

## Q.No. 1 to Q.No. 10 carry 1 mark each

Q. 1 A thin cylindrical steel shell of diameter 150 mm and wall thickness 3 mm has hemispherical ends. If there is no distortion of the junction under pressure, then the thickness of the hemispherical end will be
(a) 7.29 mm
(b) 1.235 mm
(c) 2.11 mm
(d) 3 mm
Q. 2 Two wooden members of uniform rectangular cross-section are joined by the simple glued scarf splice as shown. For the axial load $P$ of 11 kN , the normal stress in the glued splice is

(a) 0.977 MPa
(b) 0.488 MPa
(c) 0.325 MPa
(d) 0.628 MPa
Q. 3 The magnitude of slope at $x=\frac{l}{3}$ for a cantilever beam of length ' $l$ ' and fluxural rigidity $E I$ is

(a) $\frac{3}{8} \frac{P l^{2}}{E I}$
(b) $\frac{2}{9} \frac{P l^{2}}{E I}$
(c) $\frac{4}{9} \frac{P l^{2}}{E I}$
(d) $\frac{9}{2} \frac{P l^{2}}{E I}$
Q. 4 Mohr's circle drawn for a point has centre at $(4 \mathrm{MPa}, 0)$ and minimum principal stress as 4 MPa compressive. The magnitude of maximum principal stress is
(a) 4 MPa
(b) 8 MPa
(c) 10 MPa
(d) 12 MPa
Q. 5 The ratio of bulk modulus to shear modulus for a rod with poisson ratio 0.25 is
(a) 1.2
(b) 0.62
(c) 1.67
(d) 1.53
Q. 6 The shear force diagram for a simply supported beam with uniformly distributed load of $2 \mathrm{kN} / \mathrm{m}$ over whole length and a point load $P$ is shown. The reaction force generated at support $A$ is

(a) 3 kN
(b) 9 kN
(c) 6 kN
(d) 4.5 kN
Q. 7 The maximum bending moment produced in the cantilever beam shown below is

(a) 48.66 kNm
(b) 51.31 kNm
(c) 53.33 kNm
(d) 60.16 kNm
Q. 8 A bar of rectangular cross-section ( $E=200$ GPa) having thickness 6 mm , width 18 mm and length 2 m is bent by a couple into circular arc subtending an angle $42^{\circ}$ at centre. The magnitude of bending couple applied is

(a) 23.76 Nm
(b) 31.88 Nm
(c) 46.84 Nm
(d) 51.31 Nm
Q. 9 Match List-I (Point on beam) with List-II (State of stress) and select the correct answer using the codes given below the lists:


## List-I

(A) Point-A
(B) Point-B
(C) Point-C
(D) Point-D

List-II

2.

3.

4.

5.


## Codes:

|  | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| (a) | 1 | 2 | 3 | 4 |
| (b) | 1 | 4 | 3 | 2 |
| (c) | 5 | 2 | 3 | 4 |
| (d) | 5 | 4 | 3 | 2 |

Q. 10 A straight bar of rectangular cross section ( $12 \mathrm{~mm} \times 6 \mathrm{~mm}$ ) is used as a strut with both ends fixed. If slenderness ratio of strut is 100, then effective length of the strut is
(a) 86.6 mm
(b) 173.21 mm
(c) 69.41 mm
(d) 346.41 mm

## Q. No. 11 to Q. No. 30 carry 2 marks each

Q. 11 A simply supported beam of length ' $l$ ' carries a point load $P$ at its centre. If beam is made of rectangular cross-section of width $b$ and depth $d$, then the shear stress at distance of $\frac{d}{6}$ from centroidal axis is
(a) $\frac{3 P}{4 b d}$
(b) $\frac{4 P}{3 b d}$
(c) $\frac{2 P}{3 b d}$
(d) $\frac{3 P}{2 b d}$
Q. 12 A load of 500 kN falls from a height of 12 mm on a rectangular bar of $50 \mathrm{~cm}^{2}$ crosssectional. area. The length of bar is 1 m and Young's modulus of elasticity is 200 GPa. If now the same load is placed gradually on the bar, then the ratio of deflection when load is fallen from height to when it is placed gradually is
(a) 6
(b) 7
(c) 8
(d) 9
Q. 13 A propped cantilever beam has internal hinge at centre as shown below. The central deflection of beam under given loading is

