CLASS TEST

Sl.: 02 SK_EC+EE_E+F_240523

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Bhubaneswar | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

SIGNALS AND SYSTEMS

EC + EE

Date of Test : 24/05/2023

ANSWER KEY

1.	(c)	7.	(b)	13.	(d)	19.	(a)	25.	(b)
2.	(b)	8.	(d)	14.	(a)	20.	(d)	26.	(c)
3.	(a)	9.	(c)	15.	(d)	21.	(b)	27.	(c)
4.	(b)	10.	(a)	16.	(c)	22.	(d)	28.	(a)
5.	(a)	11.	(a)	17.	(a)	23.	(a)	29.	(d)
6.	(d)	12.	(d)	18.	(b)	24.	(c)	30.	(d)

DETAILED EXPLANATIONS

1. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2+a^2}$$

$$\therefore \quad \cos(\pi t)u(t) = \frac{s}{s^2+\pi^2}$$

now, the given function $x(t)$ can be written as,

$$= \cos\pi t[u(t) - u(t-1)]$$

$$= \cos(\pi t)u(t) - \cos\pi t\, u(t-1)$$

$$= \cos\pi t u(t) - \cos\pi(t-1+1)u(t-1)$$

$$= \cos\pi t u(t) - \cos[\pi(t-1)+\pi]u(t-1)$$

$$x(t) = \cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2+\pi^2} + \frac{se^{-s}}{s^2+\pi^2} \qquad [\because x(t-t_0) = X(s)\cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1+e^{-s}]}{s^2+\pi^2}$$

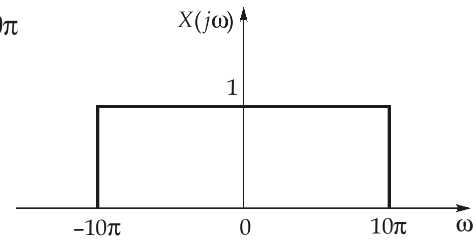
2. (b)

Given, $$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \;; & |\omega| \le 10\pi \\ 0 \;; & |\omega| > 10\pi \end{cases}$$

or



$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore \quad T_s < \frac{1}{10}$$

3. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!}z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

$$X(z) = 1+\frac{\frac{1}{2z}}{1!}+\frac{\left(\frac{1}{2z}\right)^2}{2!}+\frac{\left(\frac{1}{2z}\right)^3}{3!}+\ldots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2}=\sqrt{e}=1.648\approx 1.65$$

4. (b)

The output of the given LTI system is,

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]e^{j\omega(n-k)}+\sum_{k=-\infty}^{+\infty} h[k]e^{j2\omega(n-k)}$$

$$= e^{j\omega n}\sum_{k=-\infty}^{+\infty} h[k]e^{-j\omega k}+e^{j2\omega n}\sum_{k=-\infty}^{+\infty} h[k]e^{-j2\omega k}$$

$$= e^{j\omega n}H(e^{j\omega})+e^{j2\omega n}H(e^{j2\omega})$$

Since the input cannot be extracted from the above expression, the sum of the complex exponential is not an eigen function.

5. (a)

6. (d)

Let us consider two signals,

$$x_1(t) = 1, \quad \forall\, t$$

$$x_2(t) = -1, \quad \forall\, t$$

Clearly $x_1(t) \neq x_2(t)$ but $(x_1(t))^2=(x_2(t))^2$

Therefore different inputs gives the same output hence the system is non invertible.
And also it is non linear system.

7. (b)

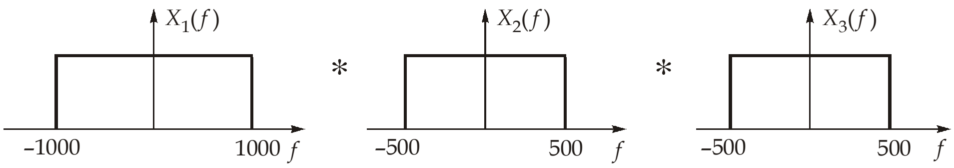
By using FFT to compute DFT, we need $\frac{N}{2}\log_2 N$ complex multiplications.

Therefore for 16-point DFT by using FFT we require $\frac{16}{2}\log_2 16 = 32$.

