CLASS TEST

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ALGORITHMS

COMPUTER SCIENCE & IT

Date of Test : 08/07/2023

ANSWER KEY

1.	(d)	7.	(b)	13.	(d)	19.	(c)	25.	(a)
2.	(b)	8.	(a)	14.	(b)	20.	(c)	26.	(d)
3.	(a)	9.	(c)	15.	(b)	21.	(c)	27	(b)
4.	(d)	10.	(b)	16.	(c)	22.	(a)	28.	(b)
5.	(a)	11.	(b)	17.	(a)	23.	(c)	29.	(c)
6.	(b)	12.	(d)	18.	(a)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (d)

Selection sort in worst and best case take same time i.e., $O(n^2)$. So all the input take same time.

2. (b)

Before merging blindly, we have to sort both array individually which will take $O(m \log m)$ and $O(n \log n)$ time respectively. Then merging will take $O(m + n)$ in worst case.

$$\text{Total number of comparisions} = m \log m + n \log n + m + n$$
$$= O(m \log m + n \log n)$$

3. (a)

$$T_1(n) + T_2(n) = O(F(n))$$

4. (d)

$$T(n) = 2T\left(\frac{n}{\sqrt{2}}\right) + n$$

Using Master's theorem, $a = 2$, $b = \sqrt{2}$, $f(n) = n$

$$n^{\log_b a} = n^{\log_{\sqrt{2}} 2} = n^{\left(\frac{\log 2}{\log \sqrt{2}}\right)}$$
$$= \frac{1}{n^{(1/2)}} = n^2$$

$\Rightarrow$ $$n^{\log_b a} = n^2$$

$\Rightarrow$ $$n^2 > f(n)$$

$\Rightarrow$ $T(n)$ is $O(n^2)$

5. (a)

The worst case of the sort arises, when all the elements fit in to the single bucket. Now, on applying sorting on that particular bucket, if Insertion sort or Bubble sort is used, it will result in $O(n^2)$ time.

Heap sort is not a stable algorithm.

Worst case behaviour of merge sort is $O(n \log n)$.

Hence, merge sort is the correct answer.

6. (b)

Prim's algorithm will pick up the edge with least weight for a particular node, [provided it does not form a cycle] weight of edge (V_{i-1}, V_i) or $(V_i, V_{i-1}) = 1$

$\therefore$ MST will be

$$V_1 \underset{1}{—} V_2 \underset{1}{—} V_3 \underset{1}{—} V_4 \cdots V_n$$

$\therefore$ Total edge weight $= 2 \times (n - 1) = 2n - 2$.

7. (b)

Consider the following array,

1	6	3	2	5	4	7
0	1	2	3	4	5	6

Here the element at array index, 0, 2, 4 and 6 are already in their correct position. Applying merge sort on the array, will take $O(n \log n)$ time.

8. (a)

We need to delete element from the array and insert into the AVL tree. Deletion from the array satisfying heap property will take $O(\log n)$ time for each element.

$$\text{Total time} = O(n \log n) + O(n \log n)$$
$$= O(n \log n)$$

Note: $O(n \log n)$ time to satisfy the heap property of the array for every deletion and $O(n \log n)$ time to construct the AVL tree.

9. (c)

The time complexity of the Huffman coding is $O(n \log n)$. Using heap tree to store weight of each node. Each iteration required $O(\log n)$ time to determine the small weight node and insert the new weight there are n iteration one for each item.

10. (b)

$$W = 8 \text{ (capacity)}$$

Feasible solutions:

(i) $\{I_1, I_3, I_4\}$,

(ii) $\{I_2, I_3\}$

Profit of $\{I_1, I_3, I_4\}$ is 23

profit of $\{I_2, I_3\}$ is 15

Optimal solution is $\{I_1, I_3, I_4\}$ with capacity of 8 and maximum profit 23 produced.

$\therefore$ I_2 is not selected in the solution.

