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Discrete Mathematics

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CLASS TEST 2019-2020

COMPUTER SCIENCE & IT

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ANSWER KEY > Discrete Mathematics

1.	(a)	7.	(d)	13.	(c)	19.	(b)	25.	(a)
2.	(b)	8.	(a)	14.	(c)	20.	(d)	26.	(b)
3.	(d)	9.	(b)	15.	(d)	21.	(a)	27.	(a)
4.	(d)	10.	(b)	16.	(c)	22.	(b)	28.	(a)
5.	(a)	11.	(c)	17.	(d)	23.	(a)	29.	(c)
6.	(d)	12.	(b)	18.	(c)	24.	(b)	30.	(c)

DETAILED EXPLANATIONS

1. (a)

$X \rightarrow Y$ is false only when X is True and Y is false. By substituting the truth values of X and Y in S_1 and S_2 we find that both S_1 and S_2 are False.

Note: $X \leftrightarrow Y$ is True only when both X and Y have same truth values.

2. (b)

Number of ways of distributing 5 blue pens to 6 children

where $n = 5$, $r = 6$

$$^{5+6-1}C_5 = {}^{10}C_5$$

Number of ways of distributing 6 black pens to 6 children

$$^{6+6-1}C_6 = {}^{11}C_6$$

$\therefore$ Total number of ways = $^{10}C_5 \times {}^{11}C_6 = 116424$

3. (d)

The statement "not every P is Q" can be written as "there exist a P which is not Q".

i.e., $\exists x(P(x) \wedge \neg Q(x))$ which is same as option (a), (b) and (c).

4. (d)

The upper bounds of {1, 3, 4, 6} are 6, 8 and 9.

Hence there are only 3 upper bounds.

5. (a)

Clearly, $a_n = n + 1$

$\Rightarrow a_{n-1} = n$

$\Rightarrow a_{n-2} = n - 1$

$\Rightarrow a_n = 2a_{n-1} - a_{n-2}$ $\quad [\because 2(n) - (n-1) = n + 1]$

6. (d)

$f : A \rightarrow B$ is bijective.

$\Rightarrow f : A \rightarrow B$ is one-one (injective) f onto (surjective)

1. $f : A \rightarrow B$ is one-one $\quad \Rightarrow f^{-1} : B \rightarrow A$ exists and it is unique.

$\Rightarrow f^{-1}$ is also one-one ...(1)

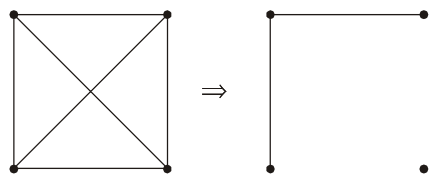
2. $f : A \rightarrow B$ is onto $\quad \Rightarrow f(A) = B$

$\Rightarrow A = f^{-1}(B)$ or $f^{-1}(B) = A \Rightarrow f^{-1} : B \rightarrow A$ is also onto ...(2)

from (1) and (2) $f^{-1} : B \rightarrow A$ is bijective.

7. (d)

Complete graph has $^nC_2$ edges (worst case) to make a connected graph atmost (n – 1) edges required. To make it disconnected graph should contain (n – 2) edges.



$$m = {}^nC_2 = {}^4C_2 = 6 \text{ edges}$$

$$^nC_2 - n + 2 = 6 - 4 + 2 = 4 \text{ edges deleted}$$

$\therefore$ (m – n + 2) edges deletion always guarantee that any graph will become-disconnected.

i.e. $\quad 10 - 6 + 2 = 6$ edges

8. (a)

Total number of element in $A \times A \times A \times A = x^4$

$\Rightarrow$ Power set of $A \times A \times A \times A = 2^{x^4}$.

9. (b)

$f : A \to B$

$g : B \to C$ is injection: $\forall b \in B, g(b) = c$ distinct images in C.

$g \circ f : A \to C$ is surjection

$$g(f(a)) = c$$

$$\Rightarrow \quad g(f(a)) = g(b)$$

$$\exists a \in A$$

$$\therefore \quad f(a) = b$$

So, $f : A \to B$ is surjection.

