CLASS TEST

S.No. : 01 SP_ME_ABCD_270523

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THEORY OF MACHINES

MECHANICAL ENGINEERING

Date of Test : 27/05/2023

ANSWER KEY

1.	(a)	7.	(d)	13.	(c)	19.	(b)	25.	(b)
2.	(a)	8.	(a)	14.	(a)	20.	(d)	26.	(c)
3.	(b)	9.	(a)	15.	(a)	21.	(c)	27.	(b)
4.	(d)	10.	(c)	16.	(c)	22.	(a)	28.	(b)
5.	(c)	11.	(b)	17.	(a)	23.	(d)	29.	(b)
6.	(d)	12.	(b)	18.	(b)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$12 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{...(i)}$$

$$6 = \frac{1}{2\pi}\sqrt{\frac{k-800}{m}} \quad \text{...(ii)}$$

Divide equation (i) by equation (ii),

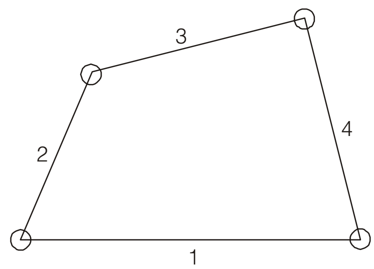
$$2 = \sqrt{\frac{k}{k-800}}$$

$$\frac{k}{k-800} = 4$$

$$\Rightarrow \quad 3k = 3200,\ k = \frac{3200}{3}$$

2. (a)

Here all are turning pairs



3. (b)

$$\text{Arc of contact} = \frac{\text{path of contact}}{\cos\phi}$$

$$\cos\phi = \frac{25.4}{27} = 0.94074$$

$$\phi = \cos^{-1}(0.94074)$$

$$\phi = 19.8°$$

4. (d)

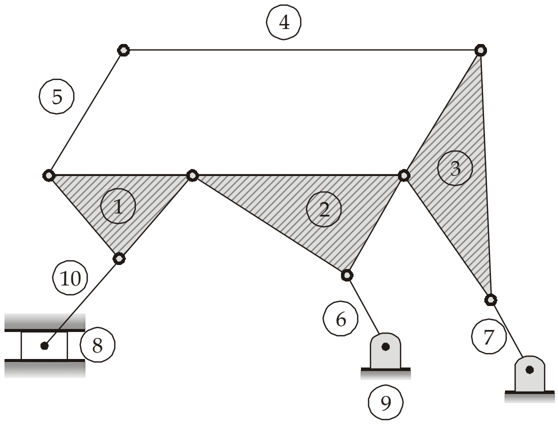
$$\text{Damping coefficient, } c = \frac{F}{v} = \frac{0.05}{0.04} = 1.25 \text{ N/m/s}$$

Critical damping coefficient,

$$c_c = 2\sqrt{mK} = 1.897 \text{ N/m/s}$$

$$\text{Damping ratio, } \xi = \frac{c}{c_c} = \frac{1.25}{1.897} = 0.658 \simeq 0.66$$

5. (c)

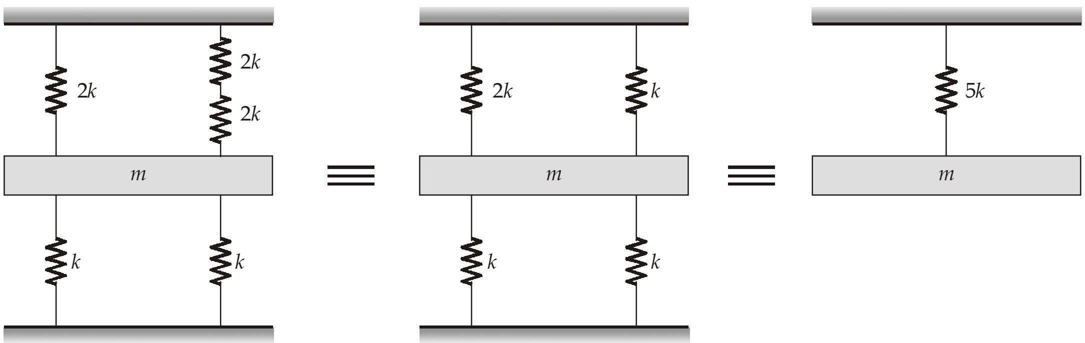


$$l = 10, \qquad h = 0$$
$$j = 12$$

By Gruebler's criterion

$$f = 3(l-1) - 2j - h$$
$$= 3(10-1) - 2 \times 12 - 0 = 27 - 24$$
$$F = 3$$

6. (d)