(a) $\frac{w a^{4}}{8 E I}+\frac{W a^{3}}{9 E I}$
(b) $\frac{w a^{4}}{8 E I}-\frac{W a^{3}}{9 E I}$
(c) $\frac{w a^{4}}{8 E I}+\frac{2 W a^{3}}{9 E I}$
(d) $\frac{w a^{4}}{8 E I}-\frac{2 W a^{3}}{9 E I}$
Q. 14 A bar made of a material with Young's modulus $E$ is subjected to a load $P$ as shown below. The percentage strain energy stored by portion $B C$ of the bar is

(a) $40 \%$
(b) $80 \%$
(c) $30 \%$
(d) $60 \%$
Q. 15 A rod of diameter ' $d$ ', length ' $l$ ' and shear modules $G$ is assembled in a tube of outer diameter $2 d$, inner diameter $d$, length ' $l$ ' and shear modulus $G / 3$. If this assembly is subjected to an external torque ' $T$ ', then angle of twist for the tube is
(a) $\frac{16 \mathrm{Tl}}{3 \pi G d^{4}}$
(b) $\frac{16 \mathrm{Tl}}{45 \pi G d^{4}}$
(c) $\frac{80 \mathrm{Tl}}{3 \pi G d^{4}}$
(d) $\frac{80 \mathrm{Tl}}{45 \pi G d^{4}}$
Q. 16 A bar $A B$ is heated non uniformly such that temperature increase at a distance $x$ from $A$ is given by $\Delta T(x)=\Delta T_{0}\left(1-\frac{x^{2}}{L^{2}}\right)$. If stiffness of spring is $K$ and cross-sectional area of bar is $A$, then the reaction generated at support $A$ is

(a) $\frac{2 \alpha \Delta T_{0} L K A E}{K L+A E}$
(b) $\frac{2}{3} \frac{\alpha \Delta T_{0} L K A E}{K L+A E}$
(c) $\frac{\alpha \Delta T_{0} L K A E}{3(K L+A E)}$
(d) $\frac{\alpha}{2} \frac{\alpha \Delta T_{0} L K A E}{(K L+A E)}$
Q. 17 A post $A B$ is tapered uniformly throughout its height $H$. The cross-section of post is square with dimensions $b \times b$ at top and $2 b \times 2 b$ at bottom. The expression for deflection of the post due to compressive load $P$, acting at its top is [Neglect self weight of post]
(a) $\frac{2}{3} \frac{P H}{b^{2} E}$
(b) $\frac{P H}{3 b^{2} E}$
(c) $\frac{P H}{2 b^{2} E}$
(d) $\frac{2 P H}{b^{2} E}$
Q. 18 A rubber cylinder of length $l$ and crosssectional area $A$ is compressed inside a steel cylinder by force $F$. If the deflection in rubber is $\alpha \frac{F l}{A E}$, then $\alpha$ is [Take Poisson's ratio as $\mu$ ]

(a) $\frac{\mu^{2}-\mu+1}{1-2 \mu}$
(b) $\frac{\mu^{2}-2 \mu+2}{1-2 \mu}$
(c) $\frac{2 \mu^{2}-2 \mu-1}{1-\mu}$
(d) $\frac{2 \mu^{2}+\mu-1}{1-\mu}$
Q. 19 A solid circular shaft of length $2 l$ consist of two segments. One segment has diameter ' $d$ ' and length $\frac{2 l}{3}$ while other segment has diameter $\frac{d}{2}$. If whole shaft is subjected to torque $T_{0}$, then the strain energy stored by shaft is

(a) $112 \frac{T_{0}^{2} l}{G d^{4}}$
(b) $118 \frac{T_{0}^{2} l}{G d^{4}}$
(c) $104 \frac{T_{0}^{2} l}{G d^{4}}$
(d) $124 \frac{T_{0}^{2} l}{G d^{4}}$
Q. 20 A composite bar made up of copper having modulus of elasticity $E$ is hanging freely under its self weight as shown in figure.


