CLASS TEST

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 14/06/2023

ANSWER KEY

1.	(b)	7.	(a)	13.	(c)	19.	(b)	25.	(d)
2.	(b)	8.	(a)	14.	(a)	20.	(d)	26.	(b)
3.	(c)	9.	(a)	15.	(c)	21.	(a)	27.	(a)
4.	(c)	10.	(b)	16.	(b)	22.	(a)	28.	(b)
5.	(b)	11.	(d)	17.	(c)	23.	(a)	29.	(d)
6.	(c)	12.	(d)	18.	(a)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (b)
Solution of laplace equation having continuous
Second order partial derivating

$$\therefore \quad \nabla^2\phi = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

$\therefore$ ϕ is harmonic function.

2. (b)
Median speed is the speed at the middle value in series of spot speeds that are arranged in ascending order. 50% of speed values will be greater than the median 50% will be less than the median.
Ascending order of spot speed studies are
32, 39, 45, 51, 53, 56, 60, 62, 66, 79

Median speed = $\frac{53+56}{2}$ = 54.5 km/hr

3. (c)

$$\int_{-\infty}^{\infty} p(x)\cdot dx = 1$$

$$\int_{-\infty}^{\infty} K\cdot e^{-\alpha|x|}\cdot dx = 1$$

$$\int_{-\infty}^{0} K\cdot e^{\alpha x}\cdot dx + \int_{0}^{\infty} K\cdot e^{-\alpha x} = 1$$

$$\Rightarrow \quad \frac{K}{\alpha}\left(e^{\alpha x}\right)_{-\infty}^{0} + \frac{K}{-\alpha}\left(e^{-\alpha x}\right)_{0}^{\infty} = 1$$

$$\Rightarrow \quad \frac{K}{\alpha} + \frac{K}{\alpha} = 1$$

$$2K = \alpha$$

$$\Rightarrow \quad K = 0.5\,\alpha$$

4. (c)
Since, $\cos 2x = \cos^2 x - \sin^2 x$, therefore $\cos 2x$ is a linear combination of $\sin^2 x$ and $\cos^2 x$ and hence these are linearly dependent.

5. (b)

$$\ddot{x} + 3x = 0$$

Auxiliary equation is

$$D^2 + 3 = 0$$

i.e. $$D = \pm\sqrt{3}\,i$$

$\therefore \quad x = A\cos\sqrt{3}t + B\sin\sqrt{3}t$

at $\quad t = 0, x = 1$

$\Rightarrow \quad A = 1$

Now, $\quad \dot{x} = \sqrt{3}(B\cos\sqrt{3}t - A\sin\sqrt{3}t)$

At $\quad t = 0, \dot{x} = 0$

$\Rightarrow \quad B = 0$

So, $\quad x = \cos\sqrt{3}t$

$$x(1) = \cos\sqrt{3} = 0.99$$

6. (c)

Intermediate value theorem states that if a function is continuous and $f(a) \cdot f(b) < 0$, then surely there is a root in (a, b). The contrapositive of this theorem is that if a function is continuous and has no root in (a, b) then surely $f(a) \cdot f(b) \geq 0$. But since it is given that there is no root in the closed interval $[a, b]$ it means $f(a) \cdot f(b) \neq 0$.

So surely $f(a) \cdot f(b) > 0$ which is choice (c).

7. (a)

$$u = 2xy$$

$$u_x = 2y \qquad u_y = 2x$$

In option (a)

$$V_x = -2x \quad u_y = -V_x$$

$$V_y = 2y$$

(–R equation are satisfied only in option a)

8. (a)

$$f(x, y) = x^2 + 3y^2$$

$$\phi = x^2 + y^2 - 2 \text{ and point } P \Rightarrow (1, 1)$$

Normal to the surface,

$$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} = 2x\hat{i} + 2y\hat{j}$$

$$\nabla\phi\big|_{\text{at P}(1,1)} = 2\hat{i} + 2\hat{j}$$

the normal vector is $\vec{a} = 2\hat{i} + 2\hat{j}$

Magnitude of directional derivative of f along $\vec{a}$ at (1, 1) is $\Rightarrow \nabla \cdot f \cdot \hat{a}$

$$\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} = 2x\hat{i} + 6y\hat{j}$$

$$\nabla f\big|_{(1,1)} = 2\hat{i} + 6\hat{j}$$

$$|\vec{a}| = \sqrt{4+4} = 2\sqrt{2}$$

$$\hat{a} = \frac{2\hat{i} + 2\hat{j}}{2\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$\therefore$ Magnitude of directional derivative

$$= (2\hat{i} + 6\hat{j})\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$$

$$= \frac{2+6}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

9. (a)

$$I = \oint_C \frac{-3z+4}{(z^2+4z+5)} dz = 2\pi i \text{ (sum of residues)}$$

Poles of $\dfrac{-3z+4}{(z^2+4z+5)}$ are given by

$$z^2 + 4z + 5 = 0$$

$$z = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

Since the poles lie outside the circle $|z| = 1$.
So $f(z)$ is analytic inside the circle $|z| = 1$.

