CLASS TEST

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 07/08/2023

ANSWER KEY

1.	(b)	7.	(b)	13.	(c)	19.	(b)	25.	(b)
2.	(d)	8.	(a)	14.	(b)	20.	(b)	26.	(c)
3.	(a)	9.	(b)	15.	(b)	21.	(a)	27.	(d)
4.	(b)	10.	(b)	16.	(a)	22.	(b)	28.	(a)
5.	(b)	11.	(c)	17.	(a)	23.	(c)	29.	(c)
6.	(b)	12.	(c)	18.	(a)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

Given matrix,
$$A = \begin{bmatrix} 21 & 17 & 7 & 10 \\ 24 & 22 & 6 & 10 \\ 6 & 8 & 2 & 3 \\ 5 & 7 & 1 & 2 \end{bmatrix}$$

Operating, $R_1 \to R_1 - R_2 - R_4$, $R_2 \to R_2 - 3R_3$

and $R_3 \to R_3 - 2R_4$

$$A = \begin{bmatrix} -8 & -12 & 0 & -2 \\ 6 & -2 & 0 & 1 \\ -4 & -6 & 0 & -1 \\ 5 & 7 & 1 & 2 \end{bmatrix}$$

$$|A| = -\begin{vmatrix} -8 & -12 & -2 \\ 6 & -2 & 1 \\ -4 & -6 & -1 \end{vmatrix} = 0$$

2. (d)

The characteristic equation of the matrix A is

$$\begin{bmatrix} (2-\lambda) & 1 & 1 \\ 0 & (1-\lambda) & 0 \\ 1 & 1 & (2-\lambda) \end{bmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

According to Cayley-Hamilton theorem, we have

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$$
$$= A^2 + A + I$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$A^2 + A + I = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

3. (a)

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$(3 - \lambda)(2 - \lambda)(5 - \lambda) = 0$$
$$\lambda = 2, 3, 5$$

4. (b)

Consider,
$$(A:B) = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & a & b \end{array}\right]$$

Applying, $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 5R_1$
$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & -6 & a-10 & b-10 \end{array}\right]$$

Applying, $R_3 \to R_3 - 2R_2$
$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & a-8 & b-6 \end{array}\right]$$

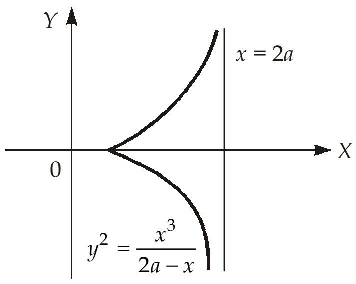
$a = 8$ and $b = 6$

$\therefore$ Infinite many solution.

5. (b)

Area between the curve and the asymptote

$$= 2\int_0^{2a} y\,dx = 2\int_0^{2a} \sqrt{\left(\frac{x^3}{2a-x}\right)}\,dx$$



Put, $x = 2a\sin^2\theta$

so that, $dx = 4a\sin\theta\cos\theta\,d\theta$

$$= 2\int_0^{\pi/2} \sqrt{\frac{(2a\sin^2\theta)^3}{2a\cos^2\theta}} \cdot 4a\sin\theta\cos\theta\,d\theta$$

$$= 16a^2 \int_0^{\pi/2} \sin^4\theta\,d\theta$$

$$= 16a^2 \cdot \frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = 3\pi a^2$$

6. (b)

Given: $x + 2y - 3z = 1$, $(\lambda + 3)z = 3$, $(2\lambda + 1)x + z = 0$.

