CLASS TEST

GATE

S.No. : 06 CH1_EE_S_230819

Control Systems

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | **Noida** | **Bhopal** | **Hyderabad** | **Jaipur** | **Lucknow** | **Indore** | **Pune** | **Bhubaneswar** | **Kolkata** | **Patna**

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

CLASS TEST 2019-2020

ELECTRICAL ENGINEERING

Date of Test : 23/08/2019

ANSWER KEY ➤ Control Systems

1.	(d)	7.	(b)	13.	(d)	19.	(b)	25.	(a)
2.	(b)	8.	(c)	14.	(b)	20.	(a)	26.	(a)
3.	(d)	9.	(c)	15.	(a)	21.	(b)	27.	(b)
4.	(b)	10.	(a)	16.	(b)	22.	(b)	28.	(b)
5.	(b)	11.	(b)	17.	(d)	23.	(a)	29.	(b)
6.	(b)	12.	(c)	18.	(a)	24.	(c)	30.	(d)

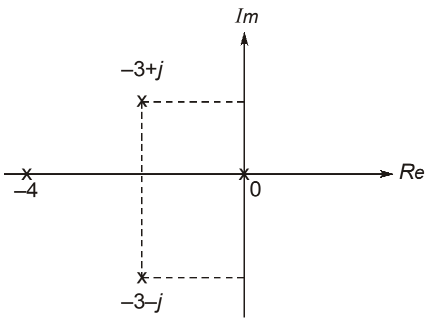
Detailed Explanations

1. (d)
From log -magnitude plot,
corner frequency at $\log \omega = -1;$
or $\omega = 0.1;$
Hence pole at $\omega = 0.1,$
and gain; $\log |G| = 1, G = 10,$
Hence transfer fuction $TF = \frac{10}{(1+s/0.1)} = \frac{1}{s+0.1}$

2. (b)
Transportation lag causes instability in a system.

3. (d)
$$(s + 3 + j4)(s + 3 - j4) = 0$$
$$(s + 3)^2 - (j4)^2 = 0;\ s^2 + 6s + 9 + 16 = 0;\ s^2 + 6s + 25 = 0$$
$$\omega_n = \sqrt{25};\quad \omega_n = 5 \text{ rad/sec};\quad 2\zeta\omega_n = 6$$
$$\zeta = \frac{6}{2 \times 5} = 0.6$$

4. (b)
$$\phi_d = 180° - \phi$$



$$\phi = 90° + 45° + 161.565° = 296.56°$$
$\therefore$ $$\phi_d = 180° - 296.5650 = \mp 116.56°$$

5. (b)
The transfer function can be
$$G(s)H(s) = \frac{K\left(1+\frac{s}{2}\right)}{s}$$
$$12 \text{ dB} = 20 \log K$$
or, $K = 4$

6. (b)
$\Rightarrow$
$$P_1 = G_1G_2G_3\ ;\qquad \Delta_1 = 1$$
$$P_2 = G_1G_2\ ;\qquad \Delta_2 = 1$$
$$L_1 = -G_1G_2G_3H_1H_2$$
$$L_2 = -G_2$$

$$L_3 = -G_1G_2H_2$$
$$L_4 = -G_1G_2H_1H_2$$

using Mason's Gain formula,

$$\frac{C}{R} = \frac{G_1G_2(1+G_3)}{1+G_2+G_1G_2H_2(1+H_1+G_3H_1)}$$

7. (b)

$$T(s) = \frac{6}{s^2+4s+6}$$

comparing with standard transfer function

we get,

$$\omega_n = \sqrt{6} \text{ and } \quad 2\xi\omega_n = 4$$

$\therefore$ $$\xi = \frac{2}{\sqrt{6}} = 0.816$$

$$\% \text{ Peak overshoot} = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\% = 1.18\%$$

8. (c)

Using Routh Array

s^5	1	2	11
s^4	1	2	10
s^3	$\lim_{\varepsilon\to 0} \varepsilon$	1	0
s^2	$\frac{2\varepsilon-1}{\varepsilon}$	10	0
s^1	$1-\frac{10\varepsilon^2}{2\varepsilon-1}$	0	0
s^0	10		

Total number of sign change in the first column is = 2 = Roots located in RHS of s-plane.

Hence, 3 poles are having negative real part.

