CLASS TEST

S.No. : 02 SP_ME_EFG_090623

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ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 09/06/2023

ANSWER KEY

1.	(d)	7.	(b)	13.	(a)	19.	(a)	25.	(c)
2.	(b)	8.	(b)	14.	(b)	20.	(c)	26.	(a)
3.	(a)	9.	(a)	15.	(a)	21.	(a)	27.	(b)
4.	(a)	10.	(b)	16.	(d)	22.	(a)	28.	(b)
5.	(a)	11.	(b)	17.	(b)	23.	(c)	29.	(a)
6.	(a)	12.	(d)	18.	(c)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (d)

For function to be differentiable i.e. continuous $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)}$$

$$= \lim_{x\to 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3}$$

$$f(0^+) = \lim_{x\to 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)}$$

$$= \lim_{x\to 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\frac{5}{3}$

2. (b)

We have $y = e^x (A\cos x + B\sin x)$

$$y' = e^x (A\cos x + B\sin x) + e^x (-A\sin x + B\cos x)$$

$$= y + e^x [-A\sin x + B\cos x]$$

$$y'' = y' + e^x (-A\sin x + B\cos x) + e^x (-A\cos x - B\sin x)$$

$$= y' + y' - y - y$$

$$= 2y' - 2y$$

$\Rightarrow$ Order $= 2$

Degree $= 1$

3. (a)

$$\frac{dy}{dx} = e^{ax} \times e^{by}$$

$$\frac{dy}{e^{by}} = e^{ax} \times dx$$

$$\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$$

$y(0) = 0$

$$\Rightarrow \quad c = -\left[\frac{1}{b} + \frac{1}{a}\right] = -\left[\frac{a+b}{ab}\right]$$

4. (a)

$$\nabla \cdot \vec{F} = 0 \qquad \text{[For solenoidal vector]}$$

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $a = 2$

5. (a)

Greatest rate of increase of ϕ is magnitude of directional derivative at that point.

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + x^2z\hat{j} + (x^2y + 8xz)\hat{k}$$

$$\nabla\phi|_{(1,-2,1)} = \hat{j} + 6\hat{k}$$

Greatest rate of increase $= \sqrt{1^2 + 6^2} = \sqrt{37} = 6.08$

6. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $y = -4$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

$\Rightarrow$ $x = 12$

$\therefore$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$

7. (b)

$$\lim_{x \to 0} \frac{\ln(1+5x)}{e^{7x} - 1} \qquad \left(\frac{0}{0} \text{ indeterminate form}\right)$$

Applying L' Hospitals rule

$$\lim_{x \to 0} \frac{\ln(1+5x)}{e^{7x} - 1} = \lim_{x \to 0} \frac{5}{(1+5x)7e^{7x}} = \frac{5}{7}$$

8. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1 P_2 P_3 = \frac{15}{50} \times \frac{14}{49} \times \frac{13}{48} = \frac{13}{560}$$

9. (a)

Given, $x = b(2 - \cos\theta)$, $y = b(\sin\theta + \theta)$

$$\therefore \quad \frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta + 1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta + 1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{b \times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

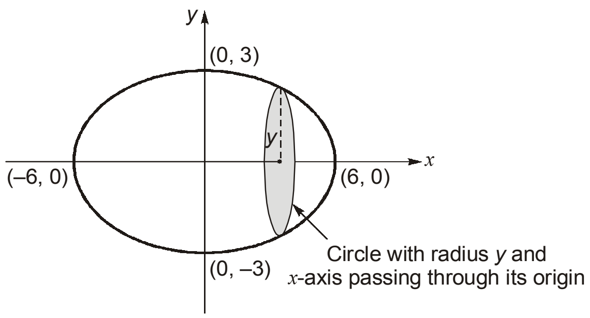
10. (b)

$$P(T) = 0.5$$

Probability of getting tails exactly 6 times is

$$8C_6(0.5)^6(0.5)^2 = \frac{7}{64}$$

11. (b)



$$\text{Volume generated} = \int_{-6}^{6} \pi y^2 dx = \int_{-6}^{6} \pi\left(\frac{36 - x^2}{4}\right) dx$$

$$= \frac{\pi \times 2}{4}\int_0^6 (36 - x^2)dx = \frac{\pi}{2}\left[36x - \frac{x^3}{3}\right]_0^6$$

$$= 72\pi \text{ unit}^3$$

12. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF \cdot f(x) \cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)} \cdot f(x) \cdot f'(x)dx$$

