CLASS TEST

S.No. : 01SKEC-EFGHIJ-230723

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Bhubaneswar | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

ELECTROMAGNETIC THEORY

ELECTRONICS ENGINEERING

Date of Test : 23/07/2023

ANSWER KEY

1.	(a)	7.	(b)	13.	(c)	19.	(b)	25.	(d)
2.	(a)	8.	(a)	14.	(c)	20.	(d)	26.	(a)
3.	(a)	9.	(d)	15.	(a)	21.	(c)	27.	(a)
4.	(b)	10.	(a)	16.	(a)	22.	(a)	28.	(d)
5.	(d)	11.	(d)	17.	(c)	23.	(c)	29.	(a)
6.	(d)	12.	(b)	18.	(b)	24.	(d)	30.	(b)

Detailed Explanations

1. **(a)**

Since $\vec{A}$ is irrotational,

$$\nabla \times \vec{A} = 0$$

$\therefore$ $\vec{A}$ should be gradient of some scalar field i.e. $\vec{A} = \nabla V$, as curl of a gradient is always zero.

2. **(a)**

3. **(a)**

The cutoff wave number for the dominant mode of rectangular waveguide is '$\frac{\pi}{a}$' where 'a' is the broader dimension of the waveguide.

$\therefore$
$$\text{Wave number} = \frac{\pi}{3.14 \times 10^{-2}} = 100$$

4. **(b)**

We know that,

For lossless transmission line, $v_p = \frac{1}{\sqrt{LC}}$

$$\text{Characteristic impedance, } z_0 = \sqrt{\frac{L}{C}}$$

$$v_p \times z_0 = \frac{1}{\sqrt{LC}} \times \sqrt{\frac{L}{C}} = \frac{1}{C}$$

$$v_p = \frac{1}{C z_0} = \frac{1}{4 \times 10^{-6} \times 25}$$

$$v_p = 10000 \text{ m/s}$$

5. **(d)**

The minimum distance between the primary and secondary source is $\frac{2d^2}{\lambda}$.

$\therefore$
$$r = \frac{2(700 \times 10^{-2})^2}{\lambda}$$

$$\lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.1$$

$$r = \frac{2(700 \times 10^{-2})^2}{0.1} = 980 \text{ m}$$

6. **(d)**

From Maxwell's equation

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\frac{\partial}{\partial x}8x - \frac{\partial}{\partial y}2ky + \frac{\partial}{\partial z}4z = 0$$

$$8 - 2k + 4 = 0$$

or,
$$k = \frac{12}{2} = 6$$

7. (b)

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 2xy & z \end{vmatrix}$$

$$= (0-0)\hat{a}_x - (0-y)\hat{a}_y + (2y-z)\hat{a}_z$$

$$= y\hat{a}_y + (2y-z)\hat{a}_z$$

At point $B(2, 0, -1)$

$$\vec{\nabla}\times\vec{A} = \hat{a}_z$$

8. (a)

For a good conductor,

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$$

$$= \sqrt{\frac{2\pi\times 500\times 10^6\times 4\pi\times 10^{-7}}{10^6}}\angle 45^\circ$$

$$= 0.0628\angle 45^\circ\ \Omega$$

$\because$

$$\eta = \frac{E}{H}$$

$\therefore$

$$|H| = \left|\frac{E}{\eta}\right| = \frac{2\,\mu\text{V/m}}{0.0628} = 31.84\,\mu\text{A/m}$$

9. (d)

The reflection coefficient,

$$\Gamma = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{P_r}{P_i} = \Gamma^2 = (0.5)^2 = 25\%$$

Thus 25% of P_i is reflected.