9. (c)

The state transition matrix

$$\phi(s) = (sI - A)^{-1}$$

or $$(sI - A) = [\phi(s)]^{-1}$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - [A] = [\phi(s)]^{-1} = \begin{bmatrix} s & -1 \\ 5 & s+6 \end{bmatrix}$$

or $$[A] = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}$$

10. (a)

A 4th order all pole system means that the system must be having no zero or 's' terms in numerator and s^4 terms in denominator or 4-poles, i.e.,

$$H(s) \;\alpha\; \frac{1}{s^4}$$

one poles exhibits –20 dB/decade slope, so 4-pole exhibits a slope of –80 dB/decade.

11. (b)

Characteristic equation

$$1 + G(s)\,H(s) = 0$$

$$\Rightarrow \quad 1+\frac{11\beta}{s^3+4s^2+3s+1} = 0$$

or, $s^3 + 4s^2 + 3s + 1 + 11\beta = 0$

Routh array

s^3	1	3
s^2	4	$(11\beta + 1)$
s^1	$\frac{12-(11\beta+1)}{4}$	0
s^0	$(11\beta + 1)$	

for stability,

$$\frac{12-(11\beta+1)}{4} \geq 0$$

or $12 \geq (11\beta + 1)$

or $\beta \leq 1$

12. (c)

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{-1}$$

$$\Rightarrow \quad \frac{\pi\xi}{\sqrt{1-\xi^2}} = 1 \qquad \text{...(i)}$$

also $t_p = \dfrac{\pi}{\omega_d} = 1$ sec

$$\frac{\pi\xi}{\sqrt{1-\xi^2}} = 1$$

$$\frac{\pi\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} = 1$$

$$\frac{\pi}{\omega_d}\times\xi\omega_n = 1$$

$$t_s = \frac{4}{\xi\omega_n} = 4\left(\frac{1}{\xi\omega_n}\right) = 4 \text{ sec}$$

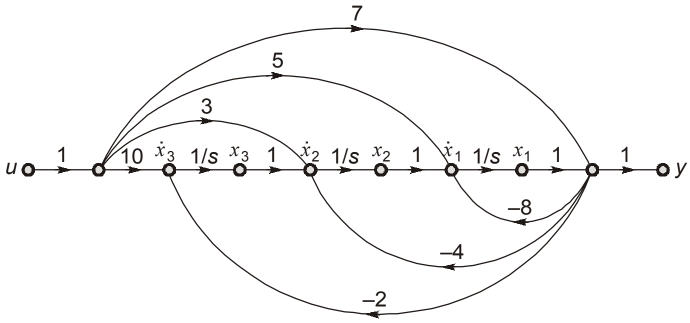
13. (d)

$$T(s) = \frac{K\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{5}+1\right)^2}$$

Also, $26\,\text{dB}\big|_{\omega=0.1} = 20\log K - 20\log\omega$

or $K = 2$

14. (b)



$$\dot{x}_3 = 10u - 2x_1$$

$$\dot{x}_2 = 3u + x_3 - 4x_1$$

$$\dot{x}_1 = 5u - 8x_1 + x_2$$

$$y = 7u + x_1$$

$$\therefore \quad \dot{x} = \begin{bmatrix} -8 & 1 & 0 \\ -4 & 0 & 1 \\ -2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 5 \\ 3 \\ 10 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]x + [7]u$$

15. (a)

The steady state error for step input

$$e_{ss} = \frac{A}{1+K_p} = \frac{5}{1+K_p} = 0.12$$

$$\because \quad r(t) = 5\,u(t)$$

or $$1 + K_p = \frac{5}{0.12} = 41.66$$

Hence, $$K_p = 41.66 - 1 = 40.66$$

16. (b)

$$\text{For GM} = 25 \text{ dB}$$

$$\text{GM} = 20 \log \frac{1}{X_1} = 25 \text{ dB}$$

$$\therefore \quad X_1 = 0.056$$

For $$GM = 18 \text{ dB}$$

$$18 \text{ dB} = 20 \log \frac{1}{X_2}$$

$$\therefore \quad X_2 = 0.125$$

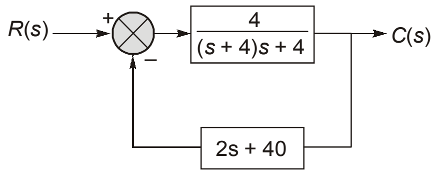
Therefore,

The gain must increase by a factor

$$\frac{0.125}{0.056} = 2.23$$

17. (d)

