CLASS TEST

S.No. : 03IGCE-B+D-01072023

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 14/06/2023

ANSWER KEY

1.	(b)	7.	(a)	13.	(a)	19.	(c)	25.	(d)
2.	(d)	8.	(c)	14.	(a)	20.	(a)	26.	(c)
3.	(d)	9.	(c)	15.	(a)	21.	(b)	27.	(c)
4.	(b)	10.	(c)	16.	(b)	22.	(b)	28.	(d)
5.	(d)	11.	(c)	17.	(d)	23.	(b)	29.	(d)
6.	(c)	12.	(b)	18.	(c)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

$$f(x) = \frac{1}{15-0} = \frac{1}{15}$$

$$P\{5 < x < 9\} = \int_5^9 \frac{1}{15}\, dx = \frac{4}{15}$$

2. (d)

Given

$$f(x) = 2x^2 - 5x - 6$$

$$f'(x) = 4x - 5$$

$$f''(x) = 4$$

For minima/maxima,

$$f'(x) = 0$$

$$4x - 5 = 0$$

$$x = \frac{5}{4}$$

$$f''(x) = 4 > 0 \Rightarrow \text{Minima}$$

3. (d)

$$y = \frac{2}{3}\frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{2}{3x}\cdot\frac{1}{x} + \frac{2}{3}\ln x\left(\frac{-1}{x^2}\right) = \frac{2}{3x^2}(1 - \ln x)$$

For maxima, $$\frac{dy}{dx} = 0$$

$$\ln x = 1 \quad \Rightarrow \quad x = e \text{ is a stationary point}$$

$$\frac{d^2y}{dx^2} = \frac{-2}{3x^3}(3 - 2\ln x)$$

At $x = e$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = \frac{-2}{3e^3} < 0$$

Hence maxima at $x = e$

4. (b)

$$y^2 = 2ax$$

$$x = \frac{y^2}{2a} \qquad \text{...(i)}$$

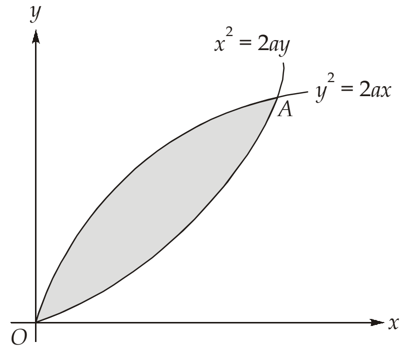
$$x^2 = 2ay$$

Using equation (i)

$$\frac{y^4}{4a^2} = 2ay$$

$$y^4 - 8a^3y = 0$$
$$y = 0, y = 2a$$

The parabolas intersect at 0 (0, 0) and A $(2a, 2a)$



$$\text{Required area} = \int_0^{2a}\int_{x^2/2a}^{\sqrt{2ax}} dy\,dx$$

$$= \int_0^{2a}\left(\sqrt{2ax} - \frac{x^2}{2a}\right)dx$$

$$= \left|\sqrt{2a}\frac{2}{3}x^{3/2} - \frac{1}{2a}.\frac{x^3}{3}\right|_0^{2a}$$

$$= \sqrt{2a}\frac{2}{3}(2a)^{3/2} - \frac{1}{2a}\frac{(2a)^3}{3}$$

$$= \frac{8a^2}{3} - \frac{4a^2}{3} = \frac{4a^2}{3}$$

5. (d)

$$1 + \left(\frac{d^3y}{dx^3}\right)^{7/5-5/7} = 0$$

$$\left(\frac{d^3y}{dx^3}\right)^{24/35} = -1$$

Raising power 35 on both sides.

$$\left(\frac{d^3y}{dx^3}\right)^{24} = -1$$

From here degree of equation is 24.

6. (c)

Eigen values of $(A + 7I)$ are $\gamma + 7$ and $\delta + 7$

Eigen values of $(A + 7I)^{-1} = \dfrac{1}{\gamma + 7}$ and $\dfrac{1}{\delta + 7}$

7. (a)

Let $\quad y = x^x$

Taking logarithm on both sides, we get

$$\log y = x \log x$$

Differentiating w.r.t. x, we get

$$\frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \quad \frac{dy}{dx} = y(1 + \log x)$$

$$= x^x(1 + \log x)$$

8. (c)

$$\text{Volume of solid} = \int_a^b \pi y^2 dx$$

Given $$y = \frac{1}{2\sqrt{x}}$$

$$\text{Volume of the solid} = \int_3^4 \frac{\pi}{4x} \cdot dx = \frac{\pi}{4}(\ln x)_3^4 = \frac{\pi}{4}\ln\left(\frac{4}{3}\right)$$