10. (b)

Let $= (1 + x + x + x^3 + \ldots\ldots + \ldots.)^2$

$$= \left\{\frac{1}{1-x}\right\}^2 = (1-x)^{-2} = \sum_{r=0}^{\infty} {}^{2-1+r}C_r x^r$$

the coefficient of x^{20} is equal to $= {}^{2-1+20}C_{20} = {}^{21}C_{20} = \dfrac{2!}{20! * 1!} = 21.$

11. (c)

$$1 + x + x^2 + x^3 + \ldots + x^n = \frac{1-x^{n+1}}{1-x} \qquad [\because\ C \to 3]$$

$$1 + x + x^2 + x^3 + \ldots\infty = \frac{1}{1-x} \qquad [\because\ B \to 1]$$

$$\sum_{r=0}^{\infty} {}^{n-1+r}C_r \cdot x^r = \frac{1}{(1-x)^n} \qquad [\because\ A \to 2]$$

12. (b)

Total number of edges in complete graph of 6 vertices $\dfrac{6(6-1)}{2} = 15.$

$\therefore$ 15 – 7 = 8 edges are there in $\bar{G}$.

13. (c)

Euler formula says

Number of regions (r) = Number of edges (e) – Number of vertices (n) + 2

$$r = e - n + 2 \qquad \ldots(1)$$

$$e = \frac{n \cdot k}{2} = \frac{8 \times 11}{2} = 44$$

$$\therefore \quad r = 44 - 8 + 2 = 38 \text{ regions.}$$

14. (c)

Dirac's theorem states that min degree(s) should be $\geq \lfloor n/2 \rfloor$. This is satisfied by only $K_{3,3}$ and $K_{3,4}$.

Note: Minimum degree for $K_{m,n}$ = min(m, n).

Every cycle in a bipartite graph is even and alternates between vertices from V_1 and V_2. Since a Hamilton cycle uses all the vertices in V_1 and V_2, we have $m = |V_1| = |V_2| = n$.

This condition is satisfied by $K_{3,3}$ only.

Therefore only $K_{3,3}$ will have Hamiltonian cycle.

15. (d)

Let $|A| = n$, and $|B| = m$

In partial function every element in domain need not have a range in co-domain.

$\therefore$ Each element in A will have $(m + 1)$ choices.

For n elements in A

$$\underbrace{(m+1)(m+1)\ldots(m+1)}_{n \text{ times}} = (m+1)^n.$$

In this question, $|A| = 4$, $|B| = 4$

The number of partial functions from A to B are $(4 + 1)^4$.

$\therefore$ $(4 + 1)^4 = 625$

16. (c)

Let a, b, c be the number of balls distributed among 3 children respectively.

$a + b + c = 8$, $a, b, c \geq 2$ and $a, b, c \leq 4$

Let $a = a' + 2$, $b = b' + 2$, $c = c' + 2$, $a', b', c' \geq 0$ and $a', b', c' \leq 2$

$\Rightarrow$ $a' + 2 + b' + 2 + c' + 2 = 8$

$\Rightarrow$ $a' + b' + c' = 2$

Since $a', b', c' \geq 0$ a', b', c', can never exceed 2, such that above equation holds true.

This is equivalent to integral solutions of

$$x_1 + x_2 + x_3 + \ldots x_n = r,$$
$$x_1, x_2, x_3, \ldots x_n \geq 0$$

which is equal to ${}^{n+r-1}C_r$

$$n = 3, r = 2 \qquad \therefore {}^{n+r-1}C_r = {}^{3+2-1}C_2 = {}^4C_2$$

$${}^4C_2 = \frac{4 \times 3}{2} = 6$$

17. (d)

The operation is not commutative as since upper and lower triangle is not same.

$q * p = p$ and $p * q = r$

The operation is not associative as $p * (q * r) \neq (p * q) * r$

LHS $p * r = s$

RHS $r * r = p$

18. (c)

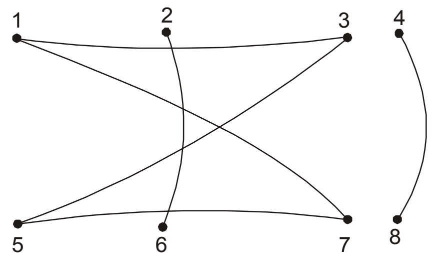
The candidate is unsuccessful if he fails in 9 or 8 or 7or 6 or 5 papers.

