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Engineering Mathematics

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CLASS TEST 2019-2020

COMPUTER SCIENCE & IT

Date of Test : 26/08/2019

ANSWER KEY ➤ Engineering Mathematics

1.	(c)	7.	(c)	13.	(c)	19.	(b)	25.	(d)
2.	(b)	8.	(b)	14.	(c)	20.	(a)	26.	(a)
3.	(c)	9.	(b)	15.	(d)	21.	(c)	27.	(d)
4.	(b)	10.	(b)	16.	(c)	22.	(c)	28.	(c)
5.	(d)	11.	(c)	17.	(a)	23.	(d)	29.	(a)
6.	(b)	12.	(a)	18.	(b)	24.	(c)	30.	(d)

DETAILED EXPLANATIONS

1. (c)
Commutative for multiplication of matrices does not hold.

$$AB \neq BA$$

2. (b)
Exact weight cannot be written but there will be limit to measure the weight.
Therefore it is continuous.
Number of questions in a test is finite and can be find easily that number of questions attempted.
Hence it is discrete.

3. (c)

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = B + C$$

[$\because$ Any square matrix can be expressed as the sum of symmetric and skew-symmetric matrices]
Here B is symmetric and C is skew-symmetric, B′ = B, C′ = – C.

4. (b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $\quad f(1) = 0$

Thus $\quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$ and $Lf'(1) = 2$, $Rf'(1) = 1$
$\Rightarrow$ f is not differentiable at $x = 1$

5. (d)

$$\text{rank of } [AB] \leq \text{rank of } [A]$$
$$\text{rank of } [AB] \leq \text{rank of } [B]$$
$$\text{rank of } [AB] \leq \min[\text{rank of } A, \text{rank of } B]$$

6. (b)
eigen values of $(A + 5I)$ are $\alpha + 5$ and $\beta + 5$

eigen values of $(A + 5I)^{-1} = \dfrac{1}{\alpha + 5}$ and $\dfrac{1}{\beta + 5}$

7. (c)

$$\text{Eigen values of } A = \text{Eigen value of } A^T$$

$\therefore \quad \begin{bmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{bmatrix} = 0 \Rightarrow (5-\lambda)(3-\lambda) = 0$

$\Rightarrow \quad \lambda = 3, 5$ are eigen values

$\therefore$ (c) is correct.

8. (b)

A is skew-symmetric,

$$\Rightarrow \quad A = -A^T$$

Now, $\quad (A \cdot A)^T = A^T \cdot A^T = (-A) \cdot (-A) = A \cdot A$

$\therefore$ $A \cdot A$ is a symmetric matrix.

9. (b)

$$\lambda = np = \frac{1}{100} \times 100 = 1$$

$$P(X > 2) = 1 - (P(X = 0) + P(X = 1))$$

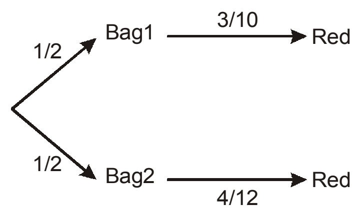
$$P(X = 0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = e^{-\lambda}$$

$$P(X = 1) = \frac{e^{-\lambda}\lambda'}{1!} = e^{-\lambda} \cdot \lambda$$

$$P(X > 2) = 1 - e^{-1}(2) = \frac{1-2}{e} = \frac{e-2}{e}$$

10. (b)

The tree diagram for above problem, is shown below:



$$P(\text{bag1} \mid \text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})} = \frac{1/2 \times 3/10}{1/2 \times 3/10 + 1/2 \times 1/3} = \frac{3/20}{3/20 + 1/6} = 0.317$$

11. (c)

$$P(x) = x^5 + x + 2$$

It has a real root at $x = -1$

$$\Rightarrow \quad P(x) = (x^4 - x^3 + x^2 - x + 2)(x + 1)$$

Now, $x^4 - x^3 + x^2 + x + 2$ will give other 4 roots

To find roots,

$$\Rightarrow \quad x^4 - x^3 + x^2 - x + 2 = 0$$

$$\Rightarrow \quad x^3(x - 1) + x(x - 1) + 2 = 0$$

$$\Rightarrow \quad x(x^2 + 1)(x - 1) + 2 = 0$$

In the above expression, $x^2 + 1$ is always positive. So, either 'x' or '$x - 1$' should be negative in order to satisfy the equation.

For $x > 1$, both (x) and $(x - 1)$ are positive and,

For $x < 0$, both (x) and $(x - 1)$ are negative

$\therefore$ x should lie within 0 and 1 in order to have real roots.

As $x \in (0, 1)$

$$\Rightarrow \quad |x| < 1$$

$$\Rightarrow \quad |x^2 + 1| < 2,\ |x| < 1 \text{ and } |x - 1| < 1$$

$\therefore$ The product of these three will be less than 2 and hence, no real value of 'x' can satisfy the equation

$$x^4 - x^3 + x^2 - x + 2 = 0$$

$\therefore$ The equation will have four imaginary roots apart from one real roots.

