ANSWER KEY ➤ Engineering Mechanics

1. (a)	7. (c)	13. (a)	19. (b)	25. (c)
2. (b)	8. (b)	14. (d)	20. (a)	26. (b)
3. (a)	9. (c)	15. (c)	21. (a)	27. (a)
4. (b)	10. (d)	16. (d)	22. (d)	28. (a)
5. (b)	11. (c)	17. (a)	23. (a)	29. (b)
6. (b)	12. (c)	18. (d)	24. (b)	30. (a)

DETAILED EXPLANATIONS

1. (a)

$$a = \frac{f}{m} = -\frac{bv}{m}$$

but, $$a = v\frac{dv}{dx}$$

∴ $$\frac{vdv}{dx} = -\frac{bv}{m}$$ (at time infinity means steady state)

$$\int_u^0 dv = -\frac{b}{m}\int_0^x dx$$

$$-u = -\frac{b}{m}\times x$$

⇒ $$x = mu/b$$

2. (b)

Resolving the forces in horizontal and vertical components.

$$\text{Horizontal components, } \Sigma F_X = 60\cos 30° - 80\cos 45° = -4.607$$

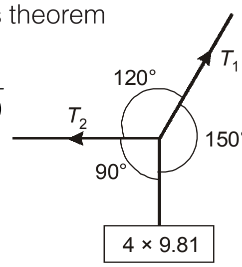
$$\text{Vertical components, } \Sigma F_Y = 80\sin 45° + 60\sin 30° = 86.568$$

$$\text{Resultant, } R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(-4.607)^2 + (86.568)^2}$$

$$= 86.69 \text{ N}$$

3. (a)

As the body is in equilibrium, using Lami's theorem



$$\therefore \quad \frac{T_1}{\sin 90°} = \frac{4 \times 9.81}{\sin(120°)}$$

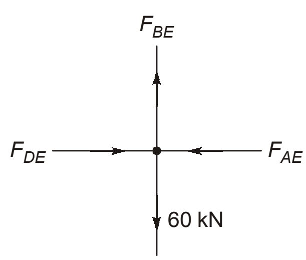
$$\therefore \quad T_1 = 45.310 \text{ N}$$

$$\frac{T_2}{\sin 150°} = \frac{4 \times 9.81}{\sin 120°}$$

$$\Rightarrow \quad T_2 = 22.65 \text{ N}$$

4. (b)

Consider joint (E)

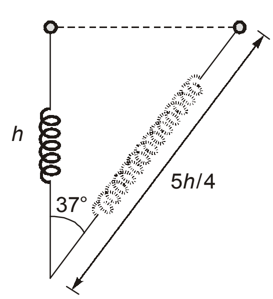


$$F_{BE} = 60 \text{ kN (Tensile)}$$

6. (b)

∴ The kinetic energy of the ring will be given by the potential energy of spring.

∴ Let V be the speed of the ring when the spring becomes vertical



$$\frac{1}{2}mV^2 = \frac{1}{2}k[X]^2$$

$$X = \frac{5h}{4} - h = \frac{h}{4}$$

$$mV^2 = k\left[\frac{h}{4}\right]^2$$

$$V = \frac{h}{4}\sqrt{\frac{k}{m}}$$

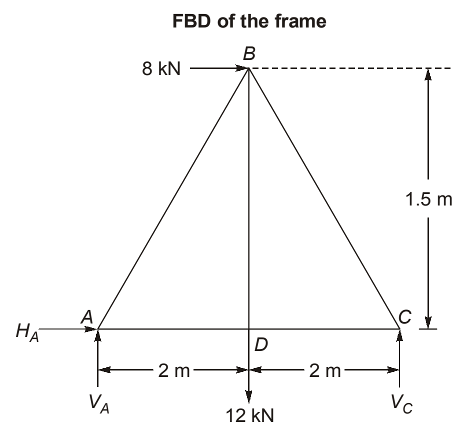
8. (b)

Using Lami's Theorem,

$$\frac{T_1}{\sin 120°} = \frac{T_2}{\sin(360° - (90° + 120°)}$$

$$\frac{T_1}{T_2} = \frac{\sin 120°}{\sin 150°} = 1.732$$

9. (c)



∵ Taking moments about A,

$$V_C \times 4 = 8 \times 1.5 + 12 \times 2$$

$$V_C = \frac{12 + 24}{4} = \frac{36}{4} = 9 \text{ kN}$$

Reaction of support C, V_C = 9 kN

10. (d)