$\therefore$ The number of ways to be unsuccessful

$$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5 = 256$$

19. (b)

Let $n = 2 \Rightarrow$ # vertices = 8 $\quad$ [$\because$ # vertices in $G = 4n$]



$\Rightarrow$ 3 components [*Note:* For any n, the #components in G = 3]

$$\left.\begin{array}{l} V(C_1) = \{1, 3, 5, 7\} \Rightarrow m_1 = 4 \\ V(C_2) = \{2, 6\} \Rightarrow m_2 = 2 \\ V(C_3) = \{4, 8\} \Rightarrow m_3 = 2 \end{array}\right\} \quad \max = 4$$

20. (d)

To check function is one-to-one:

$\Rightarrow \quad f(x_1) = f(x_2)$

$\Rightarrow \quad f(x) = x^2 + 1$

$\Rightarrow \quad x_1^2 + 1 = x_2^2 + 1$

$\Rightarrow$ $x_1 = \pm x_1$ here x_1 has to images so, it is not one-to-one function.

To check function is onto:

$$y = x^2 + 1$$

$$x = \sqrt{y-2}$$

So, range = $|y|$ for $y \geq 1 \neq z$ so, it is not onto.

21. (a)

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= {}^{10}C_3 + {}^{10}C_3 - 0 \\ &= 2 \times {}^{10}C_3 \\ &= 2 \times \frac{10 \times 9 \times 8}{1 \times 2 \times 3} \\ &= 30 \times 8 = 240 \end{aligned}$$

22. (b)

Let p : GATE rank is needed

q : I will write the GATE exam

r : I will join in MADEEASY.

Given arguments:

P$_1$: If GATE rank is needed, i will not write GATE exam, if i do not join MADEEASY.

$$p \to (\sim r \to \sim q) = (p \wedge \sim r) \to \sim q$$

P$_2$: GATE rank is needed : p

P$_3$: I will join MADEEASY : r

Q: I will write the GATE exam : q

Inference is: $(p \wedge \neg r) \to \neg q$

$$\begin{array}{c} p \\ \underline{r} \\ \underline{q} \end{array}$$

We can also write the above inference as following: $(p \wedge \neg r)$

$$\left([(p \wedge \neg r) \rightarrow \neg q] \wedge p \wedge r\right) \rightarrow q$$

If above proposition is tautology then given inference is valid.

$((pr')' + q')' + p' + r' + q$

$= pr'q + p' + r' + q$

$= p' + r' + q$ which is consistency hence invalid.

23. (a)

Total number of terms $= 8 + 1 = 9$

The middle term is : 5^{th} term

$(x + y)^n$ has $(r + 1)^{th}$ term as : ${}^nC_r x^{n-r} y^r$

$[(4 + 1)^{th}$ term] 5^{th} term is:

$${}^8C_4 \left(\frac{y\sqrt{x}}{3}\right)^{8-4} \left(\frac{-3}{x\sqrt{y}}\right)^4$$

$$= {}^8C_4 \cdot \frac{y^4 \cdot x^2}{3^4} \cdot \frac{3^4}{x^4 \cdot y^2} = {}^8C_4 \cdot \frac{y^2}{x^2} = 70\left(\frac{y}{x}\right)^2$$

24. (b)

- One graph in which $|P| < 2$ i.e. their is no edge in the graph
- Second is ${}^nC_{|P|}$ where $|P| \geq 2$ where all vertex make complete graph. So, total number of such graphs are

$$= 1 + \sum_{k=2}^{n} {}^nC_K = 1 + \sum_{k=0}^{n} ({}^nC_K) - 1 - n$$

$$= 2^n - n$$

25. (a)

Put $x = y$ and $y = x$ at the and to get inverse function

$$y = 2.2^x + 4^x$$

$$\Rightarrow \quad x = 2.2^y + 4^y$$

$$\Rightarrow \quad x = 2.2^y + (2^y)^2$$

$$\Rightarrow \quad x+1 = (2^y)^2 + 2.2^y + 1$$

$$\Rightarrow \quad x+1 = (2^y + 1)^2$$

$$\Rightarrow \quad \sqrt{x+1} = 2^y + 1$$

$$\Rightarrow \quad 2^y = \sqrt{x+1} - 1$$

$$\Rightarrow \quad \log 2^y = \log\left(\sqrt{x+1} - 1\right)$$

$$\Rightarrow \quad y \log 2 = \log\left(\sqrt{x+1} - 1\right)$$

$$\Rightarrow \quad y = \frac{\log\left(\sqrt{x+1} - 1\right)}{\log 2}$$

26. (b)