Hence $\oint_C f(z) dz = 2\pi i (0) = 0$

10. (b)

Given that the partial differential equation is parabolic.

$$\therefore \quad B^2 - 4AC = 0 \qquad \text{Here } A = 3$$

$$\therefore \quad B^2 - 4(3)(3) = 0 \qquad C = 3$$

$$B^2 - 36 = 0$$

$$B^2 = 36$$

11. (d)

The differential equation is $3y''(x) + 27y(x) = 0$
The auxillary equation is

$$3m^2 + 27 = 0$$

$$m^2 + 9 = 0$$

$$m = \pm 3i$$

Solution is $y = c_1 \cos 3x + c_2 \sin 3x$

given that $y(0) = 0$

$$\therefore \quad 0 = c_1$$

$$y' = 3c_2 \cos 3x$$

$$y'(0) = 2000$$

$$2000 = 0 + 3c_2$$

$$c_2 = \frac{2000}{3}$$

$$\therefore \quad y = \frac{2000}{3} \sin 3x$$

when $x = 1$ $\quad y = \dfrac{2000}{3} \sin 3 = 94.08$

12. (d)

$$x + y + z = 4 \qquad ...(1)$$

$$x - y + z = 0 \qquad ...(2)$$

$$2x + y + z = 5 \qquad ...(3)$$

Adding (1) and (2) & (2) and (3) gives
$2x + 2z = 4$ and $3x + 2z = 5$ which gives $x = 1$, $z = 1$ and $y = 2$
Alt: Option (b) can be eliminated since they do not satisfy 1st condition. Only (d) satisfies 3rd equation.

13. (c)

$$\text{Trace of } A = 14$$
$$a + 5 + 2 + b = 14$$
$$a + b = 7 \quad \text{...(i)}$$
$$\det(A) = 100$$
$$5\begin{vmatrix} a & 3 & 7 \\ 0 & 2 & 4 \\ 0 & 0 & b \end{vmatrix} = 100$$
$$5 \times 2 \times a \times b = 100$$
$$10\,ab = 100$$
$$ab = 10 \quad \text{... (ii)}$$

From equation (i) and (ii)

either $a = 5, \quad b = 2$

or $a = 2 \quad b = 5$

$$|a - b| = |5 - 2| = 3$$

14. (a)

Given differential equation is

$$x\frac{dy}{dx} + y = x^4$$
$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{y}{x}\right) = x^3 \quad \text{... (i)}$$

Standard form of leibnitz linear equation is

$$\frac{dy}{dx} + Py = Q \quad \text{... (ii)}$$

where P and Q function of x only and solution is given by

$$y(\text{I.F.}) = \int Q.(\text{I.F.})\,dx + C$$

where, integrating factor (I.F.) = $e^{\int P dx}$

Here in equation (i),

$$P = \frac{1}{x} \text{ and } Q = x^3$$
$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Solution $\quad y(x) = \int x^3.x\,dx + c$

$$yx = \frac{x^5}{5} + c$$

given condition

$$y(1) = \frac{6}{5}$$

means at $\quad x = 1;\ y = \frac{6}{5}$

$$\Rightarrow \quad \frac{6}{5} \times 1 = \frac{1}{5} + c$$
$$\Rightarrow \quad c = \frac{6}{5} - \frac{1}{5} = 1$$

Therefore
$$yx = \frac{x^5}{5} + 1$$
$\Rightarrow$
$$y = \frac{x^4}{5} + \frac{1}{x}$$

15. (c)

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$
$$= \int_0^1 2e^{-st}dt + \int_1^\infty 0 \cdot e^{-st}dt$$
$$= 2\left[\frac{e^{-st}}{-s}\right]_0^1 = \frac{2}{-s}[e^{-s} - 1]$$
$$= \frac{2(1-e^{-s})}{s} = \frac{2-2e^{-s}}{s}$$

