CLASS TEST

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# ELECTROMAGNETIC THEORY

## ELECTRONICS ENGINEERING

**Date of Test : 21/05/2023**

### ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (a) | 7. | (c) | 13. | (d) | 19. | (c) | 25. | (a) |
| 2. | (a) | 8. | (a) | 14. | (d) | 20. | (b) | 26. | (a) |
| 3. | (a) | 9. | (b) | 15. | (a) | 21. | (c) | 27. | (c) |
| 4. | (a) | 10. | (c) | 16. | (c) | 22. | (d) | 28. | (b) |
| 5. | (c) | 11. | (c) | 17. | (d) | 23. | (b) | 29. | (a) |
| 6. | (d) | 12. | (a) | 18. | (a) | 24. | (a) | 30. | (c) |

## Detailed Explanations

1. (a)

$$\eta_{TE} = \frac{120\pi}{\cos\theta} > 120\pi$$
$$\eta_{TM} = 120\pi \times \cos\theta < 120\pi$$
$$\eta_{TEM} = 120\pi$$

2. (a)

Antenna efficiency, $$\eta = \frac{R_{rad}}{R_{rad} + R_L} = \frac{50}{60} = \frac{5}{6}$$

$$\text{Directive gain} = \frac{\text{Power gain}}{\eta} = \frac{20}{5} \times 6 = 24$$

3. (a)

Skin depth is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\therefore \quad \frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\therefore \quad \frac{\delta}{\delta_2} = \sqrt{\frac{4}{1}}$$

$$\therefore \quad \delta_2 = \frac{6}{7} = 3\ \mu\text{m}$$

4. (a)

$$\nabla \times \vec{A} = \vec{B}$$
$$\nabla \times \vec{A} = \mu \vec{H}$$
$$\Rightarrow \quad \vec{H} = \frac{1}{\mu}\left(\nabla \times \vec{A}\right)$$

5. (c)

$$P_r = \frac{1}{2}(E_r)(H_r)$$

$$E_r = E_i\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) = 5\ \text{V/m}$$

$$H_r = -\frac{E_r}{\eta_1} = \frac{-1}{20}\ \text{A/m}$$

$$|P_r| = \frac{1}{2} \times 5 \times \frac{1}{20} = \frac{1}{8}\ \text{W/m}^2$$

6. (d)

$$v_p \times v_g = c^2$$
$$3.5 \times 10^8 \times v_g = (3 \times 10^8)^2$$
$$v_g = \frac{9 \times 10^{16}}{3.5 \times 10^8} = 2.57 \times 10^8\ \text{m/s}$$

7. (c)
The phase shift between *X* component and *Y* component is 90° and *X* component lags *Y* component hence it is left hand circular polarization.

8. (a)

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}}$$

$$\frac{\sin 30°}{\sin\theta_t} = \sqrt{3}$$

$$\Rightarrow \quad \sin\theta_t = \frac{1}{2\sqrt{3}}$$

$$\Rightarrow \quad \theta_t = \sin^{-1}\left(\frac{1}{2\sqrt{3}}\right) = 16.78°$$

9. (b)
Electrical length of the line, $\theta = \beta l$

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times 10^{-6}\times 0.1\times 10^{-12}}} = 10^9 \text{ m/s}$$

$$\lambda = \frac{v_p}{f} = \frac{10^9}{10^6} = 1000 \text{ m}$$

$$\theta = \frac{2\pi}{\lambda}\times l = \frac{2\pi}{1000}\times 250 = \frac{\pi}{2} = 90°$$

10. (c)
Given,

$$\left|\frac{J_C}{J_D}\right| = \left|\frac{\sigma E}{\omega\varepsilon E}\right| = \frac{\sigma}{\omega\varepsilon} = 10$$

or, $$\omega = \frac{\sigma}{10\varepsilon}$$

$$\therefore \quad 2\pi f = \frac{\sigma}{10\varepsilon} \Rightarrow f = \frac{\sigma}{20\pi\varepsilon} = \frac{20}{20\pi\times 81\times 8.854\times 10^{-12}}$$

$$f = 443.84 \text{ MHz}$$

11. (c)