12. (a)
To obtain maximum value of $f(x)$, first $f'(x)$ should be equated to zero.

$\Rightarrow \quad f'(x) = 6x^2 - 6x - 36 = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad f'(x) = 0 \quad$ at $x = 3$ and -2

Now, $\quad f''(x) = 12x - 6$

$f''(3) = 30 > 0$

at $x = 3$, there is local minima

and $\quad f''(2) = -30 < 0$

$\therefore$ at $x = -2$, a local maxima is observed.

13. (c)

Suppose $\quad y = \lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$

$$\Rightarrow \quad y = \lim_{x\to\infty}\left[\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{\left[\frac{5(x+4)}{x+1}\right]}$$

$$\Rightarrow \quad \ln y = \lim_{x\to\infty}\frac{5(x+4)}{(x+1)}\ln\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}} \quad \ldots(i)$$

$\lim_{x\to\infty}\frac{5(x+4)}{(x+1)}$ is in the form of $\frac{\infty}{\infty}$ and $\lim_{x\to\infty}\ln\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}$ is in the form of 0^0.

Calculating the limits of both terms separately

$$\lim_{x\to\infty}5\frac{(x+4)}{(x+1)} = \lim_{x\to\infty}5\frac{\left(1+\frac{4}{x}\right)}{\left(1+\frac{1}{x}\right)} = 5\frac{(1+0)}{(1+0)} = 5$$

14. (c)

$$\left(y+x\frac{dy}{dx}\right)^2 = 1+\left(\frac{dy}{dx}\right)^3$$

the maximum power of $\frac{dy}{dx}$ is 3.

15. (d)

$$[A]^2 = \begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}\cdot\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$$

$\alpha^2 = 1 \quad ; \quad \alpha + 1 = 5$

$\alpha = \pm 1 \quad ; \quad \alpha = 4$

Unique value of α is not possible.

16. (c)
Required probability is given by

$$P(1 \le x \le 3) = \int_1^3 f(x)\,dx = \int_1^3 2e^{-2x}\,dx = 2\left[\frac{e^{-2x}}{-2}\right]_1^3 = e^{-2} - e^{-6}$$

$\therefore$ Option (c) is correct.

17. (a)

$$\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$$

$$\lim_{x\to a}\frac{g(x)f(a)-g(a)\cdot f(a)+g(a)\cdot f(a)-g(a)\cdot f(x)}{x-a}$$

$$\lim_{x\to a}\frac{f(a)\cdot[g(x)-g(a)]-g(a)[f(x)-f(a)]}{x-a}$$

$$\lim_{x\to a}\frac{f(a)\cdot[g(x)-g(a)]}{x-a}-\lim_{x\to a}\frac{g(a)[f(x)-f(a)]}{x-a}$$

$$f(a)\times g'(a)-g(a)\times f'(a) = 2\times 2-1\times(-1)$$
$$= 5$$

Alternate Solution:

Applying L'Hospitals rule

$$\lim_{x\to a}\frac{g'(x)f(a)-g(a)f'(x)}{1}$$

$$f(a)\times g'(a)-g(a)\times f'(a) = 2\times 2-1\times(-1)$$
$$= 5$$

18. (b)

$$np = 3$$

$$npq = \sigma^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

from here $q = \frac{3}{4}$

$$p = \left(1-\frac{3}{4}\right) = \frac{1}{4}$$

$$n\times\frac{3}{4}\times\frac{1}{4} = \frac{9}{4}$$

$$n = 12$$

19. (b)

(*i*) $E(X+2Y) = E(X)+2E(Y) = 1+2\times 2 = 5$

(*ii*) $\text{Cov}(X, Y) = E(XY)-E(X)\,E(Y)$

$\Rightarrow$ $E(XY) = \text{Cov}(XY)+E(X)\,E(Y) = 1+1\times 2 = 3$

(*iii*) $\text{Var}[X-2Y+1] = \text{Var}(X-2Y) = \text{Var}(X)+(-2)^2\,\text{Var}(Y)-4\,\text{Cov}(X, Y)$

$= 1+4\times 2-4 = 5$

$\therefore$ $p = 5, q = 3, r = 5$

$\therefore$ $pq+r = 5\times 3+5 = 20$

20. (a)

Required probability $= \frac{4}{52}\times\frac{4}{52} = \frac{1}{13}\times\frac{1}{13}$

21. (c)

The matrix which has 0 determinant will not be invertible.

determinant of A_1, $|A_1| = 3 \times 2 - 4 \times 1 = 2$

determinant of A_2, $|A_2| = 1[-3 - 0] + 0 + 4[0 + 1] = 1$

determinant of A_3, $|A_3| = 1(20 - 14) - 3(8 - 8) + 1(14 - 20) = 0$

determinant of A_4, $|A_4| = 2(0 - 1) - 3(6 - 3) + 1(3 - 0) = -2 - 9 + 3 = -8$

22. (c)

The matrix formed by the coefficients is $\begin{bmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{bmatrix}$

$$\text{Determinant} = 2a^2 - 2a - 4$$

$$\therefore \quad D = 0 \text{ for } a = 2 \text{ or } a = -1$$

(A) If $D \neq 0$, then the system will have unique solution.