Let u, v, w be the components of velocity in x, y and z direction respectively.

$$u = \frac{dx}{dt} = 2\cos t$$

Similarly, $v = -3 \sin t$

$$w = \sqrt{5}\cos t$$

$$V = \sqrt{u^2 + v^2 + w^2}$$

$$= \sqrt{(2\cos t)^2 + (-3\sin t)^2 + \left(\sqrt{5}\cos t\right)^2}$$

$$V = \sqrt{4\cos^2 t + 9\sin^2 t + 5\cos^2 t}$$

$$V = \sqrt{9(\sin^2 t + \cos^2 t)} = 3 \text{ units}$$

11. (c)

$$a = \frac{dv}{dt}$$

Let resisting force, $F = Kv^2$

if m is mass of the bullet then,

$$a = \frac{F}{m} = \frac{Kv^2}{m}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{Kv^2}{m}$$

$$\Rightarrow \quad \frac{1}{v^{-2}} dv = \frac{K}{m} \cdot dt$$

$$\Rightarrow \quad \left[\frac{v^{-1}}{-1}\right]_u^v = \frac{K}{m}\int_0^t dt$$

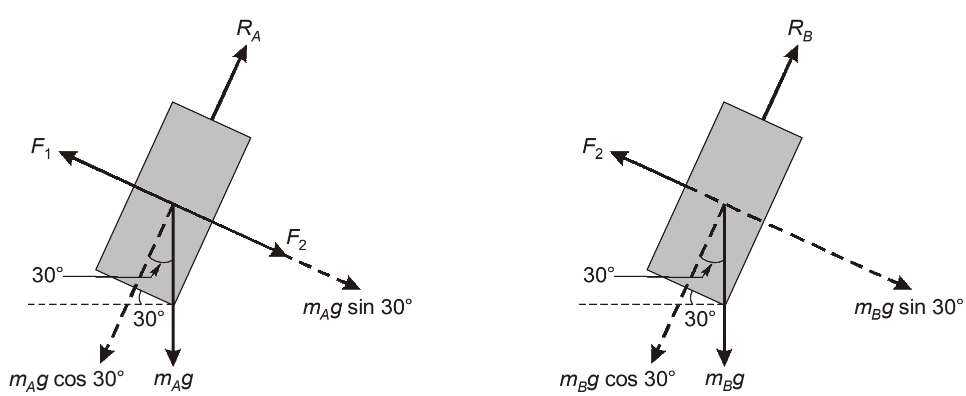
$$\Rightarrow \quad \left[\frac{v-u}{uv}\right] = \frac{K}{m} t$$

$$\Rightarrow \quad t = \frac{(u-v)}{uv} \times \frac{-m}{K}$$

$$\therefore \quad t \propto (u-v)(uv)^{-1}$$

12. (c)

The FBD of the blocks A and B are shown below



Here F_1 and F_2 are the spring forces.

$$F = k\Delta z = k(x_0 - x_{\text{unstretched}})$$

$$F_1 = 1000 \times (0.3 - 0.25) = 50 \text{ N}$$

and $$F_2 = 1000 \times (0.28 - 0.25) = 30 \text{ N}$$

At equilibrium,

ΣForces along the plane for mass $A = 0$

$\Rightarrow \quad -F_1 + F_2 + m_A g \sin 30° = 0$

$$\Rightarrow \quad m_A = \frac{F_1 - F_2}{g \sin 30°} = \frac{50 - 30}{9.81 \times 0.5} = 4.08 \text{ kg}$$

and ∑Forces along the plane for mass $B = 0$

$\Rightarrow \quad -F_2 + m_B g \sin 30° = 0$

$$\Rightarrow \quad m_B = \frac{F_2}{g \sin 30°}$$

$$= \frac{30}{9.81 \times 0.5} = 6.12 \text{ kg}$$

13. (a)

$$\text{K.E.} = \frac{1}{2} I \omega^2$$

$$I = \frac{mr^2}{2} = \frac{20 \times (0.2)^2}{2} = 0.4 \text{ kgm}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 500}{60} = 52.33 \text{ rad/s}$$

$$\text{K.E.} = \frac{1}{2} \times 0.4 \times 52.33^2 = 547.68 \text{ J}$$

14. (d)

Let speed of car moving in opposite direction is V m/s.