10. (a)

Magnetic energy density,

$$W = \frac{1}{2}\mu\left|\vec{H}\right|^2$$

$$= \frac{1}{2}\times 4\times 4\pi\times 10^{-7}\left|\sqrt{2^2+4^2+8^2}\right|^2$$

$$= \frac{1}{2}\times 4\times 4\pi\times 10^{-7}[4+16+64]$$

$$= 672\pi\times 10^{-7}$$

$\therefore$

$$W \approx 211\ \mu\text{J/m}^2$$

11. **(d)**

As we know,

$$\Psi = \beta d \cos\theta + \alpha$$

$$\theta = 60°$$

$\therefore$
$$0 = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} \cos 60° + \alpha$$

$$0 = \frac{\pi}{4} \times \frac{1}{2} + \alpha$$

or
$$\alpha = -\frac{\pi}{8}$$

12. **(b)**

$$\eta_1 = \eta_0 = 120\pi$$

$$\eta_2 = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{5}{2}}$$

For incident wave,

$$E_i = -40\eta_1 \cos(\omega t - \beta z)\hat{a}_y \text{ V/m}$$

$$= -40\eta_0 \cos(10^8 t - \beta z)\hat{a}_y \text{ V/m}$$

now,
$$\frac{E_r}{E_i} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(1.58 - 1)\eta_0}{(1.58 + 1)\eta_0} = 0.225$$

$\therefore$
$$E_r = 0.225\ E_i$$

$$= -9\eta_0 \cos(10^8 t + \beta z)\hat{a}_y \text{ V/m}$$

where,
$$\beta_1 = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

$$E_r = -9\eta_0 \cos\left(10^8 t + \frac{1}{3}z\right)\hat{a}_y \text{ V/m}$$

13. **(c)**

We know that; phase constant,

$$\beta = \omega\sqrt{\mu_0\, \mu_r\, \in_0 \in_r} \qquad \text{(for lossless dielectric)}$$

$$\beta = \omega\sqrt{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi} \times 1 \times 9}$$

$$= 2\pi \times 10.5 \times 10^6 \times 10^{-8}$$

$\therefore$
$$\beta = 0.21\pi \text{ rad/m}$$

14. **(c)**

For a good conductor,

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Also,
$$\frac{\partial\beta}{\partial\omega} = \frac{1}{2}\left(\frac{\omega\mu\sigma}{2}\right)^{-1/2} \frac{\mu\sigma}{2}$$

or $$\frac{\partial \beta}{\partial \omega} = \frac{1}{2}\sqrt{\frac{\mu\sigma}{2\omega}} = \frac{1}{v_g}$$

$\because$ $$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

$$\frac{v_g}{v_p} = \frac{2\sqrt{\frac{2\omega}{\mu\sigma}}}{\sqrt{\frac{2\omega}{\mu\sigma}}}$$

$\therefore$ $$\frac{v_g}{v_p} = 2$$

15. (a)

The electric field is given by

$$\vec{E} = -\nabla \vec{V}$$

$$= -100\frac{\partial}{\partial \rho}\left[\rho^{0.6}\hat{a}_\rho\right]$$

$$\vec{E} = -60\rho^{-0.4}\hat{a}_\rho \text{ V/m}$$

but, $$\vec{D} = \epsilon_0 \vec{E} = -60\,\epsilon_0\,\rho^{-0.4}\hat{a}_\rho \text{ C/m}^2$$

At $\rho = 0.6$ m, $$\vec{D} = -60\,\epsilon_0\,(0.6)^{-0.4}$$

$\therefore$ $$\vec{D} = -73.6\epsilon_0 \text{ C/m}^2$$

(or) $$\vec{D} = -0.65 \text{ nC/m}^2$$

Since the flux density is constant

So, the flux $$\psi = \vec{D}\cdot\vec{S}$$

$$\psi = -0.65 \times 10^{-9} \times 2\pi\rho z$$

$$= -0.65 \times 10^{-9} \times 2\pi \times 0.6 \times 1$$

$\therefore$ Charge enclosed $Q_{encl} = \psi = -2.45 \times 10^{-9}\ C$ (According to Gauss law).

