S.No. : 07 LS1_EE_S_240719
Engineering Mathematics

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CLASS TEST 2019-2020

ELECTRICAL ENGINEERING

Date of Test : 24/07/2019

ANSWER KEY ➤ Engineering Mathematics

1. (d)	7. (c)	13. (b)	19. (b)	25. (c)
2. (b)	8. (a)	14. (d)	20. (c)	26. (d)
3. (d)	9. (d)	15. (c)	21. (c)	27. (b)
4. (d)	10. (b)	16. (d)	22. (d)	28. (a)
5. (a)	11. (c)	17. (a)	23. (b)	29. (a)
6. (d)	12. (c)	18. (b)	24. (d)	30. (a)

DETAILED EXPLANATIONS

1. (d)

$$\text{rank of } [AB] \leq \text{rank of } [A]$$
$$\text{rank of } [AB] \leq \text{rank of } [B]$$
$$\text{rank of } [AB] \leq \min[\text{rank of } A, \text{rank of } B]$$

2. (b)

eigen values of $(A + 5I)$ are $\alpha + 5$ and $\beta + 5$

eigen values of $(A + 5I)^{-1} = \frac{1}{\alpha+5}$ and $\frac{1}{\beta+5}$

3. (d)

Let $$I = \lim_{t\to 0} \frac{1}{t} \cdot \int_0^t f(x)dx$$

By applying L-Hospital's rule

$$I = \lim_{t\to 0} \frac{f(t)}{1}$$
$$= f(0)$$

4. (d)

(a) If A and B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

(b) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(c) If A and B are mutually exclusive

$$P(A \cap B) = 0$$

(d) $$P(B) = \frac{P(A \cap B)}{P(A)} \leq 1$$

$$P(A \cap B) \leq P(A)$$

5. (a)

Auxillary equation

$$D^4 + 2D^2 + 1 = 0$$
$$(D^2 + 1)^2 = 0$$
$$D = \pm i, \ \pm i$$
$$CF = (C_1 + C_2 x)\cos x + (C_3 + C_4 x)\sin x$$

6. (d)

Given function is $$y = \frac{1}{x}$$ [hyperbolic function]

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

hence, option (d) is correct.

7. (c)

Since $\lim_{x\to 0}(1+0)^{\frac{1}{0}} = 1^{\infty} \rightarrow$ indeterminent

$$f(0) = \lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}$$

$$\ln f(0) = \lim_{x\to 0}\frac{\ln(1+\sin x)}{x}$$

$$= \lim_{x\to 0}\frac{\ln(1+\sin x)}{x} \quad \text{(Apply L' Hospital rule)}$$

$$\ln f(0) = \lim_{x\to 0}\frac{\cos x}{1+\sin x}\times\frac{1}{1}$$

$$= \frac{1}{1+0} = 1$$

$$\ln f(0) = 1$$
$$f(0) = e^1 = e$$

8. (a)

minimum (2, 3) $\Rightarrow$ highest possible rank = 2

if rank of A = 2, it will consistent. In order to be inconsistent, maximum rank of A is '1'.

9. (d)

$$I = \int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx$$

$$I = \int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$2I = \int_0^{\frac{\pi}{2}}\frac{\sin x+\cos x}{\sin x+\cos x}dx$$

$$I = \frac{\pi}{4}$$

10. (b)

$$\frac{\ln(1+i\sqrt{3})}{2} = \frac{\ln\left(2e^{\frac{i\pi}{3}}\right)}{2} = \frac{\ln 2+\frac{i\pi}{3}}{2}$$

$$= \frac{\ln 2}{2}+\frac{i\pi}{6} = \frac{0.693}{2}+\frac{i\pi}{6} = 0.35+\frac{i\pi}{6}$$

11. (c)

$$\frac{dy}{dx} - y\cos x = \sin x \cos x$$

$$\text{IF} = e^{-\int \cos x\, dx} = e^{-\sin x}$$

$$ye^{-\sin x} = \int \sin x \cos x\, e^{-\sin x}\, dx$$

$$ye^{-\sin x} = -(1+\sin x)e^{-\sin x} + C_0$$

$$y + 1 + \sin x = C_0 e^{\sin x}$$

12. (c)

For $f(x)$ to be probability density function $\Rightarrow \int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\frac{1}{A}\int_2^4 (2x+3)\, dx = 1$$

$$\frac{1}{A}\left[2\frac{x^2}{2} + 3x\right]_2^4 = 1$$

$$A = (4^2 - 2^2) + 3(4-2)$$

$$= 16 - 4 + 3 \times 2 = 18$$

13. (b)