16. (b)

From the diagram C is $y = x$

$$I = \int_C (x^2 + iy^2)\,dz$$
$$= \int_C (x^2 + iy^2)(dx + idy)$$
$$= \int_C (x^2 + ix^2)(dx + idx)$$
$$= \int x^2dx + ix^2dx + ix^2dx - x^2dx$$
$$= 2i\int_0^1 x^2dx = 2i\left(\frac{x^3}{3}\right)\Bigg|_0^1 = \frac{2i}{3}$$

17. (c)

$$x(ydx + xdy)\cos\frac{y}{x} = y(xdy - ydx)\sin\frac{y}{x}$$
$$\frac{ydx + xdy}{xdy - ydx} = \frac{y}{x}\tan\frac{y}{x}$$
Let
$$y = v \cdot x$$
$$dy = vdx + xdv$$
$$\frac{vxdx + vxdx + x^2dv}{vxdx + x^2dv - vxdx} = v\tan v$$
$$\frac{xdv + 2vdx}{xdv} = v\tan v$$
$$1 + \frac{2v}{x}\frac{dx}{dv} = v\tan v$$
$$\frac{2v}{x}\frac{dx}{dv} = v\tan v - 1$$

$$2\frac{dx}{x} = \left(\tan v - \frac{1}{v}\right)dv$$

Integrating both sides.

$$2\log x = \log|\sec v| - \log v + \log c$$

$$\Rightarrow \quad x^2 = \frac{c\sec v}{v}$$

$$\Rightarrow \quad x^2\frac{y}{x} = c\sec\frac{y}{x}$$

$$\Rightarrow \quad xy\cos\frac{y}{x} = c$$

19. (b)

Let P be the probability that six occurs on a fair dice,

$$\therefore \quad P = \frac{1}{6}$$

$$\therefore \quad q = \frac{5}{6}$$

Let X, be the number of times 'six' occurs,

Probability of obtaining at least two 'six' in throwing a fair dice 4 times is

$$= 1 - \{P(X = 0) + P(X = 1)\}$$

$$= 1 - \{{}^4C_0\, p^0 q^4 + {}^4C_1\, p^1 q^3\}$$

$$= 1 - \left\{\left(\frac{5}{6}\right)^4 + \left[4 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^3\right]\right\}$$

$$= 1 - \left\{\frac{125}{144}\right\} = \frac{19}{144}$$

20. (d)

Since negative and positive are equally likely, the distribution of number of negative values is binomial with

$n = 5$ and $p = \frac{1}{2}$

Let X represent number of negative values in 5 trials.

p(at most 1 negative value)

$$= p(x \le 1)$$

$$= p(x = 0) + p(x = 1)$$

$$= 5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 + 5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4$$

$$= \frac{6}{32}$$

21. (a)

$$\int_0^\pi x^2\cos x\,dx = x^2(\sin x) - 2x(-\cos x) + 2(-\sin x)\Big|_0^\pi$$

$$= \pi^2 \cdot 0 + 2\pi(-1) - 0 = -2\pi$$

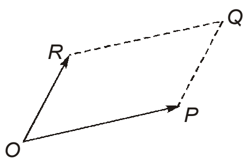
22. (a)

Let,
$$\sin^{-1}x = t$$
$$\frac{dx}{\sqrt{1-x^2}} = dt$$
$$I = \int_0^{\pi/2} t^2\,dt = \left[\frac{t^3}{3}\right]_0^{\pi/2} = \frac{\pi^3}{24}$$

23. (a)

$$(x+1)\frac{dy}{dx}+1 = 2e^{-y}$$
$$\Rightarrow \quad (x+1)\frac{dy}{dx} = (2e^{-y}-1)$$
$$\Rightarrow \quad \frac{dy}{(2e^{-y}-1)} = \frac{dx}{x+1}$$
$$\Rightarrow \quad \frac{e^y dy}{2-e^y} = \frac{dx}{x+1}$$
$$\Rightarrow \quad -\log(2-e^y) = \log(x+1)+c$$
$$\Rightarrow \quad (x+1)(2-e^y) = k$$

24. (a)