We have, $$\frac{m\pi x}{a} = \frac{2\pi x}{a}, \quad m = 2$$

$$\frac{n\pi x}{b} = \frac{3\pi y}{b}, \quad n = 3$$

$\therefore$ it is $TE_{23}$ node

Cut off frequency, $$f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{2}\right)^2} = \frac{3\times 10^8}{2}\sqrt{\left(\frac{2}{286}\right)^2 + \left(\frac{3}{1.016}\right)^2}\times 100$$

$$= 46.19 \text{ GHz}$$

$\omega = 10\pi \times 10^{10}$, $\quad f = 50$ GHz

$$\beta = \frac{2\pi f}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi \times 50 \times 10^9}{3 \times 10^8}\sqrt{1-\left(\frac{46.19}{50}\right)^2} = 400.68 \text{ rad/m}$$

$\because f > f_c$ $\quad \alpha = 0$

$\gamma = \alpha + j\beta = j400.7$ rad/m

12. (a)

Propagation constant given by

$$\tau = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(0.03 + j2\pi \times 10^3 \times 10^{-4})(0 + j2\pi \times 10^3 \times 20 \times 10^{-9})}$$

$$= 2.121 \times 10^{-4} + j8.88 \times 10^{-3} \text{ /m}$$

$\therefore$ $\quad r = \alpha + j\beta$

$\therefore$ $\quad \alpha = 2.121 \times 10^{-4}$ Np/m $= 0.21 \times 10^{-3}$ Np/m

A distortion less line operating at 120 MHz has $R = 20\ \Omega$/m, $L = 0.3\ \mu$H/m, $C = 63$ pF/m

13. (d)

Given, $\quad H_z = 5\cos(10^9 t - 4y)\hat{a}_z$ A/m

$$J_d = \nabla \times H = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix}$$

$$= \frac{\partial H_z}{\partial y}\hat{a}_x = \frac{\partial}{\partial y}(5\cos(10^9 t - 4y))\hat{a}_x$$

$$J_d = 20\sin(10^9 t - 4y)\hat{a}_x \text{ A/m}$$

But, $\quad J_d = \dfrac{\partial D}{\partial t}$

$$D = \int J_d dt = -\frac{20}{10^9}\cos(10^9 t - 4y)\hat{a}_x$$

$$D = -20\cos(10^9 t - 4y)\hat{a}_x \text{ nC/m}^2$$

14. (d)

$$\Gamma_A = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 - 100}{150 + 100} = 0.2$$

$\therefore$ $\quad S_A = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = 1.5$

$$\Gamma_B = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j150 - 100}{j150 + 100}$$

$$|\Gamma_B| = 1 = |\Gamma_C|$$

$\therefore$ $\quad S_B = S_C = \infty$

15. (a)

At junction input impedance of $\frac{\lambda}{2}$ line

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) = 0$$

$$Z_{in\,L} = Z_o\left(\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right) = Z_L = 100\ \Omega$$

For input impedance of S.C. stub

As we know for SC stub, $Z_L = 0$

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = 1$$

$$Z_{in\,S} = Z_o\left(\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right) = jZ_o = j50\ \Omega$$

At junction,
$$Y = \frac{1}{Z} = \frac{1}{j50} + \frac{1}{100} = 0.01 - j0.02$$

16. (c)

Since wave is travelling along positive $y$-direction and $E_z$ and $E_x$ components are not equal

$\therefore$
$$E_z \neq E_x$$

Also, $E_x$ leads by 90° it's left elliptical polarization

17. (d)

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{c} \times f = \frac{2\pi \times 2 \times 10^9}{3 \times 10^8} = \frac{40\pi}{3}$$

$$\beta l = \frac{40\pi}{3} \times 0.1 = \frac{4\pi}{3}$$

Input impedance of short circuited line's

$$z_{in} = jz_0 \tan\beta l = jz_0 \tan\frac{4\pi}{3} = j50 \times \sqrt{3}$$

$\therefore$ Hence inductive.