$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{5k}{m}}$$

7. (d)

Given,
$$\Delta E = 18 \text{ kJ}$$
$$N_1 = 100 \text{ rpm}$$
$$N_2 = 98 \text{ rpm}$$

We know that,
$$\Delta E = \frac{1}{2}\left(I\omega_1^2\right) - \frac{1}{2}\left(I\omega_2^2\right) = \frac{1}{2}I\left(\omega_1^2 - \omega_2^2\right)$$

$$18 \times 10^3 = \frac{I}{2} \times \left[\left(\frac{2\pi \times 100}{60}\right)^2 - \left(\frac{2\pi \times 98}{60}\right)^2\right]$$

$$I = \frac{36 \times 10^3 \times 60^2}{4\pi^2\left(100^2 - 98^2\right)}$$

$$I = 8289.915 \text{ kgm}^2$$

Kinetic energy at 140 rpm, $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 8289.915\times\left(\frac{2\pi\times 140}{60}\right)^2 = 890909.088$ J

Kinetic energy at 140 rpm, $E = 890.91$ kJ

8. (a)

$$\text{Angular speed, } \omega = \frac{2\pi N}{60} = \frac{2\pi\times 200}{60} = 20.944 \text{ rad/s}$$

$$\text{Crank radius, } r = \frac{300}{2} = 150 \text{ mm}$$

Mass to be balanced at the crank pin = $(c \times m_{reci}) + (m_{rev.}) = (0.6 \times 50) + 60 = 90$ kg

Now,
$$m_c \times r_c = mr$$
$$90 \times 0.15 = m \times 0.25$$
$$m = 54 \text{ kg}$$

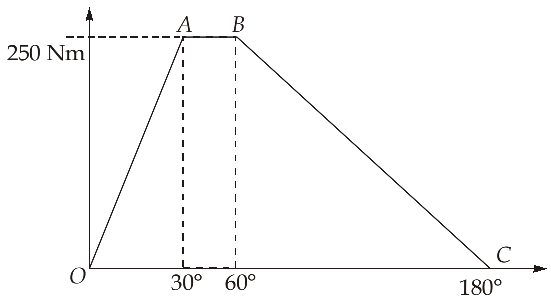
9. (a)

$$\text{Pitch line velocity } V_p = \omega_1 r_1$$
$$= 2\pi N \times \frac{mT}{2} = 2\times\pi\times 300\times\frac{80\times 8}{2}$$
$$= 603185.7895 \text{ mm/minutes} = 10053.0964 \text{ mm/s} = 10.0530 \text{ m/s}$$

10. (c)

As per given turning moment diagram,

Net energy produce in one cycle = Area of OABC = Area of trapezium OABC



$$= \frac{AB + OC}{2}\times 250 = \frac{\frac{\pi}{6}+\pi}{2}\times 250 = 145.8\pi \text{ N.m}$$

$$T_{mean} \times \pi = \text{Area of OABC}$$

$$\Rightarrow \quad T_{mean} = \frac{145.8\pi}{\pi} = 145.8 \text{ Nm}$$

11. (b)

Given:
$$d = 10 \times 16 = 160 \text{ mm}$$
$$D = 10 \times 50 = 500 \text{ mm}$$
$$\phi = 20°$$

$$r_A = \frac{d}{2} + \text{addendum} = 80 + 12 = 92 \text{ mm}$$

$$R_A = \frac{D}{2} + \text{addendum} = 250 + 8 = 258 \text{ mm}$$

$$\text{Path of approach} = \sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi$$

$$\text{Path of approach} = \sqrt{258^2 - (250\cos 20°)^2} - (250\sin 20°) = 21.15 \text{ mm}$$

$$\text{Path of recess} = \sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi$$

$$= \sqrt{92^2 - (80\cos 20°)^2} - (80\sin 20°) = 25.67 \text{ mm}$$

$$\omega_{gear} = \frac{2\pi\times 800}{60} = 83.77 \text{ rad/s}$$

$$\omega_{pinion} = \frac{T_G}{t_p}\times\omega_{gear} = \frac{50}{16}\times 83.77 = 261.799 \text{ rad/s}$$

