CLASS TEST

Sl.: 01SKEE-ABCD-20072023

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ELECTROMAGNETIC FIELDS

ELECTRICAL ENGINEERING

Date of Test : 20/07/2023

ANSWER KEY

1.	(d)	7.	(d)	13.	(c)	19.	(a)	25.	(b)
2.	(c)	8.	(d)	14.	(d)	20.	(b)	26.	(c)
3.	(c)	9.	(d)	15.	(b)	21.	(d)	27.	(b)
4.	(c)	10.	(c)	16.	(c)	22.	(a)	28.	(d)
5.	(c)	11.	(b)	17.	(c)	23.	(a)	29.	(a)
6.	(a)	12.	(c)	18.	(b)	24.	(c)	30.	(a)

1. **(d)**

It depends whether both charges are of same or opposite sign, if both are having same sign then potential energy will decrease and if having opposite signs, then it will increase.

2. **(c)**

Using the equation, $V = \dfrac{Q}{4\pi\varepsilon_0 r}$ the potential due to +10 PC is

$$V_1 = \frac{(10\times10^{-12}\text{ C})\times(9\times10^{9}\text{ Nm}^2\text{ C}^{-2})}{1\times10^{-2}\text{ m}} = 9\text{ V}$$

The potential due to +20 PC is

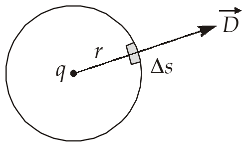
$$V_2 = \frac{(20\times10^{-12}\text{ C})\times(9\times10^{9}\text{ Nm}^2\text{ C}^{-2})}{1\times10^{-2}\text{ m}} = 18\text{ V}$$

The net potential at the given point is

$$9\text{ V} + 18\text{ V} = 27\text{ V}$$

3. **(c)**

Let us take a small element Δs on the surface of the sphere.



The electric field density here is radially outward and has the magnitude $\dfrac{q}{4\pi r^2}$.

where 'r' is the radius of the sphere. As the positive normal is also outward, $\theta = 0$ and the flux through this part is,

$$\Delta\phi = \vec{D}\cdot\Delta\vec{s} = \frac{q}{4\pi r^2}\Delta s$$

Summing over all the parts of the spherical surface,

$$\phi = \Sigma(\Delta\phi) = \frac{q}{4\pi r^2}\cdot\Sigma\Delta s = \frac{q}{4\pi r^2}4\pi r^2 = q$$

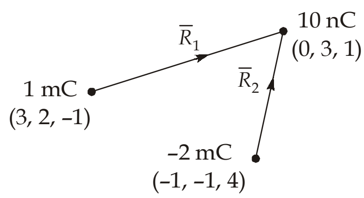
4. **(c)**

From the diagram shown below,

Required vectors are

$$\bar{R}_1 = -3\hat{a}_x + \hat{a}_y + 2\hat{a}_z$$

$$\hat{a}_{R1} = \frac{-3\hat{a}_x + \hat{a}_y + 2\hat{a}_z}{\sqrt{14}}$$



and $$\bar{R}_2 = \hat{a}_x + 4\hat{a}_y - 3\hat{a}_z$$

$$\hat{a}_{R2} = \frac{\hat{a}_x + 4\hat{a}_y - 3\hat{a}_z}{\sqrt{26}}$$

Force on 10 nC due to 1 mC charge is,

$$\bar{F}_1 = \frac{(10\times10^{-9})(1\times10^{-3})}{4\pi\varepsilon_0\left(\sqrt{14}\right)^2}\times\frac{-3\hat{a}_x+\hat{a}_y+2\hat{a}_z}{\sqrt{14}}$$

$$= 1.72\left(-3\hat{a}_x+\hat{a}_y+2\hat{a}_z\right)\times10^{-3}$$

$$\bar{F}_1 = -5.16\,\hat{a}_x+1.72\,\hat{a}_y+3.44\,\hat{a}_z\ (\text{mN})$$

Force on 10 nC due to –2 mC charge is,

$$\bar{F}_2 = 7\frac{(10\times10^{-9})\times(-2\times10^{-3})}{4\pi\varepsilon_0(\sqrt{26})^2}\times\frac{\hat{a}_x+4\hat{a}_y-3\hat{a}_z}{(\sqrt{26})}$$

$$= -1.36\left(\hat{a}_x+4\hat{a}_y-3\hat{a}_z\right)\times10^{-3}$$

The force is obtained by superposition;

$$\bar{F} = \bar{F}_1+\bar{F}_2 = \left(-6.52\hat{a}_x-3.72\hat{a}_y+7.52\hat{a}_z\right)\text{mN}$$

5. (c)

Given, $z = 0$ and $\vec{a}\neq\alpha\vec{b}$

So, it is clear that both $\vec{a},\vec{b}$ are in x-y plane.