Let $\tan^{-1}(x) = \theta$, $\quad x = \tan\theta$

$$g(x) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \frac{\theta}{2}$$

$$\frac{df(x)}{dg(x)} = \frac{d\theta}{d(\theta/2)} = 2$$

14. (d)

$$A^{-1} = \frac{(adj\, A)}{|A|}$$

$$|A| = -6 \times 3 = -18$$

$$|A| \cdot (A^{-1}) = (\text{adj}\, A)$$

$$\lambda \text{ of adj}\, A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-18}{-6}, \frac{-18}{3}$$

$$= 3, -6$$

15. (c)

Here $$u = e^{xyz}$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = e^{xyz} \cdot yz$$

$$\frac{\partial^2 u}{\partial x \partial y} = ze^{xyz} + yze^{xyz} \cdot xz$$

$$= e^{xyz}(z + xyz^2)$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}(1 + 3xyz + x^2y^2z^2)$$

16. (d)

$$f(z) = u + iv$$

$$|f(z)| = \sqrt{u^2 + v^2} = c \qquad \text{(constant given)}$$

$$u^2 + v^2 = c_1 \qquad \text{...(i)}$$

differentiating equation (i) with respect to x

$$2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x} = 0 \qquad \begin{bmatrix} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{bmatrix}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = 0 \qquad \text{...(ii)}$$

differentiating equation (i) with respect to y

similarly $$u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} = 0$$

$$-u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x} = 0 \qquad \text{...(iii)}$$

squaring and adding equation (ii) and (iii)

$$(u^2 + v^2)\left(\frac{\partial u}{\partial x}\right)^2 + (u^2 + v^2)\left(\frac{\partial v}{\partial x}\right)^2 = 0$$

$$(u^2 + v^2)\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right) = 0$$

since $$u^2 + v^2 \neq 0$$

$$\Rightarrow \quad \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 = 0$$

similarly $$\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 = 0$$

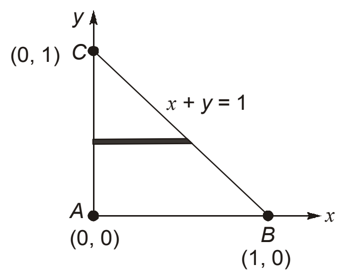
$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\overline{f'(z)} = \frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$$

$$|f'(z)|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 = 0$$

$$|f'(z)|^2 = 0$$

17. (a)



$$I = \int_0^1 \int_0^{1-x} 3y\,dy\,dx$$

$$= \int_0^1 \left[\frac{3y^2}{2}\right]_0^{1-x} dx$$

$$= \int_0^1 \frac{3}{2}(1-x)^2 dx$$

$$= -\frac{3}{2}\frac{(1-x)^3}{3}\Bigg|_0^1 = -\frac{1}{2}\frac{(1-x)^3}{1}\Bigg|_0^1 = \frac{1}{2}$$

18. (b)

$$M = (1 + xy)y$$

and $$N = (1 + xy)x$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4xy \neq 0$$

$$Mx - Ny = 2x^2 y^2 \neq 0$$

$$\frac{1}{Mx - Ny} = \frac{1}{2x^2y^2}$$

$$\text{IF (Integrating factor)} = \frac{1}{2x^2y^2}$$

$$\left(\frac{1}{x^2y} + \frac{1}{x}\right)dx + \left(\frac{1}{xy^2} - \frac{1}{y}\right)dy = 0 \qquad \text{...(i)}$$

equation (i) is exact

$$\frac{1}{y}\int \frac{1}{x^2}dx + \int \frac{1}{x}dx - \int \frac{1}{y}dy = C$$

$$\frac{-1}{xy} + \log x - \log y = C$$

or $$\log\left(\frac{x}{y}\right) - \frac{1}{xy} = C$$

19. (b)

$$Z = \frac{(2-3i)\overline{(1+i)}}{2+i} = \frac{(2-3i)(1-i)}{2+i} = \frac{-1-5i}{2+i}$$

$$= \frac{(-1-5i)(2-i)}{(2+i)(2-i)} = \frac{-7-9i}{5} = -\frac{7}{5}-\frac{9}{5}i$$

$$|Z| = \sqrt{\left(-\frac{7}{5}\right)^2+\left(-\frac{9}{5}\right)^2} = \sqrt{\frac{130}{25}} = \sqrt{\frac{26}{5}}$$