Given equations are non homogeneous system of equation of the form,

$$AX = B$$

For inconsistent, $\rho(A) \neq \rho(A/B)$

Hence,
$$[A/B] = \begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 0 & 0 & \lambda+3 & : & 3 \\ 2\lambda+1 & 0 & 1 & : & 0 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$\Rightarrow \qquad [A/B] = \begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 2\lambda+1 & 0 & 1 & : & 0 \\ 0 & 0 & \lambda+3 & : & 3 \end{bmatrix}$$

For inconsistent, $\lambda + 3 = 0$

$\Rightarrow \qquad \lambda = -3$

7. **(b)**

Given: $$I = \int_0^{\infty} \frac{dx}{e^x + e^{-x}} = \int_0^{\infty} \frac{e^x dx}{1+e^{2x}}$$

Put, $e^x = t$

$e^x dx = dt$

$$\therefore \qquad I = \int_1^{\infty} \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_1^{\infty} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

8. **(a)**

For any given $\vec{F}$,

$$\text{div}\left(\text{cur}\,\vec{F}\right) = 0 \qquad \text{(Always)}$$

Hence, $$\nabla\cdot\left(\nabla \times \vec{F}\right) = 0$$

9. **(b)**

For binomial distribution,

$$\text{Mean} = np = 9 \qquad \text{... (i)}$$

$$\text{Variance} = npq = \sigma^2 = 6 \qquad \text{... (ii)}$$

From (i) and (ii), $$q = \frac{6}{9} = \frac{2}{3}$$

$$p = 1 - q = \frac{1}{3}$$

$$n \times \frac{1}{3} = 9$$

$$n = 27$$

10. **(b)**

Given: $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

$\because$ Given matrix is upper triangular matrix, hence its eigen values are $\lambda = 1, 3, -2$

For $3A^3 + 5A^2 - 6A + 2I$

(i) First eigen value = $3(1)^3 + 5(1)^2 - 6(1) + 2(1) = 4$

(ii) Second eigen value = $3(3)^3 + 5(3)^2 - 6(3) + 2(1) = 110$

(iii) Third eigen value = $3(-2)^3 + 5(-2)^2 - 6(-2) + 2(1) = 10$

Sum of the eigen values are = 4 + 110 + 10 = 124

11. (c)

Clearly, $n(S) = 6 \times 6 = 36$

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then,

$$E = \{(1, 3), (1, 5), (2, 2), (2, 4), (2, 6)\ (3, 1)\ (3, 3), (3, 5)\ (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)\}$$

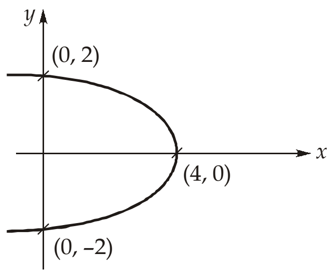
$\therefore$ $n(E) = 14$

Hence, $$P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}$$

12. (c)

Given,

equation of curve $x = 4 - y^2$



$$I = \int_C (y^3 dx + x^2 dy)$$

$\because$ $x = 4 - y^2$

$\therefore$ $dx = -2y\, dy$

So, $$I = \int_{-2}^{2} \left[y^3(-2y\, dy) + (4 - y^2)dy\right]$$

$$= \int_{-2}^{2} \left[-2y^4 + 16 + y^4 - 8y^2\right]dy$$

$$= \int_{-2}^{2} (16 - 8y^2 - y^4)dy$$

$$= 2\int_{0}^{2} (16 - 8y^2 - y^4)dy$$

$$= 2\left[16(y) - \frac{8y^3}{3} - \frac{y5}{5}\right]_0^2$$

$$= 2\left[16(2) - \frac{8}{3}(8) - \frac{1}{5}(32)\right]$$

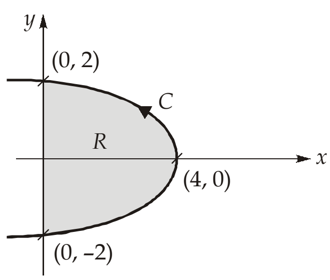
$$= 2\left[32 - \frac{64}{3} - \frac{32}{5}\right] = \frac{128}{15}$$

Alternate Method :

Given cure 'C' formed a closed region from(0, –2) to (0, 2)