$\therefore$ The vector perpendicular to both $\vec{a},\vec{b}$ will be in the direction of z-axis. □

6. (a)

A vector is said to be irrotational when its curl is zero.

For vector $\vec{A}$

$$\nabla\times\vec{A} = \begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ yz & zx & xy\end{vmatrix}$$

$$= \hat{a}_x\left[\frac{\partial}{\partial y}(xy)-\frac{\partial}{\partial z}(zx)\right]-\hat{a}_y\left[\frac{\partial}{\partial x}(xy)-\frac{\partial}{\partial z}(yz)\right]+\hat{a}_z\left[\frac{\partial}{\partial x}(zx)-\frac{\partial}{\partial y}(yz)\right]$$

$$\nabla\times\vec{A} = 0+0+0=0$$

For vector $\vec{A}$ to be solenoidal, its divergence must be zero

$$\nabla\times\vec{A} = 0$$

$$\nabla\times\vec{A} = \frac{\partial}{\partial x}(yz)+\frac{\partial}{\partial y}(zx)+\frac{\partial}{\partial z}(xy)=0$$

Therefore option (a) is correct.

7. **(d)**

From Biot savart law,

$$\vec{H} = \int_0^{2\pi} \frac{IRd\phi\hat{a}_\phi \times (-\hat{a}_\rho)}{4\pi R^2} = \left(\frac{I}{4\pi}\int_0^{2\pi}\frac{Rd\phi}{R^2}\right)\hat{a}_z$$

$$\vec{H} = \frac{I}{2R}\hat{a}_z$$

8. **(d)**

In cylindrical coordinate system:

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & A_\phi & A_z \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & \rho\sin 2\phi & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\hat{a}_\rho(0) - \rho\hat{a}_\phi(0) + \hat{a}_z\frac{\partial}{\partial\rho}(\rho\sin 2\phi)\right] = \frac{1}{\rho}(\sin 2\phi)\hat{a}_z$$

at point $P(2, \pi/4, 0)$,

$$\nabla \times \vec{A} = \frac{1}{2}\sin\left(2\times\frac{\pi}{4}\right)\hat{a}_z = 0.5\hat{a}_z$$

9. **(d)**

Let vector $\vec{B}$ in Cartesian system be;

$$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$$

Where, $$B_x = \bar{B}\cdot\hat{a}_x = \left(-\rho\hat{a}_\phi + z\hat{a}_z\right)\cdot\hat{a}_x$$

$$= +\rho\sin\phi$$

$$B_y = \bar{B}\cdot\hat{a}_y = (-\rho a_\phi + z\hat{a}_z)\cdot\hat{a}_y$$

$$= -\rho\cos\phi$$

$$B_z = \bar{B}\cdot\hat{a}_z = z$$

As we use the relation,

$$x = \rho\cos\phi$$

and $$y = \rho\sin\phi$$

then, $$B_x = \rho\times\frac{y}{\rho} = y$$

$$B_y = -\rho\times\frac{x}{\rho} = -x$$

$$B_z = z$$

then the vector, $$\vec{B} = y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$$

10. **(c)**

If the divergence of a given vector is zero, then it is said to be solenoidal.

$$\nabla.\vec{A} = 0$$

By Divergence theorem,

$$\int_V (\nabla.\vec{A})\,dv = \oint_s \vec{A}.\vec{ds}$$

So, for a solenoidal field,

$$\nabla.\vec{A} = 0 \text{ and } \oint_s \vec{A}.\vec{ds} = 0$$

11. (b)

The surface charge density of inner cylinder,

$$\rho_{s,\ in\text{-}cy} = \frac{Q_{in\text{-}cy}}{2\pi r L}$$

where, $Q_{in\text{-}cy}$ = total charge in inner cylinder
r = radius of inner cylinder
L = length

$$\rho_s = \frac{30\times10^{-9}}{2\pi\times(10^{-3})\,(0.5)} = 9.549\ \mu\text{C/m}^2$$

The inner electric flux density, (for region between 1 mm to 4 mm)

$$D_p = \frac{r\rho_s}{P} = \frac{10^{-3}\times9.549\times10^{-6}}{P} = \frac{9.549}{P}\ \text{nC/m}^2$$

$\therefore$ Electric field between cylinders,

$$E_p = \frac{D_P}{\epsilon_0} = \frac{9.549\times10^{-9}}{8.85\times10^{-12}\,P} = \frac{1078.98}{P}\ \text{V/m}$$

12. (c)

We know,
$$D = \epsilon E = \epsilon\frac{V}{d}$$

$$J = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d}\frac{dV}{dt} \quad \text{(as voltage is function of time)}$$

If 's' is area of cross-section.