8. (d)

We know that,

For $\quad X(f) = X_1(f) * X_2(f) * X_3(f)$



Sampling frequency, $\quad f_s = 2(1000 + 500 + 500)$

$$f_s = 4000 \text{ samples/sec}$$

9. (c)

Conjugate anti-symmetric part of $x[n]$ is $\dfrac{x[n]-x^*[-n]}{2}$.

$$x^*[-n] = [2, 1 + j, -2 + j5]$$

$$\therefore \quad \frac{x[n]-x^*[-n]}{2} = \frac{[-2-j5, 1-j, 2]-[2,(1+j),-2+j5]}{2} = [-2-j2.5, \underset{\uparrow}{-j}, 2-j2.5]$$

11. (a)

Given,

$$X(e^{j\omega}) = \frac{\sin\frac{3\omega}{2}}{\sin\frac{\omega}{2}} = \frac{e^{j3\omega/2}-e^{-j3\omega/2}}{e^{j\omega/2}-e^{-j\omega/2}} = \frac{e^{j3\omega/2}\left[1-e^{-j3\omega}\right]}{e^{j\omega/2}\left[1-e^{-j\omega}\right]}$$

$$= e^{j\omega}\left[\frac{1-e^{-j3\omega}}{1-e^{-j\omega}}\right]$$

$$X(e^{j\omega}) = \frac{e^{j\omega}}{1-e^{-j\omega}} - \frac{e^{-j2\omega}}{1-e^{-j\omega}}$$

by taking inverse DTFT,

$$x[n] = u[n+1] - u[n-2]$$

$$= \begin{cases} 1; & -1 \le n < 2 \\ 0; & \text{otherwise} \end{cases}$$

From parseval's theorem,

$$\sum_{n=-\infty}^{\infty} |nx[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left[\frac{d}{d\omega}X(e^{j\omega})\right]\right|^2 d\omega$$

$$\therefore \quad \frac{1}{\pi}\int_{-\pi}^{\pi}\left|\left[\frac{d}{d\omega}X(e^{j\omega})\right]\right|^2 d\omega = 2\sum_{n=-\infty}^{\infty}|nx[n]|^2\, d\omega$$

$$= 2\sum_{n=-1}^{1}|n|^2 = 2[1+0+1] = 4$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\left|\left[\frac{d}{d\omega}X(e^{j\omega})\right]\right|^2 d\omega = 4$$

12. (d)

Given,

$$X(\omega) = \frac{1}{\omega^2 + j\omega} = \frac{1}{j\omega[1-j\omega]} = \frac{2}{2}\times\frac{1}{j\omega}\cdot\frac{1}{(1-j\omega)}$$

$$= \frac{1}{2}\,\frac{2}{j\omega}\cdot\frac{1}{(1-j\omega)}$$

From the property of convolution,

$$x_1(t) * x_2(t) = X_1(\omega)\, X_2(\omega)$$

$$\text{sgn}\, t * e^t\, u(-t) \xleftrightarrow{FT} \frac{2}{j\omega}\cdot\frac{1}{1-j\omega}$$

$$\frac{1}{2}\text{sgn}\, t * e^t u(-t) \xleftrightarrow{FT} \frac{1}{j\omega}\cdot\frac{1}{1-j\omega}$$

$$\therefore \quad x(t) = \frac{1}{2}\text{sgn}\, t * e^t u(-t)$$

13. (d)

Given $\quad x(t) = \sin(150\pi t)$

Time period, $\quad T = \frac{2\pi}{\omega_0} = \frac{2\pi}{150\pi} = \frac{1}{75}$ sec

3 time periods $= 3 \times T = 3\times\frac{1}{75} = \frac{1}{25}$ sec

$\therefore$ The signal sampled at a rate of five samples is $\frac{1}{25}$ sec

So, 1 sample in $\frac{1}{125}$ sec $= T_s$ [sampling interval]

$\therefore$ Sampling frequency $= f_s = \frac{1}{T_s} = 125$ samples/sec

also, Nyquist rate $= f_N = 2f_m = 2 \times 75$ $\quad [\because \omega_m = 150\pi \Rightarrow f_m = 75 \text{ Hz}]$