18. (a)

$$Z_{in_2} = Z_{02}\left.\frac{Z_L + jZ_{02}\tan\beta l}{Z_{02} + jZ_L \tan\beta l}\right|_{\substack{l = 5\lambda/2 \\ Z_{02} = 100\ \Omega \\ Z_L = 75\ \Omega}} = 100\frac{75 + j100\tan 5\pi}{100 + j75\tan 5\pi} = 75\ \Omega$$

$$Z_{in} = Z_{01}\left.\frac{Z_{in_2} + jZ_{01}\tan\beta l}{Z_{01} + jZ_{in_2}\tan\beta l}\right|_{\substack{l = 3\lambda/4 \\ Z_{in_2} = 75\ \Omega \\ Z_{01} = 50\ \Omega}} = 50\frac{75 + j50\tan\frac{3\pi}{2}}{50 + j75\tan\frac{3\pi}{2}}$$

$$= \frac{50^2}{75} = 33.33\ \Omega$$

19. (c)

From boundary condition of dielectric - dielectric medium.

$$E_{t_1} = E_{t_2}$$

and $$D_{n_1} = D_{n_2}$$

$$\varepsilon_{r_1} E_{n_1} = \varepsilon_{r_2} E_{n_2}$$

or $$E_{n_2} = \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} E_{n_1} = \frac{2}{8} \times 100 = 25$$

∴ $$\vec{E}_2 = 25\hat{a}_x + 200\hat{a}_y - 50\hat{a}_z$$

20. (b)

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \left(2\pi \times 10^6\right) \cos\left(2\pi \times 10^6 t + \beta z\right) \hat{a}_y$$

$$= \frac{10^{-9}}{36\pi} \times 2\pi \times 10^6 \cos\left(2\pi \times 10^6 t + \beta z\right) \hat{a}_y \text{ A/m}^2$$

$$= 55.5 \cos\left(2\pi \times 10^6 + \beta z\right) \hat{a}_y \ \mu\text{A/m}^2$$

21. (c)

- Field contains orthogonal components with unequal amplitudes ⇒ Elliptical polarization
- $y$ component leads $x$ component by 90° and wave is travelling in positive-z direction ⇒ Left elliptically polarized.

22. (d)

For a distortionless line, $$RC = GL$$

$$G = \frac{RC}{L}$$

- $$Z_0 = \sqrt{\frac{L}{C}}$$
- $$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

⇒ $$R = \alpha Z_0 = 10 \times 10^{-3} \times 100 = 1 \ \Omega/\text{m}$$

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{LC}} \times \sqrt{\frac{L}{L}} = \frac{1}{L}\sqrt{\frac{L}{C}} = \frac{1}{L} \times Z_0$$

⇒ $$L = \frac{Z_0}{v} = \frac{100}{2 \times 10^8} = 50 \times 10^{-8} = 0.5 \ \mu\text{H/m}$$

23. (b)

- $$Z_{in_3} = \frac{Z_{03}^2}{Z_{L3}} = \frac{300^2}{200} = 450 \ \Omega$$
- $$Z_{in_2} = \frac{Z_{02}^2}{Z_{L2}} = \frac{100^2}{0} = \infty \ \text{(open)}$$
- $$Z_{L(\text{eff})} = Z_{in_3} \,||\, Z_{in_2} = 450 \ \Omega$$
- $$Z_{in} = Z_{in_1} = \frac{100^2}{450} = 22.22 \ \Omega$$

24. (a)

Given: $Z_L = 80 + j40$ ; $Z_o = 50\ \Omega$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{80 + j40 - 50}{80 + j40 + 50}$$

$$= \frac{30 + j40}{130 + j40} = \frac{50\angle 53.13^\circ}{136\angle 17.10^\circ} = 0.367\angle 36.03^\circ$$

$$P_{\text{load}} = P_{\text{incid}} - P_{\text{reflected}} = P_{\text{incid}}\left[1 - |\Gamma|^2\right]$$

$$P_{\text{load}} = 30\,[1 - (0.367)^2] = 25.9\text{ W}$$

25. (a)

The cut-off frequency for the $TE_{mn}$ mode is,

$$f_c = \frac{C}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

We need the frequency lie between the cut-off frequencies of the $TE_{10}$ and $TE_{01}$ modes.