Maximum sliding velocity = $(\omega_p + \omega_G) \times 25.67$

Maximum velocity of sliding = $(83.77 + 261.799) \times 25.67 = 8870.756$ mm/s = 8.87 m/s

12. (b)

$$T_A = 72$$
$$T_B = 32$$
$$N_{arm} = 18 \text{ rpm}$$
$$r_A = r_C + 2r_B$$
$$T_A = T_C + 2T_B$$
$$72 = 32 + 2T_B$$
$$T_B = \frac{40}{2} = 20$$

Condition	arm	Gear C (32)	Gear B (20)	Gear A (72)
Arm fixed	0	+1	$-\frac{32}{20}$	$-\frac{32}{20}\times\frac{20}{72}$
Gear C rotates by x revolutions	0	x	$-\frac{32}{20}x$	$-\frac{32}{72}x$
add $+y$ revolutions to all	y	$x+y$	$y-\frac{32}{20}x$	$y-\frac{32}{72}x$

$$y = 18 \text{ rpm}$$

Gear A is fixed

$$y = \frac{32}{72}x$$

$$y = \frac{32}{72}x$$

$$\frac{18\times72}{32} = x$$

$$x = 40.5$$

Speed of 'B' $N_B = y - \frac{32}{20}x$

$$= 18 - \frac{32}{20}\times 40.5$$

$$N_B = -46.8 \text{ rpm}$$

13. (c)

Given, m = 1 tonne = 1000 kg

Logarithmic decrement of n cycles is given by

$$\delta = \frac{1}{n}\log_e \frac{x_0}{x_n}$$

$n = 4$

$$\delta = \frac{1}{4}\log_e \frac{5}{0.128} = 0.916$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad \text{or} \quad 0.916 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\xi = 0.144$$

Given, T_d = 0.64 seconds

$$\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{0.64} = 9.817 \text{ rad/s}$$

$$\omega_d = \sqrt{1-\xi^2}\ \omega_n$$

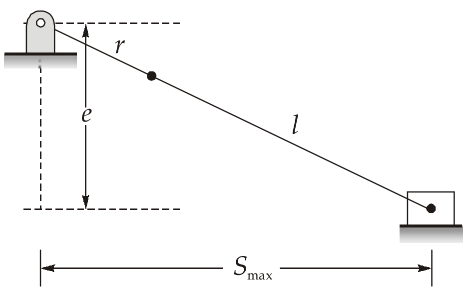
$$\omega_n = \frac{9.817}{\sqrt{1-0.144^2}} = 9.92 \text{ rad/s}$$

$$\sqrt{\frac{k}{m}} = 9.92$$

$$k = 9.92^2 \times 1000 = 98406.4 \text{ N/m} = 98.406 \text{ N/mm}$$

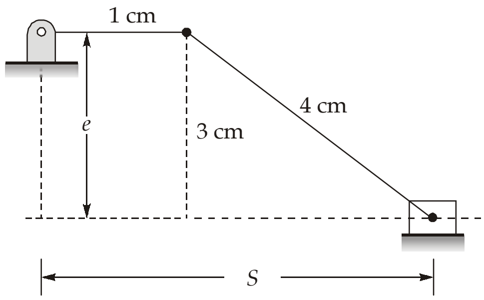
14. (a)

At $S = S_{max}$: $r + l = \sqrt{S_{max}^2 + e} = \sqrt{4^2+3^2} = 5 \text{ cm}$



Given, r = 1 cm, So, l = 5 – 1 = 4 cm

at θ = 0:



$$S = \sqrt{4^2 - 3^2} + 1 = \sqrt{7} + 1$$

So, at $\theta = 0$, $S = 3.645$ or 3.65 cm

15. (a)