From relative velocity approach

$$\frac{12}{V + 50} = \frac{5}{60}$$

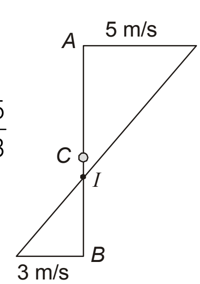
$$12 \times 60 = 5v + 250$$

$$V = 94 \text{ km/hr}$$

15. (c)

∵ Velocities are in opposite directions,

∴ I will lie between A and B,



$$\frac{IA}{IB} = \frac{V_a}{V_b} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{0.5 - IB}{IB} = \frac{5}{3}$$

$$IB = 0.1875 \text{ m}$$

$$IA = 0.3125 \text{ m}$$

$$\omega = \frac{V_A}{IA} = \frac{5}{0.3125} = 16 \text{ rad/s}$$

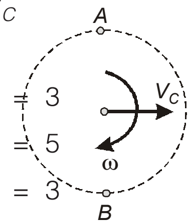
Alternatively,

$$\therefore \quad V_A = V_C + R\omega$$

$$V_B = R\omega - V_C$$

$$\therefore \quad V_C + R\omega = 5$$

$$R\omega - V_C = 3$$



$$V_C + 0.25\,\omega = 5 \quad \text{...(a)}$$

$$0.25\,\omega - V_C = 3 \quad \text{...(b)}$$

On solving (a) and (b),

$$\omega = 16 \text{ rad/s}$$

$$V_C = 1 \text{ m/s}$$

where V_C = velocity of centre C.

16. (d)

$$E = \frac{1}{2} I\omega^2$$

$$I = MR^2$$

$$E = \frac{1}{2} MR^2\omega^2$$

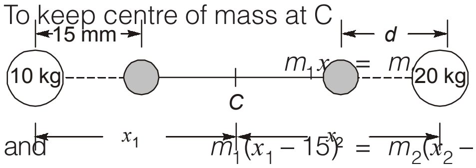
$$\frac{E_1}{E_2} = \frac{MR_1^2\omega^2}{MR_2^2\omega^2} = 4$$

17. (a)

$$I_y = I_x = \frac{1}{2} I_{\text{circle}} = \frac{1}{2} \times \pi \times \frac{D^4}{64} = \frac{\pi r^4}{8}$$

18. (d)

To keep centre of mass at C



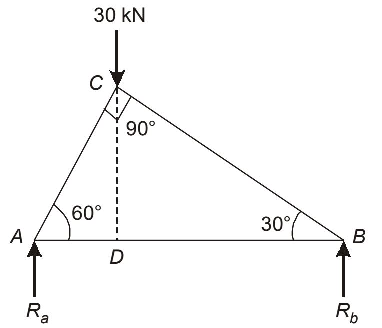
$$m_1 x_1 = m_2 x_2 \quad \rightarrow \quad \text{(Let 10 kg} = m_1,\ 20 \text{ kg} = m_2)$$

and $$m_1(x_1 - 15) = m_2(x_2 - d)$$

$$15\, m_1 = m_2 d$$

$$d = \frac{15 \times 10}{20} = 7.5 \text{ mm}$$

19. (b)



$$AC = AB \cos 60° = 2.5 \text{ m}$$

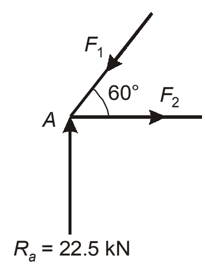
$$AD = AC \cos 60° = 2.5 \times 0.5 = 1.25$$

∴ Taking moments about A,

$$R_b \times 5 = 30 \times 1.25$$

$$R_b = 7.5 \text{ kN}, \quad R_a = 30 - 7.5 = 22.5 \text{ kN}$$

Considering joint A,



$$\Sigma F_x = 0, \quad F_2 - F_1 \cos 60° = 0$$

$$F_1 \sin 60° - R_a = 0$$

$$F_1 = \frac{R_a}{\sin 60°} = \frac{22.5}{\sin 60°} = 25.97 \text{ kN} \qquad \text{(compressive)}$$

$$F_2 = F_1 \cos 60° = 12.99 \text{ kN} \qquad \text{(tensile)}$$

∴ AB is in tension.