11. (b)

$$T(n) = T(n-1) + 1/n \Rightarrow \Theta(\log n)$$
$$T(n) = 2T(n/2) + n / \log n \Rightarrow \Theta(n \log \log n)$$
$$T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n \Rightarrow \Theta(n \log \log n)$$
$$T(n) = 4T(n/2) + n^2\sqrt{n} \Rightarrow \Theta(n^{2.5})$$

12. (d)

To build max-heap tree number of comparision combine every 2 element pair from n to 1.

$$\frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \frac{n}{16} \cdots \frac{n}{2^{\log n}} = (n) \text{ comparision.}$$

n comparision to find 1^{st} largest element, for next two elements it take O(log n) time.
So, total comparision n + O(log n).

13. (d)

Number of leaf nodes = n

Let internal nodes be K

$\therefore$ Total nodes = K + n

For m-ary tree, number of leaf nodes with K internal nodes = (m – 1) K + 1

$\therefore$ (m – 1) K + 1 = n

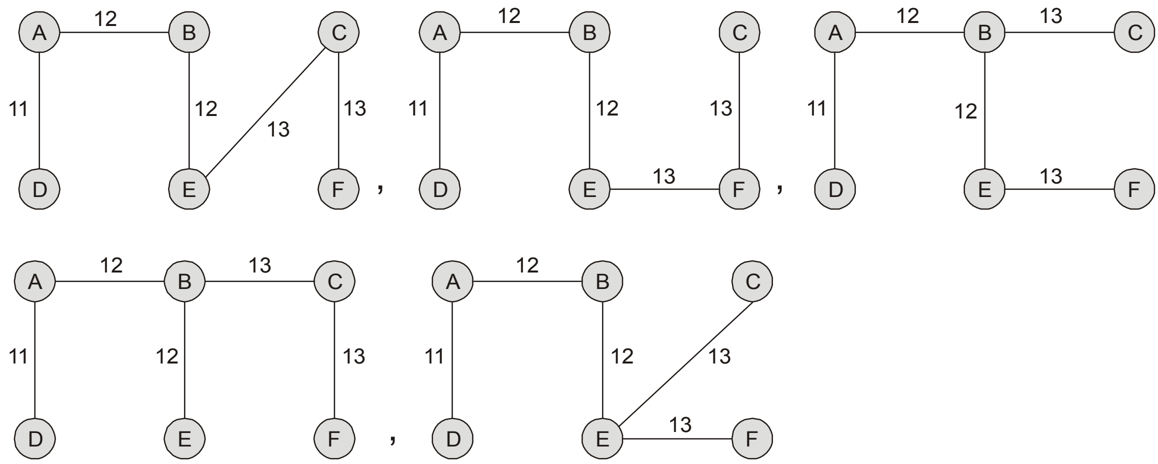
$$\therefore \qquad K = \frac{n-1}{m-1}$$

$\therefore$ Total number of nodes

$$= \frac{n-1}{m-1} + n = \frac{n-1+n(m-1)}{m-1}$$

$$= \frac{n-1 \;+nm-n}{m-1} = \frac{nm-1}{m-1}$$

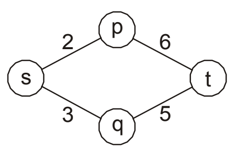
14. (b)



All above trees MST with cost: 11 + 12 + 12 + 13 + 13 = 61

15. (b)

Even if no two edges have the same weight, there could be two paths having the same weight.



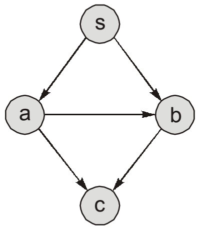
To find the shortest path between s and t.

$$s \rightarrow p \rightarrow t \Rightarrow 8$$
$$s \rightarrow q \rightarrow t \Rightarrow 8$$

So, there are two distinct paths.

