$T_s = \frac{G_s J_s \theta}{l} \quad \text{and} \quad T_b = \frac{G_b J_b \theta}{l}$$

The total torque, $T = T_s + T_b = (G_s J_s + G_b J_b)\frac{\theta}{l}$

$$\theta = \frac{Tl}{G_s J_s + G_b J_b} = \frac{\left(800 \times 10^3\right)\left(1.5 \times 10^3\right)}{\left(8 \times 10^4 \times 613592\right) + \left(4 \times 10^4 \times 2492719\right)}$$

$$= 0.008065 \text{ radian} = 0.462°$$

30. (a)

$$\text{Total strain energy} = \frac{1}{2}P_1\delta_1 + \frac{1}{2}P_2\delta_2 + \frac{1}{2}P_3\delta_3$$

$$= \frac{1}{2} \times P \times \frac{PL_1}{A_1E} + \frac{1}{2}P \times \frac{PL_2}{A_2E} + \frac{1}{2} \times P \times \frac{PL_3}{A_3E}$$

$$= \frac{P^2}{2E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

$$= \frac{(10 \times 1000)^2}{2 \times 200 \times 10^9}\left[\frac{100 \times 4 \times 1000}{\pi \times (30)^2} + \frac{120 \times 4 \times 1000}{\pi \times (20)^2} + \frac{80 \times 4 \times 1000}{\pi \times (10)^2}\right]$$

$$= 0.3855 \text{ N-m}$$

■■■■