$m = 120$ kg, $E = 200 \times 10^9$ N/m^2

$l = 0.7$ m, $d = 0.04$ m

$a = 0.25$ m, $b = 0.7 - 0.25 = 0.45$ m

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times (0.04)^4$$

$$= 0.1256 \times 10^{-6} \text{ m}^4$$

$$\Delta = \frac{mga^3b^3}{3EIl^3} = \frac{120 \times 9.81 \times (0.25)^3 \times (0.45)^3}{3 \times 200 \times 10^9 \times 0.1256 \times 10^{-6} \times (0.7)^3}$$

$$= 6.48 \times 10^{-5} \text{ m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{6.48 \times 10^{-5}}} = 61.90 \text{ Hz}$$

16. (c)

$$MF = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

According to question, $\frac{\omega}{\omega_n} = k$

So, $$MF = \frac{1}{\sqrt{(1-k^2)^2 + (2\xi k)^2}}$$

For MF to be maximum, $(1 - k)^2 + (2\xi k)^2$ should be minimum. So, for the denominator to be minimum

$$\frac{d\left((1-k^2)^2 + (2\xi k)^2\right)}{dk} = 0$$

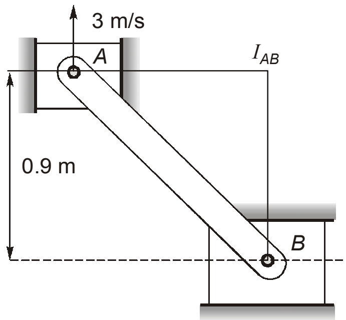
$$2(1 - k^2)(-2k) + 8\zeta^2 k = 0$$

$$k(k^2 - 1 + 2\xi^2) = 0$$

$$k = \sqrt{1 - 2\xi^2} = \sqrt{1 - 2 \times 0.32^2}$$

$$k = 0.89$$

17. (a)



$$I_{AB} \cdot B = 0.9 \text{ m}$$
$$AB = 1.5 \text{ m}$$
$$I_{AB} \cdot A = \sqrt{(AB)^2 - (I_{AB} B)^2} = \sqrt{(1.5)^2 - (0.9)^2} = \sqrt{2.25 - 0.81} = 1.2 \text{ m}$$
$$\therefore \quad \frac{V_B}{V_A} = \frac{I_{AB}.B}{I_{AB}.A} = \frac{0.9}{1.2}$$
$$V_B = V_A \times \frac{0.9}{1.2} = 3 \times \frac{9}{12} = 2.25 \text{ m/s}$$

18. (b)

$$\frac{N_F}{N_A} = \frac{T_{\text{input}}}{T_{\text{output}}} = \frac{T_E}{T_F} \times \frac{T_C}{T_D} \times \frac{T_A}{T_B} = \frac{26 \times 25 \times 20}{50 \times 75 \times 65}$$
$$N_F = 0.0533\, N_A = 0.0533 \times 975 = 52 \text{ rpm}$$

19. (b)

$$\omega_{AB} \times 0.2 = \omega_{BD} \times 0.25$$
$$\therefore \quad 15 \times 0.2 = \omega_{BD} \times 0.25$$
$$\omega_{BD} = 12 \text{ rad/s}$$

20. (d)

$$\omega_{max} = \frac{\omega_1}{\cos \alpha}$$
$$\omega_{min} = \omega_1 \cos\alpha$$

Variation of speed, $\omega_{max} - \omega_{min} = \omega_1 \left[\frac{1}{\cos\alpha} - \cos\alpha\right]$

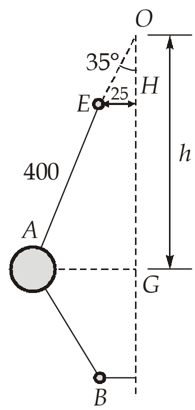
Permissible variation of speed = ±4 % of mean speed

or, $\omega_1 \left[\frac{1}{\cos\alpha} - \cos\alpha\right] = 0.08\, \omega_1$

or, $\cos^2 \alpha + 0.08 \cos \alpha - 1 = 0$

$$\cos \alpha = 0.96$$
$$\Rightarrow \quad \alpha = 16.1°$$

21. (c)



$$h = GO = GH + HO = AE\cos\theta + EH\cot\theta$$
$$h = 400\cos 35° + 25\cot 35° = 363.4 \text{ mm}$$
$$h' = 400\cos 30° + 25\cot 30° = 389.7 \text{ mm}$$