Transfer function



$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4s + 4 + 160 + 8s}$$

$$= \frac{4}{s^2 + 12s + 164}$$

$$\therefore \quad \omega_n = \sqrt{164}$$

$$\xi = \frac{12}{2\sqrt{164}} = 0.468$$

Resonant peak $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.209$

18. (a)

$$G(s) = \frac{1}{sT_1(1+sT_2)}$$

$$TF = \frac{\frac{1}{sT_1(1+sT_2)}}{1+\frac{1}{sT_1(1+sT_2)}} = \frac{1}{sT_1(1+sT_2)+1} = \frac{1}{s^2T_1T_2 + sT_1 + 1}$$

$$= \frac{1}{T_1T_2\left(s^2 + \frac{s}{T_2} + \frac{1}{T_1T_2}\right)}$$

$$\omega_n = \frac{1}{\sqrt{T_1T_2}};$$

$$\xi = \frac{1}{2}\sqrt{\frac{T_1}{T_2}}$$

for $\xi << 1$, $\Rightarrow$ $T_1 << T_2$

19. (b)

From the equations, we have

$$[A] = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}; [B] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[C] = [1 \;\; 2]$$

Check for Controllability:

$$[AB] = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$[B\ AB] = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}$$

whose determinant is zero. Hence the system is not controllable.

Test for Observability:

$$[C] = [1 \;\; 2]$$

$$C^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

Observability $$O = \begin{bmatrix} C^T & A^T C^T \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -4 \end{bmatrix}$$

whose determinant $\neq 0$, Hence, observable.

20. **(a)**

The open loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

(a) Finite poles are at $s = 0, -1, -2, (P = 3)$

Finite zeros are nil $(z = 0)$

(b) Number of asymptotes $= P - Z = 3$

The centroid σ (or the meeting point of the asympotes) is at

$$\sigma = \frac{0-1-2}{3} = -1$$

The angles of the asymptotes are given by

$$\theta_K = \frac{(2K+1)\pi}{P-Z}, K = 0,1 \; ...., (| P - Z | - 1)$$

$$= \frac{(2K+1)\pi}{3}, K = 0, 1, 2 \; = 60°, 180°, 300° \; (-60°)$$

(c) The breakaway points are given by

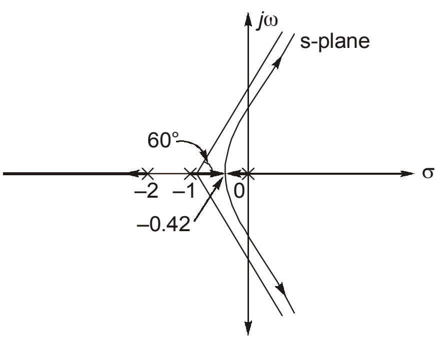
$$\frac{dK}{ds} = 0 \quad \text{or} \quad \frac{dK}{ds} = \frac{d}{ds}[-s(s+1)(s+2)] = 0$$

or $$-(3s^2 + 6s + 2) = 0$$

or $$s = \frac{-6 \pm \sqrt{36-24}}{6} = -0.42, -1.577 \text{ (invalid)}$$

Thus, $s = -0.42$ is only valid break away point

(d) The number of branches of the root loci is the greater of P and Z, viz, 3. Using all the above information, we plot the root loci,



Hence choice (a) is correct.

21. (b)

Before the switch is closed the TF,

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+a)}}{1+\frac{K}{s(s+a)}} = \frac{K}{s(s+a)+K} = \frac{K}{s^2+as+K}$$

$$\omega_n = \sqrt{K}, \quad 2\zeta\omega_n = a,$$

$$\zeta = \frac{a}{2\sqrt{K}}$$

Steady state error is

$$e_{ss} = \lim_{s\to 0} s.R(s)\frac{1}{1+G(s)H(s)} = \lim_{s\to 0} s.\frac{1}{s^2}\frac{1}{1+\frac{K}{s(s+a)}}$$

$$= \lim_{s\to 0} s.\frac{1}{s^2}\frac{s(s+a)}{s(s+a)+K} = \lim_{s\to 0}\frac{s+a}{s^2+as+K} = \frac{a}{K}$$