16. (a)

Given, $$E_y = 10\sin\left(\frac{2\pi x}{6}\right)\cos\left(\frac{3\pi y}{2}\right)\sin(\omega t - 4z) \text{ V/cm}$$

$$\frac{m\pi}{a} = \frac{2\pi}{6} \qquad \therefore \quad m = 2$$

$$\frac{n\pi}{b} = \frac{3\pi}{2} \qquad \therefore \quad n = 3$$

$\therefore$ The propagating mode is TE_{23} mode,

$\therefore$ $$f_c = \frac{3\times 10^8}{2\pi \times 10^{-2}}\sqrt{\left(\frac{2\pi}{6}\right)^2 + \left(\frac{3\pi}{2}\right)^2} = 23.05 \text{ GHz}$$

Given, $f = 40$ GHz

Since, $f > f_c; \quad \eta = \eta_0 \Big/ \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$\therefore \quad \eta = 377 \Big/ \sqrt{1 - \left(\frac{23.05}{40}\right)^2}$

$\therefore \quad \eta = 461.29\ \Omega$

17. (c)

Given, loss tangent, $\frac{\sigma}{\omega \in} = 1498.97 >> 1$

Hence, the medium is a good conductor,

$$\therefore \quad \alpha = \beta = \omega\sqrt{\frac{\mu \in}{2} \times \frac{\sigma}{\omega \in}} \quad \left(\because \frac{\sigma}{\omega \in} >> 1\right)$$

$$= 10\pi\sqrt{\frac{10\mu_0 \in_0}{2}} \times 1498.98 = 9.066 \times 10^{-6}$$

Skin depth in a good conductor is

$$\delta = \frac{1}{\beta} = \frac{1}{\alpha} = 1/9.066 \times 10^{-6} = 1.103 \times 10^5 \text{ m}$$

18. (b)

At dominant mode, TE_{10}

$$f_c = \frac{v_p}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{v_p}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2}$$

here, $v_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{1.44}} = 2.5 \times 10^8$ m/s

$$8.5 \times 10^9 = \frac{2.5 \times 10^8}{2 \times a}$$

or $a = 0.0147$ m $= 1.47$ cm

19. (b)

$$\eta_1 = \eta_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}}$$

$$\Gamma = -0.268$$

$$\tau = 1 + \Gamma = 0.732$$

$$\tau = \frac{E_{\text{Transmitted}}}{E_{\text{incident}}} \quad \text{also} \quad \beta_{\text{incident}} = 1$$

$$E_{\text{incident}} = \frac{7.32\cos(\omega t - z)}{0.732}$$

$$E_{\text{incident}} = 10 \cos(\omega t - z)\, a_y \text{ V/m}$$

20. (d)

By using boundary conditions, normal component will be $E_{N1} = E_1 \cdot n$. Taking $f = x - y + 2z$, the unit vector that is normal to the surface is

$$n = \frac{\nabla f}{|\nabla f|} = \frac{1}{\sqrt{6}}[\hat{a}_x - \hat{a}_y + 2\hat{a}_z]$$

$$\therefore \quad E_{N1} = E_1 \cdot n = (100\hat{a}_x + 200\hat{a}_y - 50\hat{a}_z)\cdot\frac{1}{\sqrt{6}}(\hat{a}_x - \hat{a}_y + 2\hat{a}_z)$$

$$= \frac{1}{\sqrt{6}}[100 - 200 - 100]$$

$$E_{N1} = -81.65 \text{ V/m}$$

$\therefore$ The normal component points into region 1 from the surface

Then, $$E_{N1} = -81.65\left(\frac{1}{\sqrt{6}}\right)\left[\hat{a}_x - \hat{a}_y + 2\hat{a}_z\right]$$

$$E_{N1} = -33.33\hat{a}_x + 33.33\hat{a}_y - 66.67\hat{a}_z \text{ V/m}$$

The tangential component will be

$$E_{T1} = E_1 - E_{N1}$$

$$E_{T1} = 133.3\hat{a}_x + 166.7\hat{a}_y + 16.67\hat{a}_z$$

From the boundary conditions,

$$E_{T2} = E_{T1} \;;\; E_{N2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}\cdot E_{N1} = \frac{1}{4}E_{N1}$$

$$E_2 = E_{T2} + E_{N2} = E_{T1} + \frac{1}{4}E_{N1}$$

$$\therefore \quad E_2 = 133.3\hat{a}_x + 166.7\hat{a}_y + 16.67\hat{a}_z - 8.3\hat{a}_x + 8.3\hat{a}_y - 16.67\hat{a}_z$$