Now, $$h = \frac{g}{\omega^2} \text{ and } h' = \frac{g}{\omega'^2}$$

$$\frac{\omega'}{\omega} = \sqrt{\frac{h}{h'}} = \sqrt{\frac{363.4}{389.7}} = 0.966$$

Decrease in speed = (1 – 0.966) × 100 = 3.44%

22. (a)

As per given data, $I = 1.5$ kg-m^2

The angular velocity of spin of the disc,

$$\omega = \frac{2\pi \times 500}{60} = \frac{100\pi}{6} \text{ rad/s}$$

The angular velocity of precession ,

$$\omega_p = \frac{2\pi}{5} \text{ rad/s}$$

Gyroscopic couple, $T = I\omega\omega_p$

$$= 1.5 \times \frac{100\pi}{6} \times \frac{2\pi}{5} = 10\pi^2 = \frac{20\pi^2}{2} \text{ kg-m}^2/\text{s}^2$$

23. (d)

24. (c)

Disturbing force, $F = (1 - c)\, mr\omega^2\cos\theta$

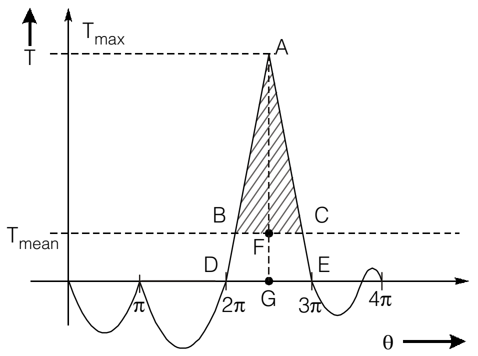
$$= (1 - 0.4) \times 6 \times 0.10 \times 15^2 \times \cos 60 = 40.5 \text{ N}$$

25. (b)

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/sec}$$

for SHM $$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi_a}\right)^2 = \frac{30}{2}\left(\frac{\pi \times 12.57}{180 \times \frac{\pi}{180}}\right)^2 = 2370.0735 \text{ mm/s}^2 = 2.37 \text{ m/s}^2$$

26. (c)



$$P = \frac{2\pi N T_{mean}}{60}$$

$$\Rightarrow \qquad T_{mean} = \frac{60 \times 40 \times 10^3}{2 \times \pi \times 130} = 2938.245 \text{ N-m}$$

$$\Rightarrow \qquad \text{Energy produced} = T_{mean} \times 4\pi = 36923.076 \text{ N-m}$$

Now, work done during the power stroke

$$= 1.5 \times \text{Energy produced per cycle}$$
$$= 1.5 \times 36923.076$$
$$= 55384.615 \text{ Nm}$$

Now, from similar triangles ABC, ADE;

$$\frac{AF}{AG} = \frac{BC}{DE}$$

$$\frac{1}{2} \times T_{max} \times \pi = 55384.6$$

$$\Rightarrow \qquad T_{max} = 35258.93 \text{ Nm} = AG$$

Now, $$\frac{35258.93 - 2938.245}{35258.93} = \frac{BC}{\pi}$$

$$\Rightarrow \qquad BC = 2.879 \text{ rad}$$

Now, maximum fluctuation of energy $= \frac{1}{2} \times AF \times BC$

$$= \frac{1}{2} \times (35258.93 - 2938.245) \times 2.879$$
$$= 46525.62 \text{ N-m}$$

27. (b)

For the Hartnell governor
spring stiffness is given by

$$k = 2\left(\frac{a}{b}\right)^2 \left(\frac{F_1 - F_2}{r_1 - r_2}\right)$$

$$k = 2\left(\frac{a}{b}\right)^2 \left(\frac{1500 - 100}{20 - 15}\right)$$

$$k = 2\left(\frac{1400}{5}\right) = 560 \text{ N/cm} \qquad (\because \text{ a and b are same})$$

28. (b)

Given,

$$\text{Lift, } h = 25 \text{ mm}$$
$$\text{Offset, } x = 12 \text{ mm}$$
$$\text{Speed, } N = 300 \text{ rpm}$$
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 10\pi \text{ rad/s}$$
$$\text{Ascent angle, } \phi_a = 60°$$
$$\text{Descent angle, } \phi_d = 90°$$
$$\text{Dwell angle, } \delta_1 = 45°$$