If the self weight of $A B$ and $A C$ is $W$ and $3 W$ respectively and cross-sectional area of section $B C$ is twice of the cross-sectional area of $A B$. The displacement of point ' $C$ ' is
(a) $\frac{5}{2} \frac{W L}{A E}$
(b) $\frac{3 W L}{A E}$
(c) $\frac{5 W L}{A E}$
(d) $\frac{7}{2} \frac{W L}{A E}$
Q. 21 A normal stress is applied to a square plate of thickness 20 mm as shown in figure, which leads to normal strain $\varepsilon_{x}=50 \times 10^{-5}$ and $\varepsilon_{y}=15 \times 10^{-5}$. If the modulus of elasticity and Poisson's ratio are 209 GPa and 0.3 respectively, the decrease in thickness of plate is

(a) $5.575 \times 10^{-3} \mathrm{~mm}$
(b) $2.391 \times 10^{-3} \mathrm{~mm}$
(c) $6.811 \times 10^{-3} \mathrm{~mm}$
(d) $4.552 \times 10^{-3} \mathrm{~mm}$
Q. 22 Mohr's circle for the state of plane stress at a point is shown in the figure. Unit of stress is MPa and the circle is not drawn to scale.

Which one of the following options (stress values in MPa ) is true?

(a) $\sigma_{A}=-50, \sigma_{B}=10, \sigma_{1}=30, \sigma_{2}=-70$
(b) $\sigma_{A}=-50, \sigma_{B}=30, \sigma_{1}=30, \sigma_{2}=-50$
(c) $\sigma_{A}=-30, \sigma_{B}=30, \sigma_{1}=30, \sigma_{2}=-10$
(d) $\sigma_{A}=-20, \sigma_{B}=10, \sigma_{1}=50, \sigma_{2}=-30$
Q. 23 Assuming Young's Modulus ( $E$ ) = 160 GPa and Shear Modulus $(G)=100$ GPa for a material, a strain tensor is given as

$$
\left[\begin{array}{ccc}
0.002 & 0.004 & 0.006 \\
0.004 & 0.003 & 0 \\
0.006 & 0 & 0
\end{array}\right]
$$

Then the value of $\tau_{x y}+\tau_{x z}$ is
(a) 1000 MPa
(b) 1600 MPa
(c) 2000 MPa
(d) 2400 MPa
Q. 24 The vertical displacement at point $A$ for the curved beam shown below, will be

(a) $\frac{\pi P R^{3}}{3 E I}$
(b) $\frac{\pi P R^{3}}{4 E I}$
(c) $\frac{\pi P R^{3}}{E I}$
(d) $\frac{\pi P R^{3}}{2 E I}$
Q. 25 An overhang beam $A C$ is loaded as shown in the figure. The location of point of contraflexure from support $A$ is

(a) 1 m
(b) 1.5 m
(c) 2 m
(d) 3 m
Q. 26 A cantilever beam of length 1.5 m is loaded by a point load $P$ at its free end as shown in figure. It has a circular cross section with two symmetrically placed longitudinal holes. The permissible bending stress is 600 MPa. The maximum value of load $P$ that the beam can carry if radius of each hole is $100 / 3 \mathrm{~mm}$

(a) 230.5 kN
(b) 236.8 kN
(c) 242.6 kN
(d) 248.1 kN
Q. 27 The state of stress at a point is shown below. The maximum normal stress induced at that point is

(a) 150 MPa
(b) 190 MPa
(c) 210 MPa
(d) 230 MPa
Q. 28 Two columns are required to have same buckling load $P_{c r}$. The column I has flexural rigidity $E I$ and height $h_{1}$ while column II has flexural rigidity $\left(\frac{4}{3} E I\right)$ and height $h_{2}$. The ratio of $\frac{h_{2}}{h_{1}}$ at which both column will buckle under same load is

(a) 0.76
(b) 0.82
(c) 0.87
(d) 0.93
Q.29 A shaft of diameter 70 mm and length 1.5 m is subjected to a twisting moment. If angle of twist at free end is $1.8^{\circ}$, then shear strain produced in shaft is
(a) $8.55 \times 10^{-4}$
(b) $6.93 \times 10^{-4}$
(c) $9.81 \times 10^{-4}$
(d) $7.33 \times 10^{-4}$
Q. 30 The maximum bending moment produced in the cantilever beam is