(B) If $a = 2$, the matrix formed by the coefficients is $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

The rank of matrix is 2.

Considering 'z' as side unknown.

The characteristic determinant will be $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this is 0.

The system will have infinite solutions when $a = 2$.

(C) If $a = -1$, the matrix formed by the coefficients is $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Its rank is 2.

Considering 'z' as side unknown.

The characteristic matrix is $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this matrix is $3b$.

The system will have no solution if $b \neq 0$

$\therefore$ For $a = -1$ and $b \neq 0$, the system will have no solution.

23. (d)

Let $\quad f(x) = [|\sin x| + |\cos x|]$

as $\quad |\sin x| + |\cos x| \geq 1$

and $\quad |\sin x| + |\cos x| \leq \sqrt{1^2 + 1^2}$

$$\Rightarrow \quad 1 \leq \left[|\sin x| + |\cos x|\right] \leq \sqrt{2}$$

Thus, $\quad [|\sin x| + |\cos x|] = 1$

$$\therefore \quad \int_0^{2\pi} \left[|\sin x| + |\cos x|\right] dx = \int_0^{2\pi} 1.dx = 2\pi$$

24. (c)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since $P(A \cap B) = p(A)\, p(B)$ (not necessarily equal to zero).

So, $P(A \cup B) = P(A) + P(B)$ is false.

26. (a)

$$P^2 + 2P + I = P^2 + 2PI + I^2 = (P + I)^2$$

Eigen values of P are $-1, \frac{1}{2}, 3$

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{eigen values of } I_{3\times3} \text{ are } 1, 1, 1$$

Eigen values of $(P + I)$ are $-1 + 1, \frac{1}{2} + 1, 3 + 1$

$$= 0, \frac{3}{2}, 4$$

Eigen values of $(P + I)^2$ are $(0)^2, \left(\frac{3}{2}\right)^2, (4)^2 = 0, \frac{9}{4}, 16$

27. (d)

$$\text{Standard deviation} = \sqrt{\text{variance}} = \sqrt{\frac{(\beta - \alpha)^2}{12}}$$

here $\beta = 3, \alpha = 1 = \sqrt{\frac{2^2}{12}} = \frac{1}{\sqrt{3}}$

28. (c)

The system may be written in matrix form as

$$\begin{bmatrix} 1 & 3 & -8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$LU = A = \begin{bmatrix} 1 & 3 & -8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\begin{aligned} \ell_{11} &= 1, \ell_{21} = \ell, \ell_{31} = \ell \\ \ell_{11} u_{12} &= 3 \Rightarrow u_{12} = 3, \\ \ell_{21} u_{12} + \ell_{22} &= 4 \Rightarrow \ell_{22} = 4 - 1.3 = 1 \\ \ell_{31} u_{12} + \ell_{32} &= 3 \Rightarrow \ell_{32} = 3 - 1 \times 3 = 0 \end{aligned}$$

$$\ell_{11} u_{13} = -8 \Rightarrow u_{13} = \frac{-8}{1} = -8$$

$$\ell_{21}\, u_{13} + \ell_{22}\, u_{23} = 3 \Rightarrow u_{23} = 11$$

$$\ell_{31}\, u_{13} + \ell_{32}\, u_{23} + \ell_{33} = 4 \Rightarrow \ell_{33} = 12$$

$$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 12 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 3 & -8 \\ 0 & 1 & 11 \\ 0 & 0 & 1 \end{bmatrix}$$

29. (a)

Given that, even number twice than an odd number

$$P(\text{showing even number}) = \frac{2}{3}$$

$$P(\text{showing odd number}) = \frac{1}{3}$$

Sum of two numbers are odd if first is even and second numbers is odd or vice versa.

$$P(\text{sum of two number odd}) = \frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3} = \frac{2}{9}+\frac{2}{9} = \frac{4}{9} = 0.444$$

30. (d)

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$C = (A, B) = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 5 & : & 10 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

After performing $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - 2R_1$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 4 & : & 4 \\ 0 & 1 & \lambda-2 & : & \mu-12 \end{bmatrix}$$

After performing $R_3 \leftarrow R_3 - R_2$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 4 & : & 4 \\ 0 & 0 & \lambda-6 & : & \mu-16 \end{bmatrix}$$

Since $R(A) = R(C)$ for unique solution

So $\lambda - 6 \neq 0$, $\lambda \neq 6$ and $\mu - 10 \neq 0$, $\mu \neq 16$.

For no solution $R(A) \neq R(C)$ then $R(A) = 2$ and $R(C) = 3$

$$\lambda - 6 = 0$$

$$\Rightarrow \quad \lambda = 6 \text{ and } \mu - 16 \neq 0 \Rightarrow \mu \neq 16$$

For infinite solution $R(A) = R(C) = 2$

then $\lambda - 6 = 0$ and $\mu - 16 = 0$

$$\lambda = 6 \text{ and } \mu = 16$$

So all of options are true.

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