Now,
$$\delta_2 = 360° - (60° + 90° + 45°) = 165°$$

During out stroke:

$$(\text{a}_{\text{uniform}})_{\text{o.s}} = \frac{4h\omega^2}{\phi_a^2} = \frac{4 \times 25 \times (10\pi)^2 \times 10^{-3}}{\left(60 \times \frac{\pi}{180}\right)^2}$$
$$= 100 \times 100 \times 10^{-3} \times 9 = 90 \text{ m/s}^2$$

During return stroke:]

$$(a_{\text{uniform}})_{\text{r.s}} = \frac{4h\omega^2}{\phi_d^2} = \frac{4 \times 25 \times (10\pi)^2 \times 10^{-3}}{\left(90 \times \frac{\pi}{180}\right)^2} = 40 \text{ m/s}^2$$

$$\frac{(a_{\text{uniform}})_{o.s}}{(a_{\text{uniform}})_{r.s}} = \frac{90}{40} = 2.25$$

29. (b)

(i) Controlling force, $F = 3r - 60$

At lower extreme radii, $F_1 = 3 \times 120 - 60 = 300$ N

Controlling force at maximum speed, $F_1 = 300 + 30 = 330$ N

Controlling force at minimum speed, $F_2 = 300 - 30 = 270$ N

$$\text{Coefficient of insensitiveness,} = \frac{N_1 - N_2}{N_{\text{mean}}} \frac{(N_1 - N_2)(N_1 + N_2)}{2 \times N_{\text{mean}}^2}$$

$$= \frac{N_1^2 - N_2^2}{2N_{\text{mean}}^2} = \frac{F_1 - F_2}{2F} = \frac{330 - 270}{2 \times 300} \qquad \{F \propto \omega^2 \propto N^2\}$$

$$\text{Coefficient of insensitiveness} = \frac{60}{600} = 0.1 = 10\%$$

(ii) At upper extreme radii:

$$F = 3r - 60 = 3 \times 190 - 60 = 510 \text{ N}$$

Controlling force at maximum speed, $F_1 = 510 + 30 = 540$ N

Controlling force at minimum speed, $F_2 = 510 - 30 = 480$ N

$$\text{Coefficient of insensitiveness,} = \frac{F_1 - F_2}{2F} = \frac{540 - 480}{2 \times 510} = \frac{60}{2 \times 510} = 0.0588 = 5.88\%$$

Coefficient of insensitiveness at upper extreme radii = 5.88%

Coefficient of insensitiveness at lower extreme radii = 10.00%

30. (a)

Given: Mass, m = 12 kg

Number of oscillations, N = 45

Time, t = 7 seconds

$$\frac{X_0}{X_4} = \left(\frac{X_0}{X_1}\right)\times\left(\frac{X_1}{X_2}\right)\times\left(\frac{X_2}{X_3}\right)\times\left(\frac{X_3}{X_4}\right) \qquad \left(\frac{X_0}{X_1}=\frac{X_1}{X_2}=\frac{X_2}{X_3}=\frac{X_3}{X_4}\right)$$

$$\frac{X_0}{X_4} = \left(\frac{X_0}{X_1}\right)^4$$

$$\frac{X_0}{X_1} = \left(\frac{X_0}{X_4}\right)^{1/4} = \left(\frac{1}{0.4}\right)^{0.25} = 1.25743$$

Logarithmic decrement, $\delta = \ln\left(\frac{X_0}{X_4}\right) = \ln(1.25743) = 0.229$

$$\therefore \qquad \delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$(0.229)^2\,(1-\xi^2) = (2\pi)^2\,\xi^2$$

$$1-\xi^2 = 752.8159\xi^2$$

$$\xi = 0.03672$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n \qquad \{\omega_d = 45\times 2\pi/7 = 40.392 \text{ rad/s}\}$$

$$40.392 = \sqrt{1-\xi^2}\,\omega_n = 0.9993255\,\omega_n$$

$$\omega_n = 40.419262$$

Damping coefficient, $C = 2\xi\omega_n m$

$= 2\times 12\times 40.419262\times 0.03642$

$C = 35.6206$ kg/s

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