$\therefore$ $$I = \oint_C y^3 dx + x^2 dy$$

By Green theorem,



$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$

$$= \iint_R (2x - 3y^2) dx\, dy$$

$$= \int_{-2}^{2} \int_{x=0}^{x=4-y^2} (2x - 3y^2)\,dx\,dy$$

$$= \int_{-2}^{2} \int_{x=0}^{x=4-y^2} (x^2 - 3y^2 x)\,dy$$

$$= \int_{-2}^{2} \left[(4-y^2)^2 - 3y^2(4-y^2)\right] dy$$

$$= \int_{-2}^{2} (16 + y^4 - 8y^2 - 12y^2 + 3y^4)\,dy$$

$$= \int_{-2}^{2} (4y^4 - 20y^2 + 16)\,dy$$

$$= 2\left[\frac{4y^5}{5} - \frac{20y^3}{3} + 16y\right]_0^2$$

$$= 2\left[\frac{4}{5}\times 32 - \frac{20}{3}\times 8 + 32\right] = \frac{128}{15}$$

13. (c)

Integrating the inside equation we get,

$$\lim_{x\to 0} \int_0^{g(x)} \frac{2t}{x}\,dt = \lim_{x\to 0}\left[\frac{t^2}{x}\right]_0^{g(x)} = \lim_{x\to 0} \frac{[g(x)^2] - 0}{x}$$

Now applying L-Hospital's rule,

$$\lim_{x\to 0} \frac{2g(x)\,g'(x)}{1}$$

Given, $g(0) = 0$ and $g'(0) = 2$

$$\lim_{x\to 0} \frac{2(0)\times 2}{1} = 0$$

14. (b)

The given data set is arranged in the increasing order as: 4, 9, 11, 15, 18, 18

$$\text{Median of these data} = \frac{(11+15)}{2} = 13$$

Mode of these data = 18

As the number 18 is having more frequency of occurrence.

15. (b)

$$\frac{dx}{dt} = 10 - \frac{x}{20} = \frac{200-x}{20}$$

$$\frac{dx}{200-x} = \frac{dt}{20}$$

$$\therefore \quad \int_0^{100} \frac{dx}{200-x} = \int_0^t \frac{dt}{20}$$

$$-[\ln(200-x)]_0^{100} = \frac{t}{20}$$

$$-\left[\ln\left[\frac{200-100}{200}\right]\right] = \frac{t}{20}$$

$$-[\ln(1) - \ln(2)] = \frac{t}{20}$$

$$\ln 2 = \frac{t}{20}$$

$$t = 20 \ln(2) \text{ minutes}$$

16. (a)

Let the point of contact is $(2t^2, 4t)$

$$y^2 = 8x$$

$$2y\frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \frac{4}{y} = \frac{4}{2 \times 2t} = \frac{1}{t}$$

Equation of tangent

$$(y - 0) = \frac{1}{t}(x+2)$$

$$4t = \frac{1}{t}(2t^2+2)$$

$$4t^2 = 2t^2 + 2$$

$$t = \pm 1$$

Since the point is in first quadrant,

$$t = 1$$

$$\text{The coordinates of point} = (2 \times 1^2, 4 \times 1) = (2, 4)$$

17. (a)

The complex number z can be represented in rectangular form as $x + iy$ and in polar form as $re^{i\phi}$,

$$z = x + iy = r\cos\phi + ir\sin\phi = re^{i\phi}$$

where, $$r = |z| = \sqrt{x^2+y^2} = \sqrt{2^2+2^2} = 2\sqrt{2}$$

and $$\phi = \tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$$

Hence, $$z = 2\sqrt{2}\, e^{i\pi/4}$$

18. (a)

(a)
$$\frac{dy}{dx} = \frac{y}{x}$$
$$\frac{dy}{y} = \frac{dx}{x}$$
$$\int \frac{dy}{y} = \int \frac{dx}{x}$$
$$\ln y = \ln x + \ln c$$
$$y = cx \text{ Equation of straight line}$$

(b)
$$\frac{dy}{dx} = -\frac{y}{x}$$
$$\frac{dy}{y} = -\frac{dx}{x}$$
$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$
$$\ln y = -\ln x + \ln c$$
$$y = \frac{c}{x} \text{ Equation of hyperbola}$$

(c)
$$\frac{dy}{dx} = \frac{x}{y}$$
$$\int y\,dy = \int x\,dx$$
$$y^2 - x^2 = c^2$$
$$\frac{y^2}{c^2} - \frac{x^2}{c^2} = 1 \text{ Equation of hyperbola}$$