20. (c)

$$(D^2+1)y = \sin x$$

$$PI = \frac{\sin x}{D^2+1}$$

putting $D^2 = -1$

$$PI = \frac{\sin x}{-1+1} \qquad \text{[Makes denominator zero]}$$

$\therefore$ Differentiating numerator and denominator

$$PI = x\cdot\frac{\sin x}{2D}$$

$$PI = \frac{1}{2}x\int \sin x\,dx$$

$$PI = -\frac{1}{2}x\cos x$$

21. (c)

$$\text{Re}\,z = \frac{z+\bar{z}}{2}$$

$$\text{Re}\{\tan^{-1}(z)\} = \frac{\tan^{-1}(x+iy)+\tan^{-1}(x-iy)}{2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x+iy+x-iy}{1-(x+iy)(x-iy)}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2x}{1-x^2-y^2}\right)$$

22. (d)

$$\vec{A}\times\vec{B}\times\vec{C} = (\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$$

$$\therefore \qquad \nabla\times\nabla\times\vec{A} = (\nabla\cdot\vec{A})\nabla-(\nabla\cdot\nabla)\vec{A} = \nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$$

23. (b)

$$x = (N)^{1/N}$$
$$x^N = N$$
$$x^N - N = 0$$
$$f(x) = x^N - N$$
$$f'(x) = N\cdot x^{N-1}$$

Newton Raphson Iteration

$$x_{k+1} = x_k - \frac{f(k)}{f'(k)} = x_k - \frac{x_k^N - N}{N x_k^{N-1}} = \frac{N x_k^N - x_k^N + N}{N x_k^{N-1}}$$

$$x_{k+1} = \left(\frac{N-1}{N}\right) x_k + x_k^{1-N}$$

24. (d)

$$[A]^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$$

$$\alpha^2 = 1 \quad ; \quad \alpha + 1 = 5$$
$$\alpha = \pm 1 \quad ; \quad \alpha = 4$$

Unique value of α is not possible.

25. (c)

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} \cdot \hat{i} = x$, $\quad \vec{r} \cdot \hat{j} = y$, $\quad \vec{r} \cdot \hat{k} = z$

$$A = x(\vec{r} \times \hat{i}) + y(\vec{r} \times \hat{j}) + z(\vec{r} \times \hat{k})$$
$$= (\vec{r} \times x\hat{i}) + (\vec{r} \times y\hat{j}) + (\vec{r} \times z\hat{k}) = \vec{r} \times (x\hat{i} + y\hat{j} + z\hat{k}) = \vec{r} \times \vec{r}$$
$$A = 0 \qquad \text{(always)}$$

26. (d)

Rearranging the equation

$$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$$

The equation is of the form

$$\frac{dy}{dx} + p(x)y = Q(x)$$

$$IF = e^{\int p(x)dx} = e^{\int \frac{-1}{x+1}dx} = e^{-[ln(x+1)]}$$
$$= \frac{1}{x+1}$$

27. (b)

By Euler theorem

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

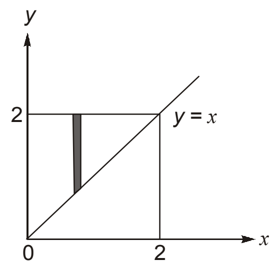
28. (a)

$$f'(x) = 2x - 1$$

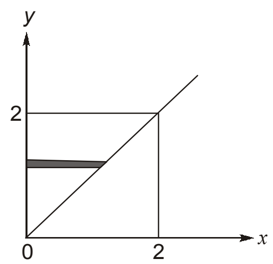
For minima/maxima

$$f'(x) = 0$$
$$2x - 1 = 0$$
$$x = \frac{1}{2}$$
$$f''(x) = 2 > 0 \quad \Rightarrow \quad \text{minima}$$

29. (a)



$x < y < 2$
$0 < x < 2$



$0 < x < y$
$0 < y < 2$

$$I = \int_0^2 \int_0^y f(x, y)\, dx\, dy$$

$$r = p = 0$$
$$q = y$$
$$s = 2$$

30. (a)

Since the probability of occurrence is very small, this follows Poisson distribution

$$\text{mean} = m = np$$
$$= 2000 \times 0.001 = 2$$

Probability that more than 2 will get a bad reaction

$$= 1 - p(0) - p(1) - p(2)$$
$$= 1 - \left[e^{-m} + \frac{e^{-m} \cdot m^1}{1!} + \frac{e^{-m} \cdot m^2}{2!} \right]$$
$$= 1 - \left[e^{-2} + \frac{e^{-2} \cdot 2}{1} + \frac{2^2 \cdot e^{-2}}{2} \right] = 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2} \right] = 1 - \frac{5}{e^2}$$

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