Let $f(x) = t$

$$f'(x)\ dx = dt$$

$$y \times e^t = \int e^t \cdot t dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log\,[y + 1 - f(x)] + f(x) = c'$$

13. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2+4)} = 96\frac{1}{4D^2\left(1+\frac{D^2}{4}\right)}x^2 = \frac{96}{4}\left[\frac{\left(1-\frac{D^2}{4}\right)x^2}{D^2}\right]$$

$$= 24\frac{\left(x^2-\frac{1}{2}\right)}{D^2}$$

$$PI = 24\left[\frac{x^4}{4\times 3} - \frac{x^2}{4}\right] = 2x^2(x^2-3)$$

$$PI\big|_{x=2} = 2 \times 2^2(4-3) = 8$$

14. (b)

$$u(x, y) = 2x(1 - y)$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

$$dv = (2x)dx + 2(1-y)dy$$

$$v = x^2 + 2y - y^2 + c$$

15. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx+6k)dx = 1$$

$$\left.\frac{kx^2}{2}\right|_0^2 + 2kx|_2^4 + \left.\left(\frac{-kx^2}{2}+6kx\right)\right|_4^6 = 1$$

$$\frac{k}{2}(2^2-0)+2k(4-2)-\frac{k}{2}(6^2-4^2)+6k(6-4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2dx + \int_2^4 \frac{1}{4}xdx + \int_4^6\left(-\frac{1}{8}x^2+\frac{3}{4}x\right)dx$$

$$= \left.\frac{1}{8}\frac{x^3}{3}\right|_0^2 + \left.\frac{1}{4}\frac{x^2}{2}\right|_2^4 - \left.\frac{1}{8}\frac{x^3}{3}\right|_4^6 + \left.\frac{3}{4}\frac{x^2}{2}\right|_4^6$$

$$= \frac{1}{3}+\frac{3}{2}-\frac{19}{3}+\frac{15}{2} = 3$$

16. (d)

$$I = \int_0^{\pi/2}\sqrt{1+\sec x}\,dx = \int_0^{\pi/2}\sqrt{1+\frac{1}{\cos x}}\,dx$$

$$= \int_0^{\pi/2}\frac{\sqrt{1+\cos x}}{\sqrt{\cos x}}dx = \int_0^{\pi/2}\frac{\sqrt{2}\cos(x/2)}{\sqrt{1-2\sin^2(x/2)}}dx$$

Let $\sin\frac{x}{2} = t,$ $\quad \begin{cases} x = 0, & t = 0 \\ x = \frac{\pi}{2} & t = \frac{1}{\sqrt{2}} \end{cases}$

$$\frac{1}{2}\cos\frac{x}{2}dx = dt$$

$$I = \int_0^{1/\sqrt{2}}\frac{2\sqrt{2}dt}{\sqrt{1-2t^2}}$$

$$= 2\sin^{-1}\left(\sqrt{2}t\right)\Big|_0^{1/\sqrt{2}} = 2\sin^{-1}\left(\sqrt{2}\times\frac{1}{\sqrt{2}}\right)-2\sin^{-1}(0)$$

$$= 2\times\frac{\pi}{2} = \pi = 3.14$$

17. (b)

$$(2y-3x)dx + xdy = 0$$

$$\frac{dy}{dx}+\frac{2}{x}y = 3$$

$$IF = e^{\int\frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$y.x^2 = 3\int x^2dx = x^3 + c$$

For $x = 0$, $y = 0$

$\Rightarrow$ $\quad 0 = 0 + c$

$\Rightarrow$ $\quad c = 0$

For $x = 2$, $\quad y \times 2^2 = 2^3$

$$y = 2$$

18. (c)

$$\frac{4C_1 \cdot 4C_1 \cdot 4C_1 \cdot 4C_1}{52C_4} = \frac{4\times4\times4\times4}{(52\times51\times50\times49)/(4\times3\times2\times1)}$$

$$= \frac{4\times4\times4\times4\times3\times2}{52\times51\times50\times49} = \frac{256}{270725}$$

19. (a)

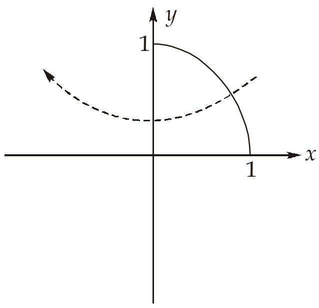
$x = \sin\left(\frac{\pi k}{2}\right), y = \cos\left(\frac{\pi k}{2}\right)$

Just by seeing, we can know that it represents a circle in $x - y$ plane, given by

$$x^2 + y^2 = 1$$

Given $0 \le k \le 1$, which gives $0 \le x \le 1$; $0 \le y \le 1$

or $$0 \le \frac{\pi k}{2} \le \frac{\pi}{2}$$



So we get a quarter circle, when this is rotated with respect to y-axis by 360 degree, it creates a hemisphere of radius 1.