After the switch is closed,

$$\text{TF} = \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+a)+K}}{\frac{K}{s(s+a)+K}.K_T s} = \frac{K}{s(s+a)+K+KK_T s}$$

$$= \frac{K}{s^2+s(a+KK_T)+K}$$

$$\omega_n = \sqrt{K}, \quad 2\zeta\omega_n = a + KK_T,$$

$$\zeta = \frac{a+KK_t}{2\sqrt{K}} = \frac{a}{2\sqrt{K}} + \frac{K_T\sqrt{K}}{2}$$

Damping factor is increased by $\frac{K_T\sqrt{K}}{2}$

Steady state error

$$e_{ss} = \lim_{s\to 0} s.R(s)\frac{1}{1+G(s)H(s)} = \lim_{s\to 0}\frac{s\times\frac{1}{s^2}}{1+\frac{K}{s[(s+a)+K_T K]}}$$

$$= \lim_{s\to 0}\frac{1}{s\left[1+\frac{K}{s[s(s+a)+K_T K]}\right]}$$

$$= \lim_{s\to 0}\frac{1}{s+\frac{K}{(s+a)+K_T K}} = \frac{1}{0+\frac{K}{a+K_T K}} = \frac{a+K_T K}{K} = \frac{a}{K}+K_T$$

Steady state error will increase by K_T.

Hence, both ζ and e_{ss} are increased.

22. (b)

From block diagram, we can write

$$\dot{x}_2(t) = u(t) - \beta x_1(t) - \alpha x_2(t) \quad ...(i)$$

$$\dot{x}_1(t) = x_2(t) \quad ...(ii)$$

and $$x_1(t) = y(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

23. (a)

The equations of performance for the system are.

$$B_1(\dot{X}_1 - \dot{X}_0) + K_1(X_1 - X_0) = K_2 X_0$$

or $$(sB_1 + K_1)X_1(s) - (sB_1 + K_1)\,X_0(s) = K_2 X_0(s)$$

$$\frac{X_0(s)}{X_1(s)} = \frac{sB_1 + K_1}{sB_1 + K_1 + K_2}$$

$$T = \frac{K_1\left(1 + \dfrac{B_1 s}{k_1}\right)}{(K_1 + K_2)\left(1 + \dfrac{sB_1}{K_1 + K_2}\right)}$$

Let $$\frac{K_1 + K_2}{K_1} = a$$

where $$a > 1$$

and $$\frac{B_1}{K_1 + K_2} = \text{T};$$

Then, $$\frac{X_0(s)}{X_1(s)} = \frac{1}{a}\left(\frac{1 + aTs}{1 + Ts}\right)$$

Therefore zero is nearer to origin than pole i.e. **Lead network.**

24. (c)

$$\text{Inner loop, } G'(s) = \frac{\dfrac{1}{s(1+4s)}}{1 + \dfrac{1}{s(1+4s)}K_o s} = \frac{1}{s(1+4s) + K_o s}$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{100}{s(1+4s)+K_o s}}{1 + \dfrac{100}{s(1+4s)+K_o s}} = \frac{100}{s(1+4s) + K_o s + 100}$$

Characteristic equation is $4s^2 + s(K_o + 1) + 100 = 0$

$$s^2 + \frac{s(K_o + 1)}{4} + 25 = 0,\ \omega_n = \sqrt{25} = 5$$

$$2\zeta\omega_n = \frac{K_o + 1}{4},$$

$$2 \times 0.5 \times 5 = \frac{K_o + 1}{4};\ K_o = 20 - 1 = 19$$

25. (a)

The gain of a lead compensator is given by

$$G_c(s) = \frac{\alpha(T_{LD}s+1)}{\alpha T_{LD}s+1} = \frac{s+\frac{1}{T_{LD}}}{s+\frac{1}{\alpha T_{LD}}}$$

Comparing the latter equation with

$$G_c(s) = \frac{1+6s}{1+2s}$$

$\Rightarrow$ $T_{LD} = 6$

and $\alpha T_{LD} = 2$

or $\alpha = \frac{2}{6} = \frac{1}{3}$

The maximum phase shift ϕ_m is given by

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2}$$

or $\phi_m = 30°$

26. (a)

$$y = 1-\frac{7}{3}e^{-t}+\frac{3}{2}e^{-2t}-\frac{1}{6}e^{-4t}$$

$$Y(s) = \frac{1}{s}-\frac{7/3}{s+1}+\frac{3/2}{(s+2)}-\frac{1/6}{(s+4)}$$

$$G(s) = s\left[\frac{1}{s}-\frac{7/3}{s+1}+\frac{3/2}{(s+2)}-\frac{1/6}{(s+4)}\right]$$