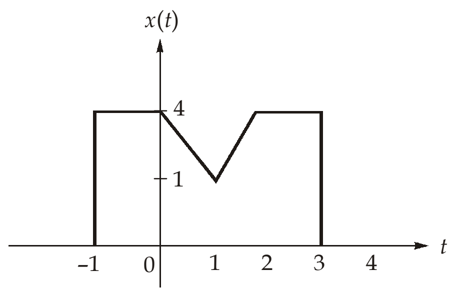
$= 150$ samples/sec

$\therefore$ The ratio, $\frac{f_s}{f_N} = \frac{125}{150} = \frac{5}{6} = 0.83$

14. (a)

Given,

By the definition of Fourier transform,



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot e^{j\omega t}\, d\omega$$

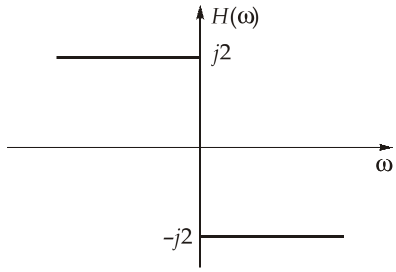
at $t = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$$\therefore \quad \int_{-\infty}^{\infty} X(j\omega)\, d\omega = 2\pi\, x(0) = 2\pi(4) = 8\pi \approx 25.13$$

15. (d)

By redrawing the given frequency response, we get,



We can write $H(\omega) = -j2\,\text{sgn}(\omega)$

We know that,

For $\qquad \text{sgn}(t) \xleftrightarrow{FT} \dfrac{2}{j\omega}$

By duality property

$$\frac{2}{jt} \xleftrightarrow{FT} 2\pi\,\text{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{FT} -2\pi\,\text{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{FT} -j2\text{sgn}(\omega)$$

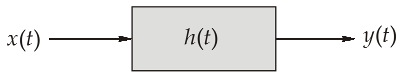
or $\qquad = 2(\pi t)^{-1}$

16. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output, $\qquad y(t) = e^{-3t}\,u(t) - e^{-4t}\,u(t)$



We know that, $\qquad H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$\therefore \qquad X(j\omega) = \dfrac{Y(j\omega)}{H(j\omega)} = \dfrac{1}{4+j\omega}$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t}\,u(t)$$

17. (a)

Given, $\qquad X(z) = \dfrac{10-8z^{-1}}{2-5z^{-1}+2z^{-2}}$

$$= \frac{2}{(2-z^{-1})} + \frac{4}{(1-2z^{-1})}$$

$$X(z) = \frac{2z}{2z-1} + \frac{4z}{z-2}$$

$$X(z) = \frac{z}{\left(z-\frac{1}{2}\right)} + \frac{4z}{(z-2)}$$

Since, ROC includes unit circle,

$\therefore$ ROC of $X(z)$ is $\frac{1}{2} < |z| < 2$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

$$\therefore \quad x(1) = \frac{1}{2} = 0.5$$

18. (b)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nK}$$

$$g[n] = x[n-2]_{\text{mod N}}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] = e^{-j\pi} X[1]$$

$$G[1] = -X[1] = -7$$

19. (a)

We know that, from the definition of DTFT,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

where, $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

by putting $n = -1$,

$$\therefore \quad x[-1] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega}\,d\omega$$

$$\therefore \quad \int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega}\,d\omega = 2\pi x[-1]$$

$$= 2\pi[-1] = -2\pi = -6.28$$

20. (d)

Given pole-zero plot can be written as,

$$X(z) = \frac{(z-1)(z+1)}{z^2} = \frac{z^2-1}{z^2} = 1 - z^{-2}$$

put $z = e^{j\omega}$,

$$|X(e^{j\omega})| = |1 - \cos 2\omega + j\sin 2\omega|$$

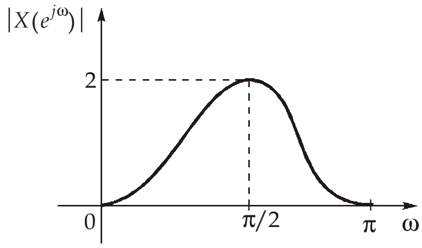
$$|X(e^{j\omega})| = \left|\sqrt{2 - 2\cos 2\omega}\right|$$

put, $\omega = 0 \Rightarrow$ $$\left|X(e^{j\omega})\right| = 0$$

$\omega = \pi/2 \quad \Rightarrow \quad |X(e^{j\pi/2})| = 2$

$\omega = \pi \quad \Rightarrow \quad |X(e^{j\pi})| = 0$

It is a bandpass filter response,



If poles are moved towards $\pm\frac{\pi}{2}$

Suppose we take poles are at $z = \pm j0.5$

$$X(z) = \frac{(z+1)(z-1)}{(z+j0.5)(z-j0.5)}$$

still response will remain same as BPF.