$\therefore$ Displacement current,

$$I_d = J_d\cdot s = \frac{\epsilon s}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

For conduction current,

$$I_c = \frac{dQ}{dt} = s\frac{d\rho_s}{dt} = s\frac{dD}{dt}$$

Also,
$$I_c = s\epsilon\frac{dD}{dt} = s\epsilon\frac{dE}{dt} = \frac{\epsilon s}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

The value of conduction current and displacement current are same,

$$\text{Ratio} = \frac{\text{Displacement current}}{\text{Conduction current}} = 1$$

$$I_d = \frac{4\times10^{-9}}{36\pi}\times\frac{10\times10^{-4}}{4\times10^{-3}}\times\frac{d}{dt}(20\sin10^3 t)$$

$$= \frac{4\times10^{-9}}{36\pi}\times\frac{10\times10^{-4}}{4\times10^{-3}}\times10^{3}\times20\cos10^{3}t$$

$$= \frac{10^{-9}}{36\pi\times10^{-3}}\times20\cos10^{3}t = 0.17683\times10^{-6}\cos10^{3}t$$

$$= 176.83\cos10^{3}t \text{ nA}$$

13. (c)

We know,
$$\rho_v = \nabla\cdot D = \frac{\partial D_z}{\partial z} = \rho\cos^2\phi$$

The total charge enclosed in cylindrical section,

$$Q = \int_v \rho_v\, dV = \int_v \rho\cos^2\phi\,\rho\, d\phi\, d\rho\, dz$$

$$= \int_{z=-2}^{2} dz \int_{\phi=0}^{2\pi}\cos^2\phi\, d\phi \int_{\rho=0}^{2}\rho^2\, d\rho$$

$$= (4)\times(\pi)\times\frac{(2)^3}{3} = \frac{32\pi}{3}\text{ C}$$

14. (d)

For boundary between two ideal dielectrics:
We can use conditions,

$$E_{t1} = E_{t2} = (100\hat{a}_y - 400\hat{a}_z)\text{ V/m}$$

and
$$D_{n1} = D_{n2}$$

$$\epsilon_{r1}E_{n1} = \epsilon_{r2}E_{n2}$$

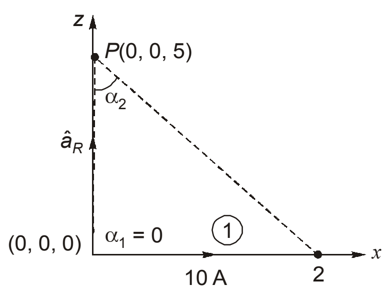
$$E_{n2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}\cdot\epsilon_{n1} = \frac{4}{6}\times300 = 200\text{ V/m}$$

$\therefore$
$$\vec{E}_2 = 200\,\hat{a}_x + 100\,\hat{a}_y - 400\,\hat{a}_z\text{ V/m}$$

Hence (d) option is correct.

15. (b)

Note here that the point of interest is on z-axis and the side-1 of the triangular is on x-axis. So no need to consider other sides.



The perpendicular from point P on the current filament is at (0, 0, 0). The vector and unit vector in the direction of perpendicular towards point P is,

$$\bar{R} = 5\hat{a}_z$$

and $$\hat{a}_R = \hat{a}_z$$

The angles made by ends of the filament with the perpendicular are (dotted line),

$$\alpha_1 = 0;$$

$$\alpha_2 = \tan^{-1}\left(\frac{2}{5}\right) = 21.80°$$

The current filament along x-axis gives

$$\hat{a}_l = \hat{a}_x$$

Now the direction of H is,

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_R = \hat{a}_x \times \hat{a}_z = -\hat{a}_y$$

The field intensity due to finite filament is,

$$\bar{H} = \frac{I}{4\pi R}[\sin\alpha_2 - \sin\alpha_1]\hat{a}_\phi$$

$$= \frac{10}{4\pi \times (5)}\left[\sin(21.80) - \sin(0)\right](-\hat{a}_y)$$

$$\bar{H} = -59.11\hat{a}_y \text{ (mA/m)}$$

16. (c)