These will be,

$$f_{c,\,10} = \frac{C}{2\sqrt{\epsilon_r}\,a} = \frac{3\times 10^8}{2\sqrt{\epsilon_r}(0.06)} = \frac{2.5\times 10^9}{\sqrt{\epsilon_r}}$$

$$f_{c,\,01} = \frac{C}{2\sqrt{\epsilon_r}\,b} = \frac{3\times 10^8}{2\sqrt{\epsilon_r}(0.04)} = \frac{3.75\times 10^9}{\sqrt{\epsilon_r}}$$

$\therefore$ The range of frequencies over which single mode operation will occur is

$$\frac{2.5}{\sqrt{\epsilon_r}}\text{ GHz} < f < \frac{3.75}{\sqrt{\epsilon_r}}\text{ GHz}$$

26. (a)

The time average power is given by,

$$P = \frac{E^2}{2\eta}$$

Where,

$$\eta = 120\pi\sqrt{\frac{\pi^2}{80}} = \frac{120\pi^2}{\sqrt{80}} = 132.414$$

$\therefore$

$$P = \frac{(15)^2}{2\times 132.414} = 0.849\ \text{W/m}^2 \approx 0.85\ \text{W/m}^2$$

27. (c)

Given: $P_{\text{rad}} = 100$ W ; $f = 100$ MHz

$\therefore$

$$\lambda = \frac{3\times 10^8}{f} = \frac{3\times 10^8}{10^8} = 3\text{ m}$$

Radiation resistance,

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 = 80\pi^2\left(\frac{0.01}{3}\right)^2$$

$$R_r = 8.77\times 10^{-3}\ \Omega$$

Also,

$$R_{\text{rad}} = \frac{2P_{\text{rad}}}{I^2}$$

$\therefore$

$$I = \sqrt{\frac{2P_{\text{rad}}}{R_{\text{rad}}}} = \sqrt{\frac{2\times 100}{8.77\times 10^{-3}}} = 151.01\text{ A}$$

28. (b)

For short circuited transmission line,

$$Z_{in} = jZ_0 \tan\beta l$$
$$j60 = j35 \tan\beta l$$
$$\tan\beta l = 1.714$$

or $$\beta l = (59.743)^\circ$$

$\because$ $$\beta = \frac{2\pi}{\lambda} = \frac{2\pi\times 1\times 10^6}{3\times 10^8}$$

$$\frac{2\pi\times 1\times 10^6}{3\times 10^8}\times l = 59.743^\circ \times \frac{\pi}{180^\circ}$$
$$l = 49.785 \text{ m}$$

29. (a)

For dominant mode, $$f_c = \frac{c}{2a} = \frac{3\times 10^{10}}{2\times 4} = 3.75 \text{ GHz}$$

and $$\eta_{TE} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{3.75}{10}\right)^2}} = 406.7\ \Omega$$

$\therefore$ $$P_{avg} = \frac{E_0^2 ab}{4\eta_{TE}} = \frac{(65)^2\times 4\times 2\times 10^{-4}}{4\times 406.7} = 2.078 \text{ mW}$$

30. (c)

$$\text{Electrical length} = \beta l = 2\pi f\sqrt{LC}\times l$$
$$92^\circ\times\frac{\pi}{180^\circ} = 2\times 40\times 10^6\times 20\times 10^{-2}\sqrt{L\times 20\times 10^{-12}}$$
$$\sqrt{L\times 20\times 10^{-12}} = \frac{92^\circ}{16\times 10^6\times 180^\circ} = 3.194\times 10^{-8}$$

On solving the above equation, we get,

or, $$L = \frac{(3.194\times 10^{-8})^2}{20\times 10^{-12}}$$
$$L = 51.008\ \mu\text{H/m}$$

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