Surface area of hemisphere,

$$A_S = 2\pi r^2$$

$$= 2\pi(1)^2 = 2\pi$$

20. (c)

$$f(y) = y^2 e^{-y}$$

$$f'(y) = y^2\left(-e^{-y}\right) + e^{-y} \times 2y$$

$$= e^{-y}\left(2y - y^2\right)$$

Putting $f'(y) = 0$

$$e^{-y}\left(2y - y^2\right) = 0$$

$$e^{-y} y(2 - y) = 0$$

$y = 0$ or $y = 2$ are the stationary points

Now, $$f''(y) = e^{-y}(2 - 2y) + \left(2y - y^2\right)\left(-e^{-y}\right)$$

$$= e^{-y}\left(2 - 2y - 2y + y^2\right)$$

$$= e^{-y}\left(y^2 - 4y + 2\right)$$

At $y = 0$, $$f''(0) = e^{-0}(0 - 0 + 2) = 2$$

Since $f''(0) = 2$ is > 0 at $y = 0$ we have a minima

$$\text{Now, at } y = 2 f''(2) = e^{-2}\left(2^2 - 4 \times 2 + 2\right)$$
$$= e^{-2}(4 - 8 + 2)$$
$$= -2e^{-2} < 0$$

$\therefore$ At $y = 2$ we have a maxima.

21. (a)

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$
$$P(x < 3) = P(0) + P(1) + P(2)$$
$$= \frac{\mu^0 e^{-\mu}}{0!} + \frac{\mu^1 e^{-\mu}}{1!} + \frac{\mu^2 e^{-\mu}}{2!}$$
$$= \frac{1}{e^{\mu}} + \frac{\mu}{e^{\mu}} + \frac{\mu^2}{2e^{\mu}}$$

As $\mu(\text{mean}) = 6.8$

$$\therefore \quad P(x < 3) = \frac{1 + 6.8 + \left(\frac{6.8^2}{2}\right)}{e^{6.8}} = \frac{30.92}{897.85} \simeq 0.034$$

22. (a)

$$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x\, dx + \tan y\, dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$
$$\log \frac{1}{\cos x \cos y} = C_1$$
$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$
$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

23. (c)

$$P(x) = x^5 + x + 2$$

It has a real root at $x = -1$

$$\Rightarrow \quad P(x) = (x^4 - x^3 + x^2 - x + 2)(x + 1)$$

Now, $x^4 - x^3 + x^2 + x + 2$ will give other 4 roots

To find roots,

$$\Rightarrow \quad x^4 - x^3 + x^2 - x + 2 = 0$$

$\Rightarrow \quad x^3(x-1) + x(x-1) + 2 = 0$

$\Rightarrow \quad x(x^2+1)(x-1) + 2 = 0$

In the above expression, $x^2 + 1$ is always positive. So, either 'x' or '$x - 1$' should be negative in order to satisfy the equation.

For $x > 1$, both (x) and $(x - 1)$ are positive and,

For $x < 0$, both (x) and $(x - 1)$ are negative

$\therefore$ x should lie within 0 and 1 in order to have real roots.

As $x \in (0, 1)$

$\Rightarrow \quad |x| < 1$

$\Rightarrow \quad |x^2 + 1| < 2, |x| < 1$ and $|x - 1| < 1$

$\therefore$ The product of these three will be less than 2 and hence, no real value of 'x' can satisfy the equation

$$x^4 - x^3 + x^2 - x + 2 = 0$$

$\therefore$ The equation will have four imaginary roots apart from one real roots.