20. (a)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = \frac{4\pi^2 L}{T^2}$$

In given problem $T = \frac{36}{20} = 1.8 \text{ s}$

∴ $$g = \frac{4 \times \pi^2 \times 0.8}{1.8^2} = 9.74 \text{ m/s}^2$$

21. (a)

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2$$

$$2.5 = \frac{1}{2}\alpha(1)^2$$

$$\alpha = 5 \text{ rad/s}^2$$

The angle rotated during 1st two second

$$= \frac{1}{2}\times 5\times 2^2 = 10 \text{ radian}$$

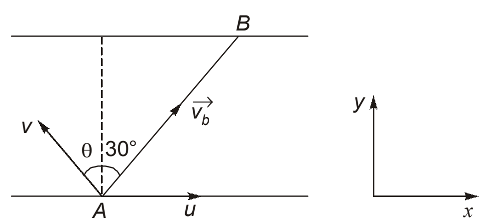
then

Angle rotated during the 2nd second is

$$10 - 2.5 = 7.5 \text{ radian}$$

22. (d)

Let v be the speed of boatman in still water



Resultant of u and v should be along AB. Components of $\vec{v}_b$ (absolute velocity of boatman) along x and y -direction are :

$$v_x = u - v\sin\theta,\ v_y = v\cos\theta$$

$$\tan 30^\circ = \frac{v_y}{v_x}$$

$$\Rightarrow \quad 0.577 = \frac{v\cos\theta}{u - v\sin\theta}$$

$$0.577u - 0.577v\sin\theta = v\cos\theta$$

$$\Rightarrow \quad v = \frac{0.577u}{0.577\sin\theta + \cos\theta}$$

$$v = \frac{(0.577\times\cos 30^\circ)u}{\sin 30^\circ\sin\theta + \cos 30^\circ\cos\theta}$$

$$v = \frac{0.49964}{\sin(\theta + 30^\circ)}$$

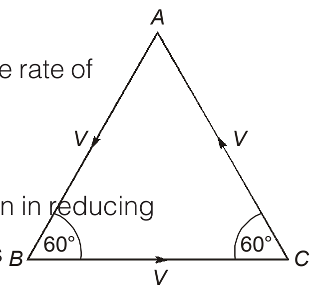
v is minimum at $\theta = 60^\circ$,

$$\Rightarrow \quad v_{\min} = 0.49964$$

$$v_{\min} \simeq 0.54$$

23. (a)

Velocity of A is v along AB and velocity of particle B is along BC, its component along BA is $v\cos 60° = \frac{v}{2}$.



Thus separation AB decreases at the rate of

$$v + \frac{v}{2} = \frac{3v}{2}$$

Since this rate is constant, time taken in reducing separation from AB from d to zero is

$$t = \frac{d}{3v/2} = \frac{2d}{3v}$$

24. (b)

$$\Sigma M_A = 0$$

$$\Rightarrow \quad P \times a \sin 60° = 2a \cdot R_{cv}$$

$$\Rightarrow \quad R_{cv} = 0.433\, P \uparrow$$

$$R_{CH} = 0$$

$$\Rightarrow \quad R_c = 0.433\, P$$

$A \rightarrow (1)$

Reaction at A

$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_{AV} = 0.433 P$$

$$\Sigma F_x = 0;\ R_{AH} = P$$

$$R_A = \sqrt{(0.433P)^2 + P^2} = 1.09\, P$$

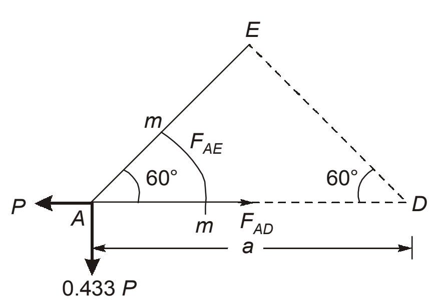
$B \rightarrow (4)$

At joint E, members AE and EB are collinear and member DE is joined at E.