(a) $\frac{q l^{2}}{30}$
(b) $\frac{11}{15} q l^{2}$
(c) $\frac{12}{30} q l^{2}$
(d) $\frac{4}{15} q l^{2}$


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## STRENGTH OF MATERIALS

## MECHANICAL ENGINEERING

## Date of Test : 08/05/2023

## ANSWER KEY

| 1. (b) | 7. (c) | 13. (c) | 19. (a) | 25. (d) |
| :---: | :---: | :---: | :---: | :---: |
| 2. (b) | 8. (a) | 14. (b) | 20. (b) | 26. (b) |
| 3. (c) | 9. (d) | 15. (a) | 21. (a) | 27. (c) |
| 4. (d) | 10. (b) | 16. (b) | 22. (a) | 28. (b) |
| 5. (c) | 11. (c) | 17. (c) | 23. (c) | 29. (d) |
| 6. (d) | 12. (c) | 18. (d) | 24. (b) | 30. (d) |

## DETAILED EXPLANATIONS

1. (b)

Thickness of cylindrical portion, $t_{1}=3 \mathrm{~mm}$
Thickness of hemispherical ends $=t_{2}$
For no distortion of the junction under pressure,

$$
\begin{aligned}
\frac{t_{2}}{t_{1}} & =\frac{1-\mu}{2-\mu}=\frac{1-0.3}{2-0.3} \\
t_{2} & =1.235 \mathrm{~mm}
\end{aligned}
$$

2. (b)

Given: $P=11 \mathrm{kN}, A=150 \times 75 \mathrm{~mm}^{2}$

$$
\begin{aligned}
\sigma_{x} & =\frac{P}{A}=\frac{11000}{150 \times 75}=0.977 \mathrm{MPa} \\
\left(\sigma_{n}\right)_{\theta=45^{\circ}} & =\frac{\sigma_{x}}{2}+\frac{\sigma_{x}}{2} \cos 2 \theta \\
& =\frac{\sigma_{x}}{2}=\frac{0.977}{2}=0.488 \mathrm{MPa}
\end{aligned}
$$


3. (c)


At $=\frac{2 l}{3}$,

$$
y=\frac{P l}{2}+\frac{2 P l}{3}=\frac{7 P l}{6}
$$

$$
\theta_{x}-\theta_{0}=\text { Area of } \frac{M}{E I} \text { diagram }
$$

$$
=\frac{1}{2}\left(\frac{3 P l}{2}+\frac{7 P l}{6}\right) \frac{l}{3 E I}
$$

$$
=\frac{4}{9} \frac{P l^{2}}{E I}
$$

4. (d)


Radius of Mohr circle $=4-(-4)=8 \mathrm{MPa}$
Maximum principal stress $=4+8=12 \mathrm{MPa}$
5. (c)

$$
\begin{aligned}
\frac{K}{G} & =\frac{2(1+\mu)}{3(1-2 \mu)} \\
& =\frac{2(1.25)}{3(0.5)}=1.67
\end{aligned}
$$

6. (d)

$$
\begin{aligned}
R_{A}+R_{B} & =P+w l \\
R_{A}+6 & =P+6 \\
R_{A} & =P
\end{aligned}
$$

$\Sigma M_{A}=0$

$$
\begin{array}{rlrl} 
& & 6 \times 3 & =P \times 2+2 \times \frac{3^{2}}{2} \\
\Rightarrow & 18 & =2 P+9 \\
\therefore & P & =4.5 \mathrm{kN} \\
\therefore & R_{A} & =4.5 \mathrm{kN}
\end{array}
$$

7. (c)

$$
\begin{aligned}
w & =5 \mathrm{kN} / \mathrm{m}, q=(10-5)=5 \mathrm{kN} / \mathrm{m} \\
\text { B.M. } & =\frac{w l^{2}}{2}+\frac{q l^{2}}{6} \\
& =\frac{5 \times 4^{2}}{2}+\frac{5 \times 4^{2}}{6}=53.33 \mathrm{kNm}
\end{aligned}
$$