(d)
$$\frac{dy}{dx} = -\frac{x}{y}$$
$$\int y\,dy = -\int x\,dx$$
$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{c^2}{2}$$
$$\frac{y^2}{2} + \frac{x^2}{2} = \frac{c^2}{2}$$
$$x^2 + y^2 = c^2 \text{ Equation of circle}$$

19. (b)

$\because$ α, β and γ are the roots of the equation $x^3 + px + q = 0$

Then, $\alpha + \beta + \gamma = 0$ and $\alpha^3 + p\alpha + q = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = p$ and $\beta^3 + p\beta + q = 0$

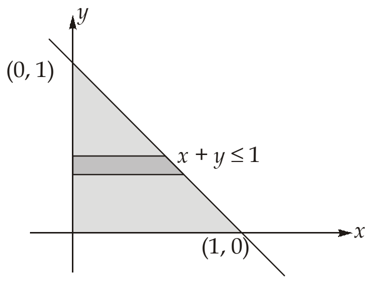
$\alpha\beta\gamma = -q$ and $\gamma^3 + p\gamma + q = 0$

$$\therefore \quad \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \alpha(\gamma\beta - \alpha^2) - \beta(\beta^2 - \alpha\gamma) + \gamma(\alpha\beta - \gamma^2)$$

$$= \alpha\beta\gamma - \alpha^3 - \beta^3 + \alpha\beta\gamma + \alpha\beta\gamma - \gamma^3$$
$$= 3\alpha\beta\gamma - (\alpha^3 + \beta^3 + \gamma^3)$$
$$= 3(-q) - [-p\alpha - q - p\beta - q - p\gamma - q]$$
$$= -3q + p\alpha + q + p\beta + q + p\gamma + q$$
$$= -3q + p(\alpha + \beta + \gamma) + 3q$$
$$= p(0) = 0$$

20. (b)

Let, $$I = \iint \left(x^2 + y^2\right) dxdy$$



$$= \int_{y=0}^{y=1} \int_{x=0}^{x=1-y} \left(x^2 + y^2\right) dxdy$$
$$= \int_{y=0}^{y=1} \left[\frac{x^3}{3} + y^2 x\right]_0^{1-y} dy$$
$$= \int_{y=0}^{1} \left[\frac{(1-y)^3}{3} + y^2(1-y)\right] dy$$
$$= \int_{y=0}^{1} \left[\frac{(1-y)^3}{3} + \left(y^2 - y^3\right)\right] dy$$
$$= \left[\frac{(1-y)^4}{3 \times 4} \times \frac{1}{-1} + \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = \left[\frac{(1-y)^4}{-12} + \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1$$
$$= \frac{1}{-12}[0-1] + \frac{1}{3} - \frac{1}{4}$$
$$= \frac{1}{12} + \frac{4-3}{12} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

21. (a)

$$P(2) = 9P(4) + 90P(6)$$

For Poisson's distribution,

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$, where λ is the mean of Poisson's distribution

Hence, $$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$\Rightarrow$ $$\frac{e^{-\lambda}\lambda^2}{2} = 9\frac{e^{-\lambda}\lambda^4}{24} + 90\frac{e^{-\lambda}\lambda^6}{720}$$

$\Rightarrow$ $$= \frac{e^{-\lambda}\lambda^6}{8} + \frac{3e^{-\lambda}\lambda^4}{8} - \frac{e^{-\lambda}\lambda^2}{2} = 0$$

$\Rightarrow$ $$\frac{e^{-\lambda}\lambda^2}{2}\left[\frac{\lambda^4}{4} + \frac{3\lambda^2}{4} - 1\right] = 0$$

Given, $\lambda \neq 0$

$$\therefore \qquad \left[\frac{\lambda^4}{4}+\frac{3\lambda^2}{4}-1\right] = 0$$

$$\Rightarrow \qquad \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\Rightarrow \qquad \lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$$

$$\Rightarrow \qquad \lambda^2(\lambda^2 + 4) - 1(\lambda^2 + 4) = 0$$

$$\Rightarrow \qquad (\lambda^2 - 1)(\lambda^2 + 4) = 0$$

$$\Rightarrow \qquad \lambda^2 = 1, \lambda^2 + 4 \neq 0$$

$$\Rightarrow \qquad \lambda = \pm 1$$

22. (b)