24. (a)

$$I = \int \sec^3\theta\, d\theta = \int \sec\theta.\sec^2\theta\, d\theta$$

$$= \sec\theta \int \sec^2\theta\, d\theta - \int \tan\theta(\sec\theta\tan\theta)\, d\theta$$

$$= \sec\theta\tan\theta - \int \tan^2\theta\sec\theta\, d\theta$$

$$\Rightarrow \quad I = \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\, d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$$

$$\Rightarrow \quad I = \sec\theta\tan\theta - I + ln|\sec\theta + \tan\theta| + c$$

$$\Rightarrow \quad I = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2} ln|\sec\theta + \tan\theta| + c$$

$$\therefore \quad a + b = \frac{1}{2} + \frac{1}{2} = 1$$

25. (c)

$$\int_C \bar{F}.\overline{dr} = \int_C x^2y^2dx + y.dy$$

For curve C, $\quad y^2 = 4x$

and $\quad 2y\, dy = 4\, dx$

$$\Rightarrow \quad \int_C \bar{F}.\overline{dr} = \int_0^4 x^2(4x)dx + 2dx$$

$$= \int_0^4 (4x^3 + 2)dx = 264$$

26. (a)

To obtain maximum value of $f(x)$, first $f'(x)$ should be equated to zero.

$\Rightarrow \quad f'(x) = 6x^2 - 6x - 36 = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad f'(x) = 0 \quad$ at $x = 3$ and -2

Now, $\quad f''(x) = 12x - 6$

$f''(3) = 30 > 0$

at $x = 3$, there is local minima

and $\quad f''(2) = -30 < 0$

$\therefore$ at $x = -2$, a local maxima is observed.

27. (b)

$$\text{Length of curve} = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

Curve: $3x^2 = y^3$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sqrt{3y}}{2}$$

$$\therefore \quad \text{Length} = \int_0^1 \sqrt{1+\frac{3y}{4}}\, dy$$

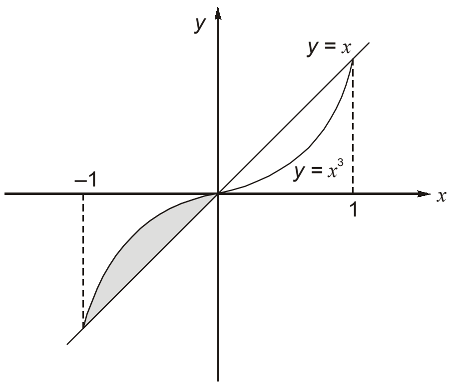
$$= \frac{1}{2}\int_0^1 \sqrt{4+3y}\, dy$$

$$= \frac{1}{2}\left[\frac{(4+3y)^{3/2}}{\frac{3}{2}\times 3}\right]_0^1$$

$$= \frac{1}{9}\left(7\sqrt{7}-8\right)$$

28. (b)

Point of inter-section of the two curves are $x = 0, 1, -1$



$$\text{Area} = \int_{-1}^{0}\left(x^3 - x\right)dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} = \frac{0-(-1)^4}{4} - \frac{0-(-1)^2}{2} = \frac{1}{4}$$

29. (a)

$$(x+1)\frac{dy}{dx}+1 = 2e^{-y}$$

$$\Rightarrow \quad (x+1)\frac{dy}{dx} = (2e^{-y}-1)$$

$$\Rightarrow \quad \frac{dy}{(2e^{-y}-1)} = \frac{dx}{x+1}$$

$$\Rightarrow \quad \frac{e^{y}dy}{2-e^{y}} = \frac{dx}{x+1}$$

$$\Rightarrow \quad -\log(2-e^{y}) = \log(x+1)+c$$

$$\Rightarrow \quad (x+1)(2-e^{y}) = k$$

30. (c)

$$\frac{d^2y}{dx^2} = y$$

$$\Rightarrow \quad D^2y = y \qquad (\therefore d/dx = D)$$

$$(D^2-1)y = 0$$

$$D^2-1 = 0$$

$$D = \pm 1$$

$$y = C_1 e^{x} + C_2 e^{-x}$$

Given point passes through origin

$$\Rightarrow \quad 0 = C_1 + C_2$$

$$C_1 = -C_2 \qquad \text{...(i)}$$

Also, point passes through (ln 2, 3/4)

$$\Rightarrow \quad \frac{3}{4} = C_1 e^{\ln 2} + C_2 e^{-\ln 2}$$

$$\frac{3}{4} = 2C_1 + \frac{C_2}{2}$$

$$\Rightarrow \quad C_2 + 4C_1 = 1.5 \qquad \text{...(ii)}$$

From (i) $C_1 = -C_2$, putting in (ii), we get

$$\Rightarrow \quad -3C_2 = 1.5$$

$$C_2 = -0.5$$

$$\therefore \quad C_1 = 0.5$$

$$\Rightarrow \quad y = 0.5(e^{x} - e^{-x})$$

$$y = \frac{e^{x}-e^{-x}}{2}$$

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