9. (c)

We know, $$AA^{-1} = I,$$

$$A \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I$$

$$\frac{A}{6} \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = I$$

$$\frac{A}{6} = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

10. (c)

$$AA^{-1} = I$$

$$\therefore \quad \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2x = 1$$

$$\therefore \quad x = \frac{1}{2}$$

11. (c)

The given equation will be consistent, if

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ 0 & \lambda - 3 & 3 - \lambda \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$C_3 \rightarrow C_3 + C_2$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 5\lambda+1 \\ 0 & \lambda-3 & 0 \\ 2 & 3\lambda+1 & 6\lambda-2 \end{vmatrix} = 0$$

$$(\lambda-3)\begin{vmatrix} \lambda-1 & 5\lambda+1 \\ 2 & 2(3\lambda-1) \end{vmatrix} = 0$$

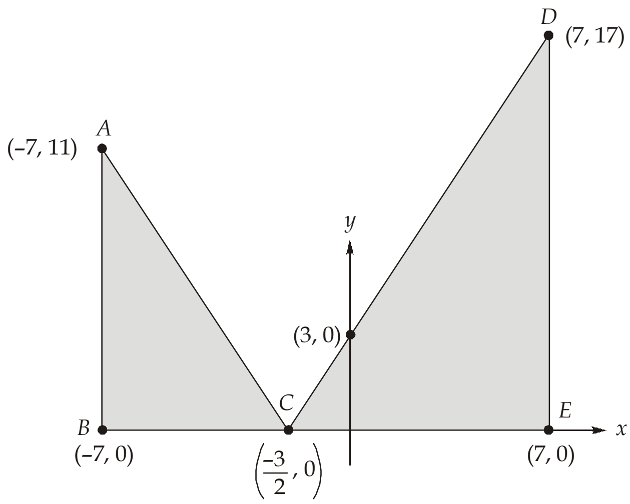
$$2(\lambda - 3)[(\lambda - 1)(3\lambda - 1)-(5\lambda + 1] = 0$$

$$6\lambda(\lambda - 3)^2 = 0$$

$$\lambda = 0 \text{ or } 3$$

The largest value $\lambda = 3$

12. (b)



The value of integral is equal to area of shaded region.

$$\text{Area} = \frac{1}{2}\times AB\times BC+\frac{1}{2}\times DE\times EC = \frac{1}{2}\times 11\times\frac{11}{2}+\frac{1}{2}\times 17\times\frac{17}{2}$$

$$= \frac{410}{4} = 102.5$$

13. (a)

Parabola given : $x^2 = 4y$ ...(i)

Straight line is $x - 2y + 4 = 0$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$

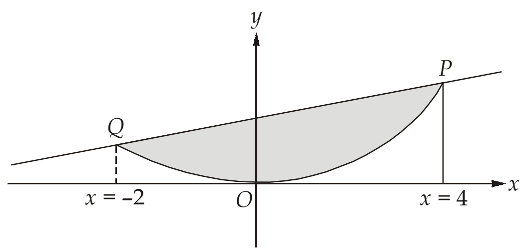
$\Rightarrow$ $x^2 = 2(x + 4)$

$\Rightarrow$ $x^2 - 2x - 8 = 0$

$\Rightarrow$ $x^2 - 4x + 2x - 8 = 0$

$\Rightarrow$ $x(x - 4) + 2(x - 4) = 0$

$\Rightarrow$ $x = 4, -2$



Required area = POQ

$$= \int_{-2}^{4} y\,dx \text{ from straight line} - \int_{-2}^{4} y\,dx \text{ from parabola}$$

$$= \int_{-2}^{4}\left(\frac{x+4}{2}\right)dx - \int_{-2}^{4}\frac{x^2}{4}dx = \frac{1}{2}\left|\frac{x^2}{2}+4x\right|_{-2}^{4} - \frac{1}{4}\left|\frac{x^3}{3}\right|_{-2}^{4}$$

$$= \frac{1}{2}\{8+16-(-6)\} - \frac{1}{12}(64+8) = \frac{1}{2}\times 30 - \frac{1}{12}\times 72 = 15-6 = 9 \text{ unit}^2$$

14. (a)