Because of zeros at the same position.

21. (b)

Given signals, $\quad x(t) = \sin\omega_0 t$

$\quad h(t) = \text{sgn}t$

from the multiplication property of Fourier transform,

$$x(t)h(t) = \frac{1}{2\pi}[X(\omega) * H(\omega)]$$

Fourier transform of $x(t)$ is $X(\omega)$,

$$X(\omega) = \frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$$

Fourier transform of $h(t)$ is $H(\omega)$,

$$H(\omega) = \frac{2}{j\omega}$$

$$\therefore \quad x(t)\ h(t) \xleftrightarrow{\text{FT}} \frac{1}{2\pi}\left[\frac{\pi}{j}(\delta(\omega-\omega_0) - \delta(\omega-\omega_0)) * \frac{2}{j\omega}\right]$$

$$\xleftrightarrow{\text{FT}} \frac{1}{2\pi}\left[\left[\frac{\pi}{j} \times \frac{2}{j(\omega-\omega_0)}\right] - \left[\frac{\pi}{j} \times \frac{2}{j(\omega+\omega_0)}\right]\right]$$

$$(\because X(\omega) * \delta(\omega-\omega_0) = X(\omega-\omega_0))$$

$$\xleftrightarrow{\text{FT}} \left[\frac{-1}{\omega-\omega_0} + \frac{1}{\omega+\omega_0}\right] = \frac{-\omega-\omega_0+\omega-\omega_0}{\omega^2-\omega_0^2}$$

$$\therefore \quad x(t)h(t) \xleftrightarrow{\text{FT}} \frac{-2\omega_0}{\omega^2-\omega_0^2}$$

22. (d)

Given signal, Let $x(t) = \frac{1}{\pi(1+t^2)}$

We know that,

$$e^{-a|t|} \xleftrightarrow{\text{FT}} \frac{2a}{a^2+\omega^2}$$

Put $a = 1$

$$e^{-|t|} \xleftrightarrow{\text{FT}} \frac{2}{1+\omega^2}$$

By using duality property,

$$\frac{2}{1+t^2} \xleftrightarrow{\text{FT}} 2\pi e^{-|-\omega|}$$

$$\frac{2}{1+t^2} \xleftrightarrow{\text{FT}} 2\pi e^{-|\omega|}$$

$$\frac{1}{\pi(1+t^2)} \xleftrightarrow{\text{FT}} e^{-|\omega|}$$

23. (a)

By the definition of Fourier series,

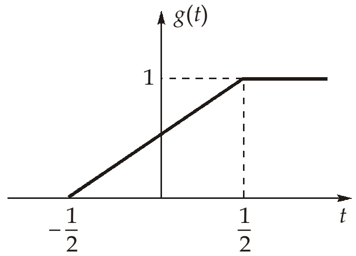
We can write $C_{N_0/2}$ for N_0 is even,

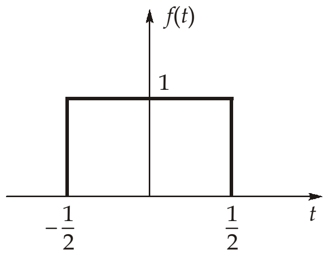
$$C_{N_0/2} = \frac{1}{N_0}\sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{N_0}{2}\right)\left(\frac{2\pi}{N_0}\right)n}$$

$$= \frac{1}{N_0}\sum_{n=0}^{N-1} x[n]\, e^{-j\pi n} = \frac{1}{N_0}\sum_{n=0}^{N-1} (-1)^n\, x[n] = \text{real}$$

24. (c)

Given,





We can write,

$$g(t) = \int_{-\infty}^{t} f(t) = \begin{cases} 0\ ; & t < -\frac{1}{2} \\ t+\frac{1}{2}\ ; & -\frac{1}{2} < t < \frac{1}{2} \\ 1\ ; & t > \frac{1}{2} \end{cases}$$

$\therefore$ $g(t) \xleftrightarrow{FT} G(\omega)$

$$g(t) = \int_{-\infty}^{t} f(t)\,dt \xleftrightarrow{FT} \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