The area of parallelogram *OPQR* in figure shown above, is the magnitude of the vector product
$$= \left|\overrightarrow{OP}\times\overrightarrow{OR}\right|$$
$$\overrightarrow{OP} = a\hat{i}+b\hat{j}$$
$$\overrightarrow{OR} = e\hat{i}+d\hat{j}$$
$$\overrightarrow{OP}\times\overrightarrow{OR} = \begin{vmatrix} i & j & k \\ a & b & 0 \\ c & d & 0 \end{vmatrix} = 0\hat{i}+0\hat{j}+(ad-bc)\hat{k}$$
$$\left|\overrightarrow{OP}\times\overrightarrow{OR}\right| = \sqrt{0^2+0^2+(ad-bc)^2} = ad-bc$$

25. (d)

$$f = u+iv$$
$$u = 3x^2-3y^2$$

for f to be analysis, we have Cauchy-Riemann conditions,
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \ldots(i)$$
$$\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x} \quad \ldots(ii)$$

From (i) we have,
$$6x = \frac{\partial v}{\partial y}$$

$\Rightarrow$
$$\int \partial v = \int 6x \partial y$$
$$v = 6xy + f(x)$$

i.e.
$$v = 6xy + f(x) \qquad \ldots(iii)$$

Now applying equation (ii) we get,
$$\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

$\Rightarrow$
$$-6y = -\left[6x + \frac{df}{dx}\right]$$

$\Rightarrow$
$$6x + \frac{df}{dx} = 6y$$
$$\frac{df}{dx} = 6y - 6x$$

By integrating,
$$f(x) = 6yx - 3x^2 + K$$

Substitute in equation (iii)
$$v = 3x^2 + 6yx - 3x^2 + K$$

$\Rightarrow$
$$v = 6yx + K$$

26. (b)

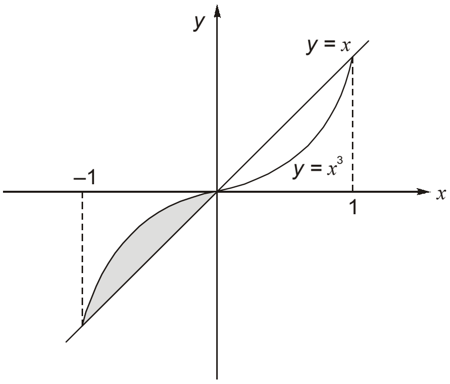
Result, Rank $(A^T A)$ = Rank (A)

27. (a)

$$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} r^2 \sin\phi \cdot dr \cdot d\phi \cdot d\theta = \int_0^{2\pi}\int_0^{\pi/3}\left[\frac{r^3}{3}\right]_0^1 \sin\phi \, d\phi \, d\theta$$
$$= \frac{1}{3}\int_0^{2\pi}[-\cos\phi]_b^{\pi/3} d\theta = \frac{1}{3}\times\frac{1}{2}\times\int_0^{2\pi} d\theta = \frac{1}{3}\times\frac{1}{2}\times 2\pi = \frac{\pi}{3}$$

28. (b)

Point of inter-section of the two curves are $x = 0, 1, -1$



$$\text{Area} = \int_{-1}^{0}(x^3 - x)dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} = \frac{0-(-1)^4}{4} - \frac{0-(-1)^2}{2} = \frac{1}{4}$$

29. (d)

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

Given that,
$$F(s) = \left[\frac{3s+1}{s^3 + 4s^2 + (K-3)s}\right]$$

$$\lim_{t \to \infty} f(t) = 1$$

$$\Rightarrow \quad \lim_{s \to 0} s\left[\frac{3s+1}{s^3 + 4s^2 + (K-3)s}\right] = 1$$

$$\Rightarrow \quad \lim_{s \to 0} \left[\frac{3s+1}{s^2 + 4s + (K-3)}\right] = 1$$

$$\Rightarrow \quad \frac{1}{K-3} = 1$$

$$\Rightarrow \quad K - 3 = 1$$

$$\Rightarrow \quad K = 4$$

30. (b)

The augmented matrix for the given system is $\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right]$.

Performing Gauss-Elimination on the above matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right] \xrightarrow[R_3 - 1/2\,R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 3/2 & -6 & 7-\alpha/2 \end{array}\right]$$

$$\xrightarrow{R_3 - 3/2\,R_2} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 0 & 0 & \dfrac{5\alpha-1}{2} \end{array}\right]$$

Now for infinite solution it is necessary that at least one row must be completely zero.

$$\therefore \quad \frac{5\alpha - 1}{2} = 0$$

$\alpha = 1/5$ is the solution

$\therefore$ There is only one value of α for which infinite solution exists.

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