$$\Rightarrow \quad F_{DE} = 0$$

$D \rightarrow (3)$

Taking section mm as shown,



$$\Sigma M_E = 0$$

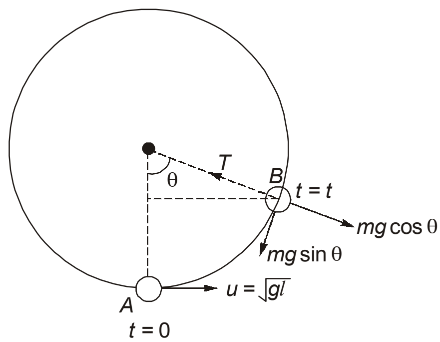
$$\Rightarrow \quad P \times a \times \sin 60° = 0.433\, P \times a \sin 30° + F_{AD} \times a \sin 60°$$

$\Rightarrow \quad 0.866P = 0.2165P + 0.866\ F_{AD}$

$\Rightarrow \quad F_{AD} = P - 0.25P = 0.75P$

$C \rightarrow (2)$

25. (c)



Let $\quad T = mg$ at angle θ shown in figure

$$h = l(1 - \cos\theta) \quad ...(1)$$

Apply conservation of mechanical energy between points A and B,

$$\frac{1}{2}m\left(u^2 - v^2\right) = mgh$$

$$u^2 = gl \quad ...(2)$$

v = Speed of particle in position on B

$$v^2 = u^2 - 2gh \quad ...(3)$$

$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$mg - mg\cos\theta = \frac{mv^2}{l}$$

$$\Rightarrow \quad v^2 = gl(1 - \cos\theta) \quad ...(4)$$

Substituting the values of v^2, u^2 and h from equations (4), (2) and (1) in equation (3).

$$gl(1 - \cos\theta) = gl - 2gl(1 - \cos\theta)$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

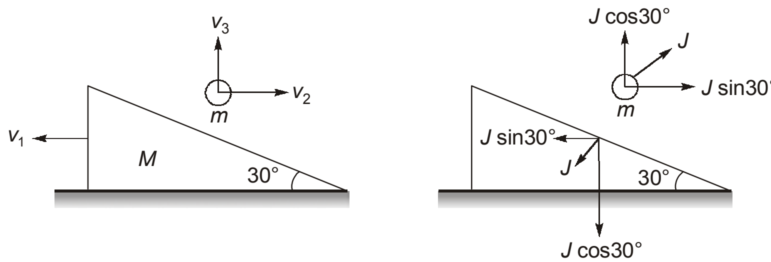
Substituting $\cos\theta = \frac{2}{3}$ in equation (4),

$$v = \sqrt{\frac{gl}{3}}$$

26. (b)

Given:

$$M = 2 \text{ kg and } m = 1 \text{ kg}$$



Let J be the impulse between ball and the wedge during collision and v_1, v_2 and v_3 be the components of the velocity of the wedge and the ball in horizontal and vertical directions respectively.

$$\text{Impulse} = \text{Change in momentum}$$

$$J\sin 30° = Mv_1 - mv_2$$

$$\Rightarrow \quad \frac{J}{2} = 2v_1 - v_2 \quad \text{...(1)}$$

$$J\cos 30° = m(v_3 + v_o)$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}J = v_3 + 2 \quad \text{...(2)}$$

$$\frac{\text{Relative speed of separation}}{\text{Relative speed of approach}} = \text{Coefficient of restitution}$$

$$\frac{(v_1 + v_2)\sin 30° + v_3\cos 30°}{v_o\cos 30°} = \frac{1}{2}$$

$$\Rightarrow \quad v_1 + v_2 + \sqrt{3}v_3 = \sqrt{3} \quad \text{...(3)}$$

Solving equations (1), (2) and (3),

$$v_1 = \frac{-1}{\sqrt{3}}\ \text{m/s}$$

$$v_2 = \frac{2}{\sqrt{3}}\ \text{m/s and } v_3 = 0$$

$$\text{Thus velocity of wedge} = \frac{-1}{\sqrt{3}}\hat{i}\ \text{m/s}$$

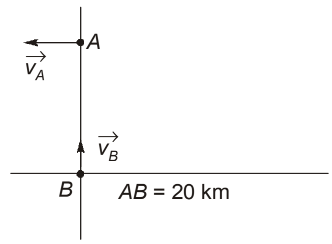
$$\text{Velocity of ball} = \frac{2}{\sqrt{3}}\hat{i}\ \text{m/s}$$

27. (a)

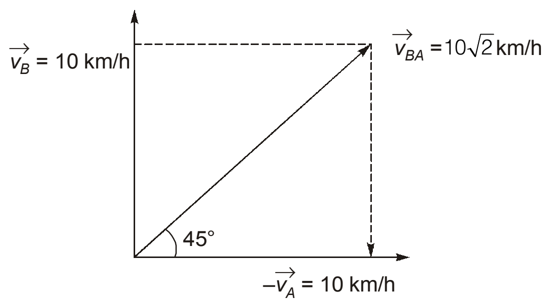
Boats A and B are moving with same speed 10 km/h in the directions shown in figure. It corresponds to a 2-dimensional, 2 body problem with zero acceleration.