Given differential equation is

$$\frac{d^2y}{dx^2}+\frac{7dy}{dx}+12y = 0$$

$$\Rightarrow \qquad m^2 + 7m + 12 = 0$$

$$\Rightarrow \qquad (m+3)(m+4) = 0$$

$$\therefore \qquad m = -3, -4$$

$$\text{C.F. is } y = c_1 e^{-3x} + c_2 e^{-4x}$$

Given, $\qquad y(0) = 1$

$$\Rightarrow \qquad y(0) = 1 = c_1 + c_2 \qquad \text{...(i)}$$

$$y'(x) = -3c_1e^{-3x} - 4c_2e^{-4x}$$

$$y'(0) = -3c_1 - 4c_2 \qquad \text{...(ii)}$$

From (i) and (ii)

$$c_2 = -3, c_1 = 4$$

Hence, solution is $\qquad y = 4e^{-3x} - 3e^{-4x} = 0$ (given)

23. (c)

$$\int_c \frac{dz}{(z^2+4z+13)}$$

Poles $\qquad z^2 + 4z + 13 = 0$

$$z = \frac{-4\pm\sqrt{16-52}}{2}$$

$$z = -2 \pm 3i$$

Pole $\qquad z = -2 + 3i$ lies inside the circle $|z+1-2i| = 2$

$$\underset{z=-2+3i}{\text{Res}} = \lim_{z\to -2+3i}(z+2-3i)\frac{1}{(z+2-3i)(z+2+3i)}$$

$$= \frac{1}{-2+3i+2+3i} = \frac{1}{6i}$$

Hence, $\qquad \int_c \frac{dz}{z^2+4z+13} = 2\pi i\left(\frac{1}{6i}\right) = \frac{\pi}{3}$

24. (d)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin(nx)dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\sin(nx)dx + \int_{-\pi}^{\pi} x^2\sin(nx)dx\right]$$

$$= \frac{2}{\pi}\int_0^{\pi} x\sin(nx)dx$$

$$= \frac{2}{\pi}\left[x\left(\frac{-\cos(nx)}{n}\right) - 1\left(\frac{-\sin(nx)}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-(\pi)\frac{\cos n\pi}{n}\right]$$

$$= \frac{-2}{n}(-1)^n = \frac{2}{n}(-1)^{n+1}$$

25. (b)

Curve 1: $y^2 = 16x$

Curve 2: $x^2 = 16y$

Intersection points of curve 1 and 2,

$$y^2 = 16x = 16\sqrt{16y} = 64\sqrt{y}$$

$$y^4 = 64 \times 64 \times y$$

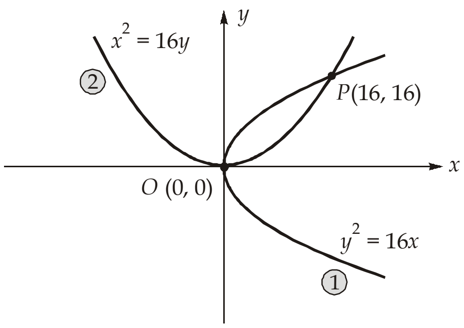
$$y^3 = 64 \times 64$$

$$y = 16$$

and $y = 0$

then $x = 16$ and $x = 0$

Therefore intersection points are $P(16, 16)$ and $O(0, 0)$. The area enclosed between curves 1 and 2 are given by



$$\text{Area} = \int_{x_1}^{x_2} y_1 dx - \int_{x_1}^{x_2} y_2 dx$$

$$= \int_0^{16}\sqrt{16x}\,dx - \int_0^{16}\frac{x^2}{16}dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^{16} - \left[\frac{x^3}{48}\right]_0^{16} = \frac{8}{3}[64-0] - \frac{1}{48}\left[16^3 - 0\right]$$

$$= 170.66 - 85.33 = 85.33$$

26. (c)