For $f(x)$ to be probability density function $= \int_{-\infty}^{\infty} f(x)dx = 1$

$$\frac{1}{A}\int_{3}^{6}(3x+5)\,dx = 1 \Rightarrow \frac{1}{A}\left|\frac{3x^2}{2}+5x\right|_{3}^{6} = 1$$

$$A = \left(\frac{3}{2}\cdot 6^2 - \frac{3}{2}\cdot 3^2\right) + 5(6-3) = \frac{3}{2}\cdot 27 + 15 = 55.5$$

15. (a)

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 2 & 6 \\ 2 & 10-\lambda & 2 \\ 6 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 14\lambda^2 + 288 = 0$$

From here

$$p+q+r = 14$$
$$pq+qr+rp = 0$$
$$pqr = -288$$
$$pq+qr+rp-pqr = 0-(-288) = 288$$

16. (b)

Let X be the number of rejections

$$n = 8$$
$$p = 0.16$$
$$q = 0.84$$

Probability of at least one rejection

$$= 1 - P(X \le 0)$$
$$= 1 - P(X_0)$$

$$P(X_0) = {}^nC_r p^r q^{n-r} = {}^8C_0(0.16)^0(0.84)^8 = 0.2479$$

Probability of at least one rejection = 1 – 0.2479 = 0.7521

17. (d)

$$f_x(X) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$E(X^4) = \int_{-\infty}^{\infty} x^4 \cdot f_x(X)\,dx$$

$$= \int_{0}^{1} x^4 = \left.\frac{x^5}{5}\right|_{0}^{1} = \frac{1}{5} = 0.2$$

18. (c)

$$A^{-1} = \frac{(\text{adj}\, A)}{|A|}$$

$$|A| = -8 \times 5 = -40$$

$$|A| \cdot \left(A^{-1}\right) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-40}{-8}, \frac{-40}{5}$$

$$= 5, -8$$

19. (c)

$$y^2 = 9x$$

$$2y\frac{dy}{dx} = 9$$

$$\frac{dy}{dx} = \frac{9}{2y} = \text{slope of tangent } (m_1)$$

$$\text{Slope of normal } (m_2) = \frac{-2y}{9} \qquad [as\ m_1 m_2 = -1]$$

Slope of the given line is $\frac{-2}{3}$

$$\frac{-2y}{9} = -\frac{2}{3}$$

$$y = 3$$

For $y = 3$

$$3^2 = 9x$$

$$x = 1$$

For (1, 3) to lie on the given line

$$\lambda = 2x + 3y = 2 + 9 = 11$$

20. (a)

$$D^2 - 4D + 4 = 0$$

$$(D - 2)(D - 2) = 0$$

$$D = 2, 2$$

$$y = (C_1 + C_2 x)e^{2x}$$

$$y(0) = C_1 = 0$$

$$y(1) = e^2 = C_2 \cdot e^2 \Rightarrow C_2 = 1$$

$$y = xe^{2x}$$

$$y(2) = 2e^4 = 109.196$$

21. (b)

Probability of drawing a card = $\frac{1}{80}$

$$E(x_i) = 1 \times \frac{1}{80} + 2 \times \frac{1}{80} + ... + 80 \times \frac{1}{80}$$

$$= \frac{1}{80} \times \frac{(80)(80+1)}{2} = \frac{81}{2}$$

Expected value of the sum of numbers on the ticket drawn:

$$E(x_1 + x_2 + x_3 + ... + x_{30}) = E(x_1) + E(x_2) + ... + E(x_{30})$$

$$30\,E(x_i) = 30 \times \frac{81}{2} = 1215$$

22. **(b)**

Since the probability of occurrence is very small, this follows Poisson distribution.

$$\text{mean} = m = np$$
$$= 1500 \times 0.002$$
$$= 3$$

Probability that more than 2 will get a hanging problem

$$= 1 - P(0) - P(1) - P(2)$$

$$= 1 - \left[e^{-m} + \frac{e^{-m}\cdot m^1}{1!} + \frac{e^{-m}\cdot m^2}{2!}\right]$$

$$= 1 - \left[e^{-3} + \frac{e^{-3}\cdot 3}{1} + \frac{e^{-3}\cdot 3^2}{2}\right]$$

$$= 1 - \left[\frac{1}{e^3} + \frac{3}{e^3} + \frac{9/2}{e^3}\right] = 1 - \frac{17}{2e^3}$$