8. (a)

$$
\begin{aligned}
\frac{M}{I} & =\frac{E}{R} \\
M & =\frac{200 \times 10^{3}}{2000} \times 42 \times \frac{\pi}{180} \times \frac{18 \times 6^{3}}{12} \\
& =23760 \mathrm{Nmm} \simeq 23.76 \mathrm{~N} . \mathrm{m}
\end{aligned}
$$

9. (d)
10. (b)

Minimum moment of inertia $=\frac{12 \times 6^{3}}{12}=216 \mathrm{~mm}^{4}$

$$
\begin{aligned}
\text { Radius of gyration } & =\sqrt{\frac{I}{A}}=\sqrt{\frac{216}{12 \times 6}}=\sqrt{3} \\
\text { Slenderness ratio } & =\frac{L_{e}}{k}
\end{aligned}
$$

$$
L_{e}=100 \times \sqrt{3}=173.21 \mathrm{~mm}
$$

11. (c)


Maximum shear force in beam $=\frac{P}{2}$

$$
\text { Shear stress, } \tau=\frac{F A \bar{y}}{I b}=\frac{\frac{P}{2} \times \frac{b d}{3} \times \frac{d}{3}}{\frac{b d^{3}}{12} \times b}=\frac{2 P}{3 b d}
$$

12. (c)

$$
\begin{aligned}
\delta_{\text {impact }} & =\delta_{s t}\left(1+\sqrt{1+\frac{2 h}{\delta_{s t}}}\right) \\
\delta_{\text {st }} & =\frac{P l}{A E}=\frac{500 \times 10^{3} \times 1000}{50 \times 10^{2} \times 200 \times 10^{3}}=0.5 \mathrm{~mm} \\
\frac{\delta_{\text {impact }}}{\delta_{\text {gradually }}} & =\left(1+\sqrt{1+\frac{2 \times 12}{0.5}}\right)=8
\end{aligned}
$$

13. (c)


$$
\delta_{\text {centre }}=\frac{w a^{4}}{8 E I}+\frac{2 W a^{3}}{9 E I}
$$

14. (b)

Strain energy stored in $A B, C D$

$$
=\frac{P^{2} L}{2 \frac{\pi}{4} D^{2} E}
$$

Strain energy stored in $\mathrm{BC}=\frac{P^{2}(2 L)}{2 \frac{\pi}{4} \frac{D^{2}}{4} E}=\frac{8 P^{2} L}{2 \frac{\pi}{4} \times D^{2} E}$

$$
\begin{gathered}
\text { \% Strain energy stored in } \mathrm{BC}=\frac{\frac{8 P^{2} L}{2 \frac{\pi}{4} D^{2} E}}{\frac{P^{2} L}{2 \frac{\pi}{4} D^{2} E}+\frac{8 P^{2} L}{2 \frac{\pi}{4} D^{2} E}+\frac{P^{2} L}{2 \frac{\pi}{4} D^{2} E}}=\frac{8}{1+8+1}=\frac{8}{10} \\
=80 \%
\end{gathered}
$$

15. (a)

As rod and tube are in parallel, so angle of twist will be same

$$
\begin{aligned}
\frac{T_{R} l}{G \frac{\pi}{32} d^{4}} & =\frac{T_{T} l}{\frac{G}{3} \frac{\pi}{32}\left((2 d)^{4}-d^{4}\right)} \\
5 T_{R} & =T_{T} \\
T_{R}+T_{T} & =T \\
T_{R}+5 T_{R} & =T \\
T_{R} & =\frac{T}{6} \\
\theta_{R} & =\theta_{T}=\frac{T}{6} \times \frac{32 l}{G \pi d^{4}}=\frac{16 T l}{3 \pi G d^{4}}
\end{aligned}
$$

16. (b)

Change of length of bar $=$ Compression of spring

$$
\begin{aligned}
\int_{0}^{L} \alpha \Delta T(x) d x-\frac{R_{A} L}{A E} & =\frac{R_{A}}{K} \\
\alpha \Delta T_{0}\left(x-\frac{x^{3}}{3 L^{2}}\right)_{0}^{L} & =\frac{R_{A} L}{A E}+\frac{R_{A}}{K} \\
\frac{2}{3} \alpha \Delta T_{0} L & =R_{A}\left(\frac{L}{A E}+\frac{1}{K}\right) \\
R_{A} & =\frac{2}{3} \frac{\alpha \Delta T_{0} L K A E}{(K L+A E)}
\end{aligned}
$$