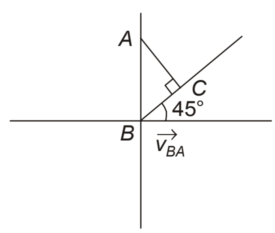
$$\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A}$$

$$\left|\overrightarrow{V_{BA}}\right| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ km/h}$$



In can be assumed that A is at rest and B is moving with $\overrightarrow{v_{BA}}$ in the direction shown





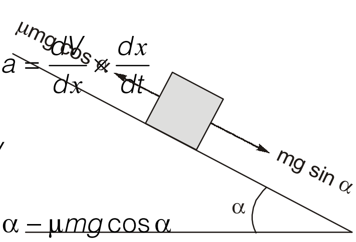
$$\text{Minimum distance} = AC = AB\sin 45° = \frac{20}{\sqrt{2}}\text{ km} = 10\sqrt{2} \text{ km}$$

$$\text{time is } t = \frac{BC}{\left|\overrightarrow{V_{BA}}\right|} = \frac{10\sqrt{2}}{10\sqrt{2}} = 1 \text{ hr}$$

28. (a)

Here, $\alpha = 45°$

We have: $a = \dfrac{dV}{dt} \Rightarrow a = \dfrac{dV}{dx} \times \dfrac{dx}{dt}$



$\therefore \quad a = \dfrac{dV}{dx} \times V$

Also, $a = \dfrac{mg\sin\alpha - \mu mg\cos\alpha}{m}$

$a = g[\sin\alpha - \mu\cos\alpha]$

$\therefore \quad g[\sin\alpha - \mu\cos\alpha] = \dfrac{dV}{dx} \times V$

$\therefore \quad g[\sin\alpha \cdot dx - 5x\cos\alpha\, dx] = V \cdot dV$

On integrating,

$$g\left[\sin\alpha\cdot x - 5\cos\alpha\times\frac{x^2}{2}\right] = \left[\frac{V^2}{2}\right]_0^0$$

$$g\left[\sin\alpha\cdot x - 5\cos\alpha\times\frac{x^2}{2}\right] = 0$$

$$\Rightarrow \quad \sin\alpha\cdot x = 5\cos\alpha\times\frac{x^2}{2}$$

$$x = \frac{2\tan\alpha}{5} \Rightarrow \frac{2\tan 45°}{5} = 0.4\,\text{m}$$

29. (b)

We have, $\quad \text{Torque} = I\alpha$

$\therefore \quad 3F\sin 30° \times 0.5 = I\alpha$

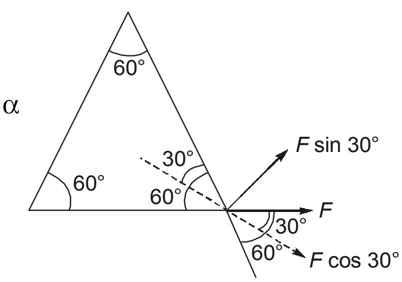
$$3\times 0.5\times\frac{1}{2}\times 0.5 = 1.5\times\frac{0.5^2}{2}\times\alpha$$

$\therefore \quad \alpha = 2\text{ rad/s}^{-1}$

$\therefore \quad \omega = \omega_0 + \alpha t$

$\omega = 0 + 2\times 1$

$\omega = 2\text{ rad s}^{-1}$



30. (a)

$$a = \frac{dV}{dt}$$

$$\Rightarrow \quad \alpha\sqrt{v} = \frac{dv}{dt}$$

$$\Rightarrow \quad \alpha\int_{t=0}^{t} dt = \int_{V_o}^{0}\frac{dv}{\sqrt{v}}$$

$$\Rightarrow \quad \alpha t = \frac{v_0^{-1/2+1}}{\frac{-1}{2}+1}$$

$$\Rightarrow \quad t = \frac{2\sqrt{v_o}}{\alpha}$$

■■■■