The problem corresponds to the number of non negative integral solutions to

$$x_1 + x_2 + x_3 = 10 \text{ with the conditions,}$$
$$0 \le x_1 \le 10$$
$$0 \le x_2 \le 5$$
$$0 \le x_3 \le 3$$

Generating functions are required, since the variables have an upper constraint

The generating function is

$(1 + x + x^2 ...)(1 + x + x^2 + x^3 ...+ x^5)(1 + x + ... x^3)$

$$= \left(\frac{1}{1-x}\right)\left(\frac{1-x^6}{1-x}\right)\left(\frac{1-x^4}{1-x}\right)$$

$$= \frac{(1-x^6)(1-x^4)}{(1-x)^3}$$

$$= (1-x^4-x^6+x^{10})\sum_{r=0}^{\infty} 3-1+rC_r x^r$$

$$= (1-x^4-x^6+x^{10})\sum_{r=0}^{\infty} r+2C_r x^r$$

The coefficient of x^{10} in above generating function is ${}^{12}C_{10} - {}^{8}C_6 - {}^{6}C_4 + {}^{2}C_0 = 24$.

27. (a)

There are n courses i.e. $c_1, c_2, c_3 \ldots c_n$.

The no. of ways to select toppers of course 1 = $2nc_2$ ways

The no. of ways to select toppers of course 2 = $(2n-2)c_2$ ways

The no. of ways to select toppers of course 3 = $(2n-4)c_2$ ways

The no. of ways to select toppers of course 4 = $(2n-6)c_2$ ways

$\vdots$

The no. of ways to select toppers of course n = $2c_2$ ways

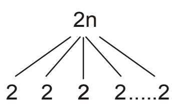
So total number of ways to assign $2n$ toppers for n courses are =

$$2n_{c_2} \times (2n-2)_{c_2} \times (2n-4)_{c_2} \ldots \times 2_{c_2}$$

$$= \frac{(2n)!}{2^n}$$

OR

This is ordered problem two divided '2n' toppers to 'n' cource with each cource '2' toppers



$$= \frac{(2n)!}{2^n}$$

28. (a)

$$T(n) - 9T(n-1) + 20T(n-2) = 0$$

Let $a_n = T(n)$

$\Rightarrow a_n - 9a_{n-1} + 20a_{n-2} = 0$

$$t^2 - 9t + 20 = 0$$
$$t^2 + 5t - 4t + 20 = 0$$

$$t(t-5) - 4(t-5) = 0$$
$$(t-4)(t-5) = 0$$
$$t = 4, 5$$

Homogenous equation become

$$a_n = c_1 . 5^n + c_2 . 4^n \quad \text{...(1)}$$

Put $n = 0$ in eq. (1)

$$a_0 = c_1 . 5^0 + c_2 . 4^0$$
$$-3 = c_1 + c_2 \quad \text{...(2)}$$

Put n = 1 in eq. (1)

$$a_1 = c_1 . 5^1 + c_2 . 4^1$$
$$-10 = 5c_1 + 4c_2 \quad \text{...(3)}$$

Solving equation (2) and (3) and get c_1 and c_2

$$(c_1 + c_2 = -3) \times 5$$
$$5c_1 + 4c_2 = -10$$
$$5c_1 + 5c_2 = -15$$
$$\underline{5c_1 + 4c_2 = -10}$$
$$c_2 = -5 \quad \text{and} \quad c_1 = 2$$

Put value of c_1 and c_2 in eq. (1)

$$a_n = 2.5^n - 5.4^n$$

29. **(c)**

S1 is true but converse of S1 is not true. (Dirac theorem)

S2 is true and converse of S2 is also true because G is connected graph. (Eular graph theoram)

30. **(c)**

Conjunction ($\wedge$) is commutative. Hence I is True.

Existential Quantifier ($\exists$) is distributive over disjunction ($\vee$) and not distributive over conjunction ($\wedge$). Hence II is false.

If we simplify III we get $\neg\forall x\ (\neg S(x) \vee \neg P(x))$ which is equal to $\exists x\ [S(x) \wedge P(x)]$ (same as given expression).

Hence only I and III are equivalent.

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