23. **(b)**

Rearranging the equation,

$$\frac{dy}{dx} - \frac{y}{(2x+1)} = e^{4x}(2x+1)$$

The equation is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$IF = e^{\int P(x)dx} = e^{\int \frac{-1}{2x+1}dx}$$

$$= e^{-\frac{\ln(2x+1)}{2}} = \frac{1}{\sqrt{2x+1}}$$

24. **(c)**

$$AX = B$$

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} \frac{9}{2} & -6 \\ \frac{3}{2} & \frac{-3}{2} \end{bmatrix} = \begin{bmatrix} \frac{15}{2} & 3 \\ -3 & \frac{3}{2} \end{bmatrix}$$

$$\frac{9}{2}p + \frac{3}{2}q = \frac{15}{2} \qquad -6p - \frac{3}{2}q = 3$$

$$p = -7 \qquad q = 26$$

$$\frac{9}{2}r + \frac{3}{2}s = -3 \qquad -6r - \frac{3}{2}s = \frac{3}{2}$$

$$r = 1 \qquad s = -5$$

$$\Rightarrow \qquad A = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -7 & 26 \\ 1 & -5 \end{vmatrix} = 35 - 26 = 9$$

25. (d)

We parameterize the curve using $t = y$

$$x = 2 - 3t^2 \qquad -1 \le t \le 1$$

$$y = t$$

Then $$dx = -6t\,dt$$

$$dy = dt$$

$$\int_c 2y^3dx + 3x^2dy = \int_{-1}^{1}\left[2t^3(-6t) + 3\left(2 - 3t^2\right)^2\right]dt$$

$$= \int_{-1}^{1}\left(15t^4 - 36t^2 + 12\right)dt$$

$$= \left[\frac{15t^5}{5} - \frac{36t^3}{3} + 12t\right]_{-1}^{1} = \left[3t^5 - 12t^3 + 12t\right]_{-1}^{1}$$

$$= 3 + 3 = 6$$

26. (c)

Output produced by A = 40%

$\therefore$ $P(A) = 0.4$

Output produced by B = 60%

$\therefore$ $P(B) = 0.6$

Let, $P\left(\frac{D}{A}\right)$ = probability that item produced by A is defective

$$\therefore \quad P\left(\frac{D}{A}\right) = \frac{9}{1000} = 0.009$$

similarly, $$P\left(\frac{D}{B}\right) = \frac{1}{250} = 0.004$$

$P\left(\frac{A}{D}\right)$ = Probability that product is produced by A given that it is defective.

$$P\left(\frac{A}{D}\right) = \frac{P(A) \times P\left(\frac{D}{A}\right)}{P(A) \times P\left(\frac{D}{A}\right) + P(B) \times P\left(\frac{D}{B}\right)}$$

$$= \frac{0.4 \times 0.009}{0.4 \times 0.009 + 0.6 \times 0.004}$$

$$P\left(\frac{A}{D}\right) = \frac{0.0036}{0.0036 + 0.0024} = \frac{0.0036}{0.006} = 0.6$$

$$\therefore \quad P\left(\frac{A}{D}\right) = 0.6$$

27. (c)

$$\text{Volume of solid} = \int_a^b \pi y^2 dx$$

Given $$y = \frac{1}{2\sqrt{x}}$$

$$\text{Volume of the solid} = \int_3^4 \frac{\pi}{4x} \cdot dx = \frac{\pi}{4}(\ln x)_3^4 = \frac{\pi}{4}\ln\left(\frac{4}{3}\right)$$

28. (d)

$$P(W \cup L) = P(W) + P(L) - P(W \cap L)$$

$$P(W \cup L) = 0.45 + 0.25 = 0.70$$

$$P(W' \cup L') = 1 - P(W \cup L)$$

$$= 1 - 0.70 = 0.3$$

29. (d)

$$\frac{d^2y}{dx^2} + y = \cos x$$

$$(D^2 + 1)y = \cos x$$

$$PI = \frac{\cos x}{D^2 + 1}$$

Putting $$D^2 = -1$$

$$PI = \frac{\cos x}{-1+1} \quad [\text{Makes denominator zero}]$$

∴ Differentiating numerator and denominator

$$PI = x \cdot \frac{\cos x}{2D}$$

$$= \frac{1}{2} x \int \cos x \, dx = \frac{1}{2} x \sin x$$

30. (c)

$$S = 160t - 20t^2$$

For maximum height

$$\frac{dS}{dt} = 160 - 40t = 0$$

and $$t = 4 \text{ sec}$$

$$\frac{d^2S}{dt^2} = -40 < 0 \quad \Rightarrow \text{Maxima}$$

$$S_{max} = 160 \times 4 - 20 \times 4^2$$

$$\text{Maximum height} = 320 \text{ m}$$

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