17. (c)


$$
\delta=\int_{0}^{H} \frac{P d x}{A E}=\int_{0}^{H} \frac{P d x}{\left(b+\frac{b x}{H}\right)^{2} E}
$$

Let

$$
\begin{aligned}
b+\frac{b x}{H} & =t \\
\frac{d t}{d x} & =\frac{b}{H} \\
d x & =d t\left(\frac{H}{b}\right)
\end{aligned}
$$

when $x=0, t=b ; x=H, t=2 b$

$$
\begin{aligned}
& \delta=\frac{H}{b} \int_{b}^{2 b} \frac{P}{t^{2}} \frac{d t}{E}=\frac{H P}{b E} \times\left(-\frac{1}{t}\right)_{b}^{2 b} \\
& \delta=\frac{H P}{E b}\left(\frac{1}{b}-\frac{1}{2 b}\right)=\frac{H P \times 1}{E b \times 2 b} \\
& \delta=\frac{H P}{2 b^{2} E}
\end{aligned}
$$

18. (d)

$$
\begin{aligned}
\sigma_{x} & =-\sigma \\
\sigma_{z} & =-\sigma \\
\varepsilon_{x} & =0=\frac{\sigma_{x}}{E}-\frac{\mu}{E}\left(\sigma_{y}+\sigma_{z}\right) \\
& =-\sigma-\mu\left(\frac{-F}{A}-\sigma\right) \\
\sigma & =+\mu \frac{F}{A}+\mu \sigma=\frac{\mu}{1-\mu} \frac{F}{A} \\
\varepsilon_{y} & =-\frac{F}{A E}-\frac{\mu}{E}(-2 \sigma)
\end{aligned}
$$

$$
\begin{aligned}
& \varepsilon_{y}=-\frac{F}{A E}+\frac{\mu}{E}\left(\frac{2 \mu}{1-\mu} \frac{F}{A}\right) \\
& \varepsilon_{y}=\frac{F}{A E}\left(\frac{2 \mu^{2}}{1-\mu}-1\right) \\
& \varepsilon_{y}=\frac{F}{A E}\left(\frac{2 \mu^{2}-1+\mu}{1-\mu}\right) \\
& \delta=\frac{F l}{A E}\left(\frac{2 \mu^{2}+\mu-1}{1-\mu}\right)
\end{aligned}
$$

19. (a)

$$
\begin{aligned}
\text { Strain energy } & =\frac{T^{2} l}{2 G J} \\
& =\frac{T_{0}^{2}}{2 G} \times\left(\frac{2 l}{3}\right) \times \frac{32}{\pi\left(d^{4}\right)}+\frac{T_{0}^{2}}{2 G} \times\left(\frac{4 l}{3}\right) \times \frac{32 \times 16}{\pi \times d^{4}} \\
& =\frac{T_{0}^{2} l}{2 G d^{4}}\left[\frac{2}{3} \times \frac{32}{\pi}+\frac{4}{3} \times \frac{32}{\pi} \times 16\right] \\
& =112 \frac{T_{0}^{2} l}{G d^{4}}
\end{aligned}
$$

20. (b)

The displacement of point ' $C$ '

$$
\begin{aligned}
\delta_{C} & =\left(\delta_{A B}\right)_{\text {self weight }}+\left(\delta_{A B}\right)_{\text {weightof } B C}+\left(\delta_{B C}\right)_{\text {Self weight }} \\
& =\left.\frac{W L}{2 A E}\right|_{A B}+\left.\frac{P_{\text {ext }} L}{A E}\right|_{A B}+\left.\frac{W L}{2 A E}\right|_{B C}
\end{aligned}
$$

From given data,

$$
\begin{aligned}
W_{B C} & =W_{A C}-W_{A B} \\
& =3 W-W=2 W \\
W_{A B} & =W \\
\left(P_{\mathrm{ext}}\right)_{B} & =2 W
\end{aligned}
$$