($\because$ from time integration property)

$$\therefore \quad G(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

but,

$$F(\omega) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= -\frac{1}{j\omega}\left[e^{-j\frac{\omega}{2}} - e^{+j\frac{\omega}{2}}\right] = \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right)$$

$$F(\omega) = Sa\left(\frac{\omega}{2}\right)$$

$$\therefore \quad G(\omega) = \frac{Sa\left(\frac{\omega}{2}\right)}{j\omega} + \pi\delta(\omega) \qquad [\because F(0) = 1]$$

25. (b)

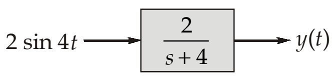
Given system is,

$$\frac{dy(t)}{dt} + 4y(t) = 2\,x(t)$$

$$(s+4)\,Y(s) = 2\,X(s)$$

$$\therefore \quad H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s+4}$$

$$\frac{e^{j4t} - e^{-j4t}}{j} = 2\sin 4t$$



$$y(t) = 2\left[\frac{2}{j4+4}\right]\sin\left[4t - \tan^{-1}\left(\frac{4}{4}\right)\right]$$

$$y(t) = \frac{1}{\sqrt{2}}\sin[4t - 45^\circ]$$

$$\therefore \quad A = \frac{1}{\sqrt{2}} = 0.707$$

26. (c)

$$x_1(t) \xrightarrow{\text{L.T.}} \frac{1}{s+2}$$

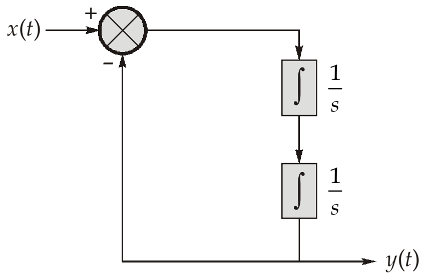
$$x_2(t) \xrightarrow{\text{L.T.}} \frac{1}{s+3}$$

$$x_1(t-2) \xrightarrow{\text{L.T.}} \frac{e^{-2s}}{s+2}$$

$$x_2(t+3) \xrightarrow{\text{L.T.}} \frac{e^{3s}}{s+3}$$

$$x_2(-t+3) \xrightarrow{\text{L.T.}} \frac{e^{-3s}}{3-s}$$

27. (c)



$$H(s) = \frac{\frac{1}{s^2}}{1+\frac{1}{s^2}} = \frac{1}{s^2+1}$$

$$h(t) = \sin t\, u(t)$$

28. (a)

The difference equation is,

$$y[n] + 0.5\, y[n-1] = x[n]$$

Characteristic equation,

$$\lambda + 0.5 = 0, \qquad \text{root, } \lambda = -0.5$$

$$y_H[n] = C_1(-0.5)^n$$

$$y_P[n] = K$$

$$K + 0.5\,K = 1$$

$$\Rightarrow \quad K = \frac{2}{3}$$

So, $$y_p[n] = \frac{2}{3}u[n]$$

Complete response,

$$y[n] = y_h[n] + y_p[n]$$

$$= C_1(-0.5)^n + \frac{2}{3}$$

Using given initial conditions,

$$y[-1] = C_1(-0.5)^{-1} + \frac{2}{3} = 0$$

$$\Rightarrow \quad C_1 = \frac{1}{3}$$

and, $$y[n] = \left[\frac{1}{3}\left(\frac{-1}{2}\right)^n + \frac{2}{3}\right]u[n]$$

Alternative Solution:

$$x[n] = 0.5y[n-1] + y[n]$$

Taking z-transform,

$$X(z) = Y(z) + 0.5\left[z^{-1}Y(z) + 0\right]$$

$\Rightarrow$ Put, $X(z) = \dfrac{z}{z-1}$ $\quad [\because x[n] = u(n)]$

$$Y(z) = \frac{z}{(z-1)(z+0.5)}$$

by partial fraction method,

$$\frac{A}{(z+0.5)} + \frac{B}{(z-1)} = \frac{z}{(z-1)(z+0.5)}$$

Sovling, $A = \dfrac{1}{3}$,

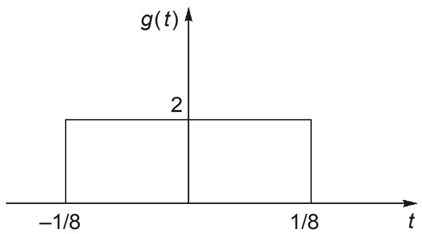
$$B = \frac{2}{3}$$

Hence, $y[n] = \left[\dfrac{1}{3}[-0.5]^n + \dfrac{2}{3}\right]u[n]$

29. (d)