16. (c)

Consider the following graph:



- Vertices 'a' and 'b' both will be on the stack at the same time if at least one of them have an edge on the other vertex. Hence, If there is no directed path from 'a' to 'b' then there exist a directed path from 'b' to 'a'.
- Since, vertices 'a' and 'b' are traversed while performing DFS at vertex 's', so it can be said that there exist directed path from 's' to 'a' and 's' to 'b'. Hence, There exist directed path from 's' to 'a' and directed path from 's' to 'b'.

17. (a)

$$f(n) = O(n),\ g(n) = \Omega(n),\ h(n) = \theta(n)$$

$$g(n) + f(n).h(n) = \Omega(n) + \underbrace{O(n).\theta(n)}_{}$$

$$= \Omega(n) + [\theta(n) \leftrightarrow \theta(n^2)]$$

$$= \Omega(n)$$

18. (a)

Starting vertex is D

	A	B	C	D	E	F	G	H
D	∞	∞	∞	0 Nill	∞	∞	∞	∞
F	16	17_D	13_D	–	∞	3_D	∞	8_D
B	16	7_E	13_D	–	∞	–	∞	8_D
H	16	–	13_D	–	∞	–	12_B	8_D
E	16	–	13_D	–	10_H	–	12_B	
G	16	–	13_D	–	–	–	12_B	
C	16	–	–	–	–	–	–	–
A	–	–	–	–	–	–	–	–

So the order of relaxed the vertices by using Dijkastra's algorithm is DFBHEGCA.

19. (c)

$$R^{(k)}[i, j] = R^{(k-1)}[i, j] \;||\; (R^{(k-1)}[i, k] \;\&\&\; R^{(k-1)}[k, j])$$

Transitive closure of directed graph computed using floyd warshall's algorithm. Which computes using all pairs shortest path algorithm.

$$R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array},$$

$$R^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R^{(2)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} (1, 2) \;\&\; (2, 3) \Rightarrow (1, 3)$$

$$R^{(3)} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, (1, 3) \;\&\; (3, 4) \Rightarrow (1, 4)$$

$$R^{(4)} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{Transitive closure of } R$$

20. (c)

Finding minimum edge each time will take (v) time, this will done for $(V - 1)$ time. Will take worst case time

$$= O(|V|) \times (V - 1)$$
$$= O(|V|^2)$$

21. (c)

Here Master's theorem is not applicable directly.

$$T(n) = 2.T\left(\sqrt{n}\right) + \log\left(\sqrt{n}\right)$$

put $n = 2^k$

$$T(2^k) = 2.T\left(2^{\frac{k}{2}}\right) + \log_2\left(2^{\frac{k}{2}}\right)$$

$$T(2^k) = 2.T\left(2^{\frac{k}{2}}\right) + \frac{k}{2}$$

put $S(k) = T(2^k)$

$$S(k) = 2.S\left(\frac{k}{2}\right)+\frac{k}{2}$$

Now apply Master's theorem: $S(k) = \Theta(k \log k)$

$$T(2^k) = \Theta(\log n \,.\, \log \log_2 n)$$

22. (a)

Function MHY (A, i, key) represent increase key or decrease key operation which will take log n time to replace the value of key and maintaining the Max heap property.

So, it will take $\Theta(\log n)$ time.

23. (c)

$$f(n) = 2^{\log_2 n} = n^{\log_2 2} = n$$

$$g(n) = n^{\log n}$$

$$h(n) = n^{1/\log n} = \sqrt[\log n]{n}\left[n > \sqrt[\log n]{n} \text{ for all large value of } n\right]$$

[It is less than n since max power of n is always less than 1 for large value of n]

So, $g(n) \geq f(n \geq h(n)$

So, $f(n) = O(g(n))$ and $g(n) = \Omega(h(n))$

24. (b)

For n time, inner loop will execute for n times.

For $\frac{n}{2}$ time, inner loop will execute for $\frac{n}{2}$ times.

For $\frac{n}{4}$ time, inner loop will execute for $\frac{n}{4}$ times.

and do on ....