Force, $$\vec{F} = I(\vec{L} \times \vec{B})$$

$$= 10(2\hat{a}_z \times 0.02(\hat{a}_y - \hat{a}_x))$$

$$= 10(0.04(-\hat{a}_x) - 0.04\hat{a}_y)$$

$$= -0.4\hat{a}_x - 0.4\hat{a}_y$$

Force acting per unit length,

$$\frac{\vec{F}}{L} = \frac{-0.4\hat{a}_x - 0.4\hat{a}_y}{2}$$

$$= -0.2\hat{a}_x - 0.2\hat{a}_y$$

$$\frac{\vec{F}}{L} = -0.2(\hat{a}_x + \hat{a}_y)$$

17. (c)

The flux Φ_1, at $i_1 = 5$ A is

$$\Phi = B \times A$$

$$= 1 \times (30 \times 10^{-4})$$

$$\Phi_1 = 30 \times 10^{-4} \text{ Wb}$$

$$\Phi_2 \text{ at } i_2 = 10 \text{ A is,}$$

$$\Phi_2 = 1.5 \times 30 \times 10^{-4}$$

Increase of flux when current is increased from 5 to 10 A

$$= 0.5 \times 30 \times 10^{-4} \text{ Wb}$$

$$\frac{d\phi}{dt}\text{(average)} = \frac{0.5 \times 30 \times 10^{-4}}{5}$$

$$\frac{d\phi}{dt}\text{(average)} = 3 \times 10^{-4} \text{ Wb/A}$$

∴ Mean value of inductance,

$$L = N\frac{d\phi}{di}$$

$$L = 2000 \times 3 \times 10^{-4}$$

$$L = 0.6 \text{ Henry.}$$

18. (b)

$$\nabla \times \vec{H} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & (z^2 - x^2) & 3y \end{vmatrix}$$

$$= \hat{i}(3-2z) + \hat{k}(-2x-2)$$

$$= (3-2z)\hat{i} - (2x+2)\hat{k}$$

At the origin , $x = 0$, $z = 0$

$$\nabla \times \vec{H} = 3\hat{i} - 2\hat{k}$$

$$|\nabla - \vec{H}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

19. (a)

In cylindrical coordinate system:

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho\sin 2\phi & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\hat{a}_\rho(0) - \rho\hat{a}_\phi(0) + \hat{a}_z\frac{\partial}{\partial \rho}(\rho\sin 2\phi)\right] = \frac{1}{\rho}(\sin 2\phi)\hat{a}_z$$

at point $P(4, \pi/6, 0)$,

$$\nabla \times \vec{A} = \frac{1}{4}\sin\left(2 \times \frac{\pi}{6}\right)\hat{a}_z = \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\hat{a}_z = \frac{\sqrt{3}}{8}\hat{a}_z$$

20. (b)

The given vector $\vec{A}$ is in spherical coordinates

$$\vec{A} = 2r\cos\theta\cdot\cos\phi\hat{a}_r + r^{1/2}\hat{a}_\phi$$

$$\text{div}.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot A_r) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial \theta}(\sin\theta \cdot A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(2r^3\cos\theta\cos\phi\right) + 0 + \frac{1}{r\sin\theta}(0)$$

$$= 6\cos\theta\cos\phi$$

At point $\left(1, \frac{\pi}{4}, \frac{\pi}{3}\right)$,

$$\nabla \cdot \vec{A} = 6\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) = 2.121$$

21. **(d)**

The incremental work is given by:

$$dW = -Q \cdot \bar{E} \cdot dl$$

Now dl in the direction of $\hat{a}_\phi$;

$$d\bar{l} = r\,d\phi\,\hat{a}_\phi = 6 \times 10^{-6}\hat{a}_\phi$$

Thus $$\vec{E} \cdot d\bar{l} = -200 \times 6 \times 10^{-6} = -1200 \times 10^{-6}$$

$$dW = -40 \times 10^{-6} \times (-1200 \times 10^{-6})$$

$$= 48 \text{ nJ}$$

22. **(a)**

$$C = 4\pi\varepsilon a \qquad \text{where } \varepsilon = \varepsilon_0,$$

$$a = 18 \text{ cm}$$

$$C = 4\pi \times \frac{10^{-9}}{36\pi} \times 18 \times 10^{-2}$$

$$= 2 \times 10^{-11} = 20 \text{ pF}$$

23. **(a)**

$$I = \int \vec{J}.d\vec{s} = \int_{-0.01}^{0.01}\int_{-\pi/4}^{\pi/4} 100\cos 2y\hat{a}_x d_y d_z\ \hat{a}_x$$

$$= [0.01 + 0.01] \times 100 \times \left.\frac{\sin 2y}{2}\right|_{-\pi/4}^{\pi/4}$$

$$= (50 \times 0.02) \times \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{2}\right)\right] = 2\,\text{A}$$