$$\therefore \quad E_2 = 125\hat{a}_x + 175\hat{a}_y \text{ V/m}$$

21. (c)

$$E_{1n} = 3\, a_z$$

$$E_{1t} = 5\, a_x - 2\, a_y$$

Applying boundary conditions,

$$D_{n1} = D_{n2}$$

$$E_{t1} = E_{t2} = 5a_x - 2a_y$$

$$\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$$

$$4 \times 3a_z = 3\, E_{n2}$$

$$\Rightarrow \quad E_{n2} = 4\, a_z$$

$$E = E_{n2} + E_{t2}$$

$$= 5\, a_x - 2\, a_y + 4\, a_z \text{ V/m}$$

22. (a)

Given, $$G = 2x^2yz\hat{a}_x - 20y\hat{a}_y + (x^2 - z^2)\hat{a}_z$$

$$\nabla \cdot G = 4xyz - 20 - 2z$$

$$\nabla(\nabla \cdot G) = 4yz\hat{a}_x + 4xz\hat{a}_y + (4xy-2)\hat{a}_z$$

$$\nabla\times[\nabla(\nabla\cdot G)] = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 4yz & 4xz & 4xy-2 \end{vmatrix}$$

$$= \hat{a}_x\left[\frac{\partial}{\partial y}(4xy-2) - \frac{\partial}{\partial z}(4xz)\right] - \hat{a}_y\left[\frac{\partial}{\partial x}(4xy-2) - \frac{\partial}{\partial z}4yz\right] + \hat{a}_z\left[\frac{\partial}{\partial x}(4xz) - \frac{\partial}{\partial y}(4yz)\right]$$

$$= (4x-4x)\hat{a}_x - (4y-4y)\hat{a}_y + (4z-4z)\hat{a}_z$$

$$= 0$$

23. (c)

$$\vec{P}_{\text{avg}} = \frac{E_0^2}{2\eta}a_x \text{ W/m}^2$$

$$\text{Power (watts)} = \int \vec{P}_{avg}\cdot\vec{dS} = \frac{E_0^2}{2\eta}a_x\cdot(S\cdot a_N)$$

$$a_N = \frac{a_x + a_y}{\sqrt{2}}$$

$$S = \frac{10\times 10}{100\times 100} = 10^{-2}\text{ m}^2$$

$$\text{Power (watts)} = \frac{(6\sqrt{\pi})^2}{2\eta}a_x\cdot\left(10^{-2}\left(\frac{a_x+a_y}{\sqrt{2}}\right)\right) = \frac{36\pi}{2\eta}\times 10^{-2}\left(\frac{1}{\sqrt{2}}\right)$$

Calculation of η, $\omega = 2\pi\times 10^8$, $\beta = \dfrac{8\pi}{3}$

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \in_0 \in_r}}$$

($\mu = \mu_0$ non-magnetic)

$$3\times\frac{2\pi\times 10^8}{8\pi} = \frac{3\times 10^8}{\sqrt{\in_r}}$$

$$\sqrt{\in_r} = 4$$

$$\eta = \frac{\eta_0}{\sqrt{\in_r}} = \frac{120\pi}{4} = 30\,\pi$$

$$\text{Power} = \frac{36\pi}{60\pi}\times 10^{-2}\times\frac{1}{\sqrt{2}}$$

$$= \left(\frac{6}{\sqrt{2}}\right)\text{ mW} = 4.24\text{ mW}$$

24. (d)

$$\frac{J_c}{J_d} = \frac{\sigma}{\omega\in} = \frac{I_c}{I_d}$$

$$\omega = \frac{\sigma}{\in}\times\frac{J_d}{J_c} = \frac{\sigma}{\in}\times\frac{I_d}{I_c}$$

$$2\pi f = \frac{60}{2\times \epsilon_0} \times \frac{8.8\times 10^{-11}}{1}$$

$$2\pi f = 298.16$$

$$\Rightarrow \quad f = 47.45 \text{ Hz}$$

25. (d)