$$g(t) = \text{rect}(4t) * 4\delta(-2t)$$
$$= 4\ \text{rect}(4t) * \delta(-2t) \qquad (\because \delta(-t) = \delta(t))$$
$$= 2\ \text{rect}(4t) \qquad \left(\because \delta(at) = \frac{1}{|a|}\delta(t)\right)$$

thus $g(t)$ is given as



now,

$$\text{rect}\,(t) \xleftrightarrow{F.T} \text{sinc}(f)$$

$$2\text{rect}\,(t) \xleftrightarrow{F.T} 2\,\text{sinc}(f)$$

$$2\text{rect}\,(4t) \xleftrightarrow{F.T} 2\cdot\frac{1}{4}\text{sinc}\left(\frac{f}{4}\right) \qquad \text{(scaling property)}$$

$$\therefore \quad 2\text{rect}\,(4t) \xleftrightarrow{F.T} \frac{1}{2}\text{sinc}\left(\frac{f}{4}\right)$$

30. (d)

Given, $x(t) = 2 + \cos(50\pi t)$

Frequency of signal $\omega_{sig} = 50\pi$

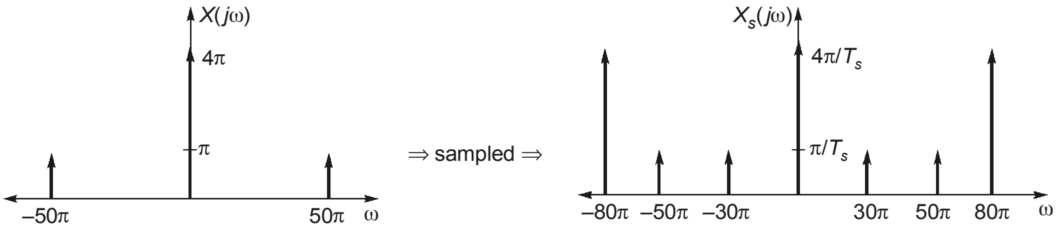
$T_s = 0.025$ sec

$\therefore$ sampling frequency $\omega_s = \frac{2\pi}{T_s} = 80\,\pi$ rad/sec

then, $X(j\omega) = 4\,\pi\,\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

Let the sampled signal be represented as $X_s(j\omega)$, where $X_s(j\omega)$ is given as

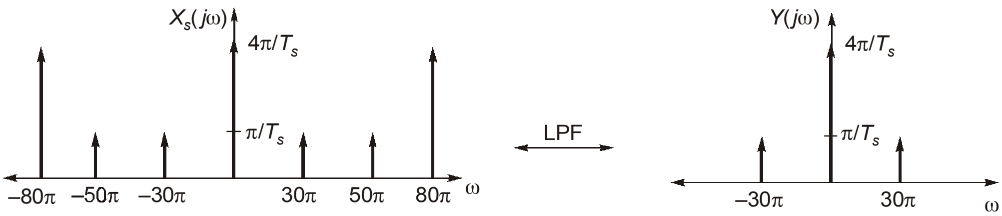
$$X_s(j\omega) = \frac{1}{T_s}\sum_{m=-\infty}^{\infty} X(j(\omega - n\omega_s))$$



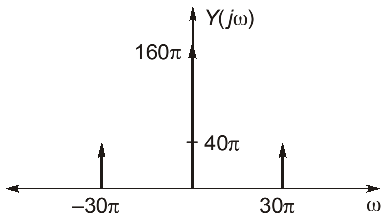
$$X_s(j\omega) = 40\sum_{m=-\infty}^{\infty}[4\pi\delta(\omega - 80\pi)] + \pi\,\delta(\omega - 50\pi - 80\pi n) - \pi\,\delta(\omega + 50\pi - 80\pi n)]$$

now, the sampled input $X_s(j\omega)$ is passed through a low passed filter having cut-off frequency at $\omega = 40\pi$.

Therefore the output $Y(j\omega)$ will contain only the components which are less than $\omega = 40\,\pi$.



Now by putting $T_s = 0.025$, we will get



■■■■