$$= 1-\frac{7s}{3(s+1)}+\frac{3s}{2(s+2)}-\frac{s}{6(s+4)}$$

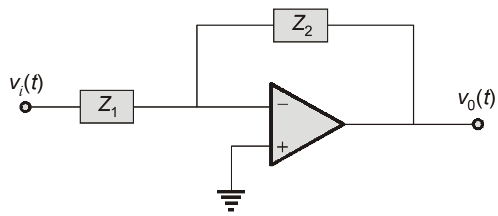
$$= 1-\frac{7}{3}+\frac{3}{2}-\frac{1}{6}+\frac{7}{3(s+1)}-\frac{3}{(s+2)}+\frac{2}{3(s+4)}$$

$$= \frac{7/3}{(s+1)}-\frac{3}{(s+2)}+\frac{2/3}{(s+4)} = \frac{(s+8)}{(s+1)(s+2)(s+4)}$$

$\Rightarrow$ $a+b+c+d = 15$

27. (b)

Equivalent impedance circuit diagram is shown in figure.



Here, $$Z_1(s) = R_1+\frac{1}{sC_1} = 10^5+\frac{1}{s\times10^{-6}} = 10^5\left(\frac{s+10}{s}\right)$$

and $$Z_2(s) = R_2 + \frac{1}{sC_2 + \frac{1}{R_2}} = 10^5 + \frac{1}{10^{-6}s + 10^{-5}}$$

$$= 10^5\left(\frac{s+20}{s+10}\right)$$

Given op-amp circuit is an inverting amplifier, therefore transfer function is

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{10^5\left(\frac{s+20}{s+10}\right)}{10^5\left(\frac{s+10}{s}\right)}$$

$$= \frac{-s(s+20)}{(s+10)^2}$$

28. (b)

For low frequencies

$$e^{-s} = (1-s)$$

$$G(s)\,H(s) = \frac{K(1-s)}{s(s^2+2s+1)}$$

$$1 + G(s)\,H(s) = 1 + \frac{K(1-s)}{s(s^2+2s+1)}$$

$$\therefore\ s(s^2 + 2s + 1) + K(1 - s) = 0$$

$$s^3 + 2s^2 + s + K - Ks = 0$$

$$s^3 + 2s^2 + s(1 - K) + K = 0$$

The Routh's array is

s^3	1	$1-K$
s^2	2	K
s^1	$\frac{2(1-K)-K}{2}$	
s^0	K	

For stability $K > 0$ and $2(1 - K) - K > 0$

or $2 - 3K > 0$ or $K < \frac{2}{3}$.

Hence, the restriction on K is $0 < K < \frac{2}{3}$.

29. (b)

For given system,

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix},$$

$$(sI - A) = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix};$$

$$|sI - A| = (s+2)(s+3)$$

and $$(sI - A)^{-1} = \frac{1}{(s+2)(s+3)}\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

Now, $$e^{At} = L^{-1}\{(sI - A)^{-1}\}$$

$\therefore$ $$e^{At} = L^{-1}\begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

or, $$e^{At} = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

30. (d)

OLTF is given by $$G(s) = \frac{K(1+sT_d)}{s^2(1+sT_1)}$$

putting $s = j\omega$

$$G(j\omega) = \frac{K(1+j\omega T_d)}{-\omega^2(1+j\omega T_1)}$$

Magnitude, $$|G(j\omega)| = \frac{K\sqrt{1+(\omega T_d)^2}}{\omega^2\sqrt{1+(\omega T_1)^2}}$$

Phase angle $$\angle G(j\omega) = -180 + \tan^{-1}(\omega T_d) - \tan^{-1}(\omega T_1).$$

$$\angle G(j\omega)|_{\omega=0} = -180° \quad \text{...(a)}$$

$$\angle G(j\omega)|_{\omega=\infty} = -180° \quad \text{...(b)}$$

Since at $\omega = \infty$,

$\angle G(j\omega) = -90°$, which is possible only if $\tan^{-1}(\omega T_d) - \tan^{-1}(\omega T_1) > 0$

or, $\tan^{-1}(\omega T_d) > \tan^{-1}(\omega T_1)$

$\Rightarrow$ $T_d > T_1$

■■■■