For quarter wave transmission line,

$$Z'_{in} = \frac{Z_0^2}{Z_L} = \frac{50\times 50}{100} = 25\,\Omega = Z''_{in}$$

As the two quarter wave lines are connected in parallel,

$$\therefore \quad Z'_L = Z'_{in} \parallel Z''_{in} = 12.5\,\Omega$$

Hence,
$$Z_{in} = \frac{Z_0^2}{Z'_L} = \frac{50\times 50}{12.5} = 200\ \Omega$$

26. (a)

We known that $V_p = f\lambda$

$$\frac{C}{\sqrt{\epsilon}} = f\lambda$$

$$\lambda \propto \frac{1}{\sqrt{\epsilon}}$$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

$$\Gamma = \frac{\sqrt{\epsilon_1}-\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}}-1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}}+1} = \frac{\left(\frac{3}{5}-1\right)}{\left(\frac{3}{5}+1\right)}$$

$$\Gamma = -\frac{1}{4}$$

$$\frac{P_r}{P_i}\times 100 = -|\Gamma|^2 = \frac{1}{16}\times 100 = 6.25\%$$

27. (a)

$$\eta_1 = \eta_0$$

$$\eta_2 = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{\eta_o}{\sqrt{5}} = 0.447\eta_0$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = -0.382$$

$$\tau = 1 + \Gamma = 0.618$$

$$E_t = \tau E_i$$

$$E_t = 92.7\cos(\omega t - 8\sqrt{5}\,y)\ \hat{a}_z \ \text{V/m}$$

28. (d)

Given,
$$H_z = 5\cos(10^9 t - 4y)\hat{a}_z \text{ A/m}$$

$$J_d = \nabla \times H = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix}$$

$$= \frac{\partial H_z}{\partial y}\hat{a}_x = \frac{\partial}{\partial y}(5\cos(10^9 t - 4y))\hat{a}_x$$

$$J_d = 20\sin(10^9 t - 4y)\hat{a}_x \text{ A/m}$$

But,
$$J_d = \frac{\partial D}{\partial t}$$

$$D = \int J_d dt = -\frac{20}{10^9}\cos(10^9 t - 4y)\hat{a}_x$$

$$D = -20\cos(10^9 t - 4y)\hat{a}_x \text{ nC/m}^2$$

29. (a)

The cut-off frequency for the TE_{mn} mode is,

$$f_c = \frac{C}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

We need the frequency lie between the cut-off frequencies of the TE_{10} and TE_{01} modes.

These will be,
$$f_{c,\,10} = \frac{C}{2\sqrt{\epsilon_r}\,a} = \frac{3\times 10^8}{2\sqrt{\epsilon_r}(0.06)} = \frac{2.5\times 10^9}{\sqrt{\epsilon_r}}$$

$$f_{c,\,01} = \frac{C}{2\sqrt{\epsilon_r}\,b} = \frac{3\times 10^8}{2\sqrt{\epsilon_r}(0.04)} = \frac{3.75\times 10^9}{\sqrt{\epsilon_r}}$$

$\therefore$ The range of frequencies over which single mode operation will occur is

$$\frac{2.5}{\sqrt{\epsilon_r}}\text{GHz} < f < \frac{3.75}{\sqrt{\epsilon_r}}\text{GHz}$$

30. (b)

Given,
$$G_d(\text{dB}) = 3 = 10\log_{10} G_d$$

or
$$G_d = (10)^{0.3} = 1.995$$

$\because$
$$P_{\text{avg}} = \frac{G_d \times P_{\text{rad}}}{4\pi R^2} = \frac{|E|^2}{2\eta}$$

or
$$|E|^2 = \frac{\eta \times G_d \times P_{\text{rad}}}{2\pi R^2} = \frac{120\pi \times 1.995 \times 30\times 10^3}{2\pi \times (10\times 10^3)^2}$$

$$= \frac{60\times 1.995\times 30\times 10^3}{10^8} = 0.03591$$

$$E = 0.189 \text{ V/m}$$

■■■■