So time complexity: $$T(n) = O\left(n+\frac{n}{2}+\frac{n}{4}+.....+1\right)$$

$$= O(n)$$

25. (a)

Since by looking through options, we get to know 'b' will be the start vertex.

	a	b	c	d	e	f	g	h
	∞	0	∞	∞	∞	∞	∞	∞
b	5	–	∞	1	3	∞	∞	∞
(b – d) d	5	–	∞	–	3	6	∞	∞
(d – e) e	5	–	4	–	–	6	9	∞
(e – c) c	4	–	–	–	–	6	9	7
(c – a) a	–	–	–	–	–	6	9	7
(d – f) f	–	–	–	–	–	–	9	7
(c – h) h	–	–	–	–	–	–	9	–
(e – g) g	–	–	–	–	–	–	–	–

So, correct sequence will be (b – d), (b – e), (e – c), (c – a), (d – f), (c – h), (e – g).

26. (d)

A. Matrix chain multiplication : (n^3)

B. Travelling salesman problem : (n^n)

C. 0/1 knapsack : (mn)

D. Fibonacci series : $O(n)$

27 (b)

$$\begin{cases} l[i, j], & \text{if } k = 0 \\ \min\left\{d_{i,j}^{k-1},\ d_{i,k}^{k-1} + d_{k,j}^{k-1}\right\}, & \text{if } 1 \le k \le n \end{cases}$$

$\therefore$ Option (b) is correct.

28. (b)

$$A_1\,A_2\,A_3 = A_1 \times (A_2 \times A_3)$$

$3 \times 100,\ 100 \times 2,\ 2 \times 2$

By **Person X applying Greedy:**

$A_1 \times (A_2 \times A_3)$

$3 \times 100,\ 100 \times 2,\ 2 \times 2$

$(A_2A_3) \rightarrow 100 \times 2,\ 2 \times 2 = 200 \times 2 = 400$

$A_1 \times (A_2A_3) \rightarrow 3 \times 100,\ 100 \times 2 = 300 \times 2 = 600$

Total number of multiplication required = 600 + 400 = 1000

Person Y with Dynamic:

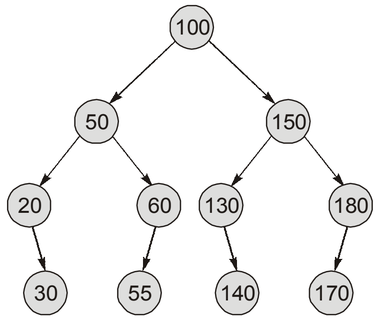
$A_1 \times A_2 \times A_3$

$(A_1 \times A_2) \times A_3$	$A_1 \times (A_2 \times A_3)$
$3 \times 100,\ 3 \times 100,\ 2 \times 2 \times 2$	$\downarrow$
$(A_1 \times A_2) \times A_3)$	$A_1 \times (A_2\,A_3)\ 100 \times 2$
	$3 \times 100 = 400$
$\Rightarrow 600 + 12$	$\Rightarrow 600 + 400$
$\Rightarrow 612$	$\Rightarrow 1000$

Number of multiplication saved by Person Y = 1000 – 612 = 388

29. (c)

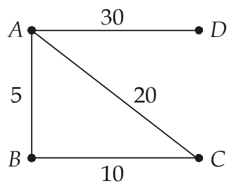
Inorder traversal of Binary Search Tree is in sorted order so,
Inorder traversal: 20, 30, 50, 55, 60, 100, 130, 140, 150, 170, 180
With inorder and preorder, unique Binary Search Tree can be determined.



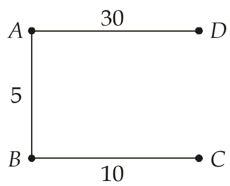
Postorder traversal will be: 30, 20, 55, 60, 50, 140, 130, 170, 180, 150, 100

30. (a)

Since edges costs are distinct, so cheapest edge must be present in every minimum spanning tree while expensive edge is may not excluded from every minimum spanning tree.
Statement S_2 is false
Example:



MST will be:



So, most expensive edge is not excluded.

■■■■