Comparing the given equation with general form of second order partial differential equation,

$$A = 1,$$

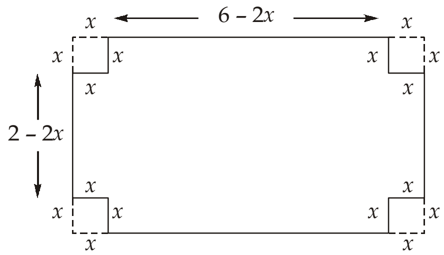
$$B = \frac{1}{2},$$

$$C = 0$$

$$\Rightarrow \quad B^2 - 4AC = \frac{1}{4} > 0$$

$\therefore$ PDE is hyperbolic.

27. (d)



Let the side of each of the square cut off be x m and the sides of the base are $6 - 2x$, $2 - 2x$ m.

$$\therefore \quad \text{Volume } V \text{ of the box} = x(6 - 2x)(2 - 2x)$$

$$= 4(x^3 - 4x^2 + 3x)$$

Then, $$\frac{dV}{dx} = 4(3x^2 - 8x + 3)$$

For volume to be maximum

$$\frac{dV}{dx} = 0 \text{ and } \frac{d^2V}{dx^2} < 0$$

$$3x^2 - 8x + 3 = 0$$

$$\therefore \quad x = \frac{8 \pm \sqrt{64 - 36}}{6} = \frac{8 \pm \sqrt{28}}{6} = 0.45 \text{ m or } 2.2 \text{ m}$$

2.2 m is not possible

For $x = 0.45$ m.

$$\frac{d^2V}{dx^2} = 4(6x - 8)$$

$$= 4(6 \times 0.45 - 8) = -21.2 < 0$$

Hence the volume of the box is maximum when its height is 45 cm.

28. (a)

$$u = \sin(3x - y)$$

$$u_x = 3\cos(3x - y)$$

$$u_{xx} = -9\sin(3x - y)$$

$$u_y = -\cos(3x - y)$$

$$u_{yy} = -[-\sin(3x - y) \times -1]$$

$$= \sin(y - 3x)$$

29. (c)

Definate intergals of form $I = \int_0^\infty e^{-at} f(t)dt$ can be solved using Laplace transform defination as shown below :

$$L(t\cos 3t) = (-1)\frac{d}{ds}\left(\frac{s}{s^2+9}\right) = -\left[\frac{-s.2s}{(s^2+9)^2} + \frac{1}{(s^2+9)}\right]$$

$$= -\left(\frac{-2s^2+s^2+9}{(s^2+9)^2}\right) = \frac{s^2-9}{(s^2+9)^2}$$

$$\because \quad L(t\cos 3t) = \int_0^\infty e^{-st} t\cos 3t\, dt$$

$$\int_0^\infty e^{-st} t\cos 3t\, dt = \frac{s^2-9}{(s^2+9)^2}$$

Putting, $s = 4$

$$\int_0^\infty e^{-4t} t\cos 3t\, dt = \frac{(4)^2-9}{(4^2+9)^2} = \frac{16-9}{(16+9)^2} = \frac{7}{625}$$

30. (c)

Let, $f(x) = x\log_{10} x - 1.2$

So that, $f(1) = -\text{ve}$,

$f(2) = -\text{ve}$

and $f(3) = +\text{ve}$

$\therefore$ the root lies between 2 and 3

Taking $x_0 = 2$

and $x_1 = 3$,

$f(x_0) = -0.59794$

and $f(x_1) = 0.23136$

Using method of false position,

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) = 2.7210 \qquad \text{...(i)}$$

$f(x_2) = f(2.7210) = -0.0171$

i.e. the root lies between 2.7210 and 3

$\therefore$ Taking $x_0 = 2.7210$,

$x_1 = 3$

$f(x_0) = -0.0171$

and $f(x_1) = 0.2313$ in equation (i), we get

$$x_3 = 2.720 + \frac{0.279}{0.2313 + 0.0171} \times 0.0171 = 2.74020$$

Representing this process, successive approximations are

$x_4 = 2.74024$

and $x_5 = 2.7406$

$\therefore$ root is 2.7406, correct to 4 decimal places.

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