24. **(c)**

The potential is given by:

$$V_{AB} = -\int_B^A \bar{E} \cdot d\bar{l}$$

Now, we know that $\vec{E} = \dfrac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$ for infinite line charge

$$\vec{E} = \frac{10^{-9}}{2\pi\left(\frac{10^{-9}}{36\pi}\right)\rho}\hat{a}_\rho = \frac{18}{\rho}\hat{a}_\rho \text{ V/m}$$

$$d\bar{l} = d\rho \cdot \hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

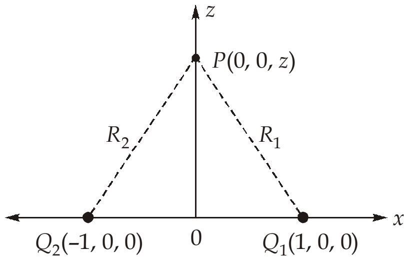
$$\bar{E} \cdot d\bar{l} = \frac{18}{\rho}d\rho$$

$$V_{AB} = -\int_4^2 \frac{18}{\rho}d\rho = \left[-18\ln\rho\right]_4^2 = -18\left[\ln 2 - \ln 4\right]$$

$$= 18 \ln 2 = 12.48 \text{ volts}$$

25. (b)

Consider the diagram shown below:



Distance of Q_1 and Q_2 from point P are:

$$R_1 = R_2 = \sqrt{z^2+1}$$

Since, $\quad Q_1 = Q_2$

and $\quad R_1 = R_2$

Now, potential at P is twice that of single charge

$$V = 2\times\frac{Q}{4\pi\varepsilon_0 R_1}$$

$$= 2\times\frac{8\times10^{-9}}{4\pi\times\left(\frac{10^{-9}}{36\pi}\right)\sqrt{z^2+1}} = \frac{144}{\sqrt{z^2+1}} \text{ Volts}$$

$$\frac{dV}{dz} = \frac{d}{dz}\left(\frac{144}{(z^2+1)^{1/2}}\right)$$

$$= 144\times\left[-\frac{1}{2}(z^2+1)^{-3/2}\right]\times 2z$$

$$\frac{dV}{dz} = \frac{-144z}{(z^2+1)^{3/2}};$$

$$\left|\frac{dV}{dz}\right| = \frac{144z}{(z^2+1)^{3/2}} \text{ V/m}$$

26. (c)

$$E = \frac{\Delta V}{\Delta Z} = \frac{250-100}{5\times10^{-3}} = 3\times10^4 \text{ V/m}$$

$$\vec{E} = -\nabla V = -3\times10^4\,\hat{a}_z \text{ V/m}$$

$$\vec{D} = \epsilon_0\epsilon_r\,\vec{E} = \left(\frac{10^{-9}}{36\pi}\right)\times2.4\times(-3\times10^4)\,\hat{a}_z$$

$$\vec{D} = -6.366\times10^{-7}\,\hat{a}_z \text{ C/m}^2$$

Since $\vec{D}$ is constant between the disks, and $D_n = \rho_s$ at a conductor surface.

$\therefore \quad \rho_s = \pm6.366\times10^{-7}$ C/m^2

Positive sign on the upper plate and negative sign on the lower plate.

27. **(b)**

Magnitude of electrical flux density,

$$|\vec{D}| = \frac{Q}{4\pi|R|^2}$$

$$R = \sqrt{1^2 + (3)^2 + (-4)^2} = 5.099 \text{ m}$$

$$|\vec{D}| = \frac{40 \times 10^{-9}}{4\pi(5.099)^2} = 122.43 \text{ pC/m}^2$$

28. **(d)**

As we know;

$$\nabla \times \vec{A} = \vec{B}$$

Where $\vec{B}$ is magnetic flux density:

$$\text{Units of } \vec{B} = \frac{Wb}{m^2}$$

Therefore, $$\text{unit of } \vec{A} = \frac{Wb}{m}$$

$\because$ $$\text{units of } \nabla = \frac{1}{m}$$

29. **(a)**

At the centre of the loop, assuming loop is horizontal,

$$\vec{H} = \frac{I}{2r}\hat{a}_z$$

Magnetic flux density, $$\vec{B} = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1 \times 10^{-3}}{2 \times 2}\hat{a}_z$$

$$\vec{B} = 0.314\hat{a}_z \text{ (n Wb/m}^2\text{)}$$

30. **(a)**

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t} \text{ is continuity equation}$$

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