So,

$$
\begin{aligned}
& \delta_{C}=\frac{W L}{2 A E}+\frac{2 W L}{A E}+\frac{(2 W) \times L}{2(2 A) E} \\
& \delta_{C}=\frac{W L}{A E}\left[\frac{2+8+2}{4}\right]=\frac{12 W L}{4 A E} \\
& \delta_{C}=\frac{3 W L}{A E}
\end{aligned}
$$

21. (a)

The applied stress in the direction of thickness of plate,

$$
\sigma_{z}=0
$$

Strain along thickness direction,

$$
\begin{equation*}
\Rightarrow \quad \varepsilon_{z}=\frac{\sigma_{z}}{E}-\frac{\mu}{E}\left(\sigma_{x}+\sigma_{y}\right) \tag{i}
\end{equation*}
$$

As we know that,

$$
\sigma_{1}=\frac{E}{1-\mu^{2}}\left(\varepsilon_{1}+\mu \varepsilon_{2}\right)
$$

So, we can write, $\quad \sigma_{x}=\frac{209 \times 10^{3}}{1-0.3^{2}}\left[50 \times 10^{-5}+0.3 \times 15 \times 10^{-5}\right]$

$$
=125.17 \mathrm{MPa}
$$

Similarly,

$$
\begin{aligned}
& \sigma_{y}=\frac{209 \times 10^{3}}{1-0.3^{2}}\left[15 \times 10^{-5}+0.3 \times 50 \times 10^{-5}\right] \\
& \sigma_{y}=68.9 \mathrm{MPa}
\end{aligned}
$$

From equation (i)

$$
\begin{aligned}
\varepsilon_{z} & =0-\frac{0.3}{209 \times 10^{3}}[125.17+68.9] \\
& =2.785 \times 10^{-4}
\end{aligned}
$$

Reduction in thickness of plate,

$$
\begin{aligned}
\delta t & =t \times \varepsilon_{z} \\
& =20 \times 2.785 \times 10^{-4} \\
& =5.571 \times 10^{-3} \mathrm{~mm}
\end{aligned}
$$

22. (a)

Point $A$ and $B$ on the Mohr's circle represents the complementary planes. So shear stress will be same, i.e. $\tau_{A}=\tau_{B}$
Radius of the Mohr's circle, $C A$

$$
\begin{aligned}
C A & =\sqrt{30^{2}+40^{2}}=50 \\
C B^{\prime} & =\sqrt{50^{2}-40^{2}}=30 \\
O B^{\prime} & =C B^{\prime}-C O=30-20=10 \\
\sigma_{B} & =O B^{\prime}=10 \\
\sigma_{1} & =50-20=30 \\
\sigma_{2} & =-(50+20)=-70 \\
\sigma_{A} & =O A^{\prime}=30+20=50
\end{aligned}
$$


23. (c)

$$
\begin{array}{rlrl} 
& & \\
\text { Strain tensor } & =\left[\begin{array}{ccc}
\epsilon_{x x} & \frac{\gamma_{x y}}{2} & \frac{\gamma_{x z}}{2} \\
\gamma_{y x} & & \epsilon_{y y} \\
\frac{\gamma_{y z}}{2} \\
\frac{\gamma_{z x}}{2} & \frac{\gamma_{z y}}{2} & \epsilon_{z z}
\end{array}\right] \\
\Rightarrow & \gamma_{x y} & =0.004 \times 2=0.008 \\
\Rightarrow & \gamma_{x z} & =0.006 \times 2=0.012 \\
\text { and } & \tau_{x y} & =G \gamma_{x y}=100 \times 0.008=0.8 \mathrm{GPa} \\
\text { So, } & \tau_{x z} & =G \gamma_{x z}=100 \times 0.012=1.2 \mathrm{GPa} \\
\tau_{x y}+\tau_{x z} & =800+1200=2000 \mathrm{MPa}
\end{array}
$$

24. (b)


Now,

$$
\begin{aligned}
\delta_{v} & =\frac{\partial U}{\partial P} \\
\frac{\partial U}{\partial P} & =\int_{0}^{\pi / 2} \frac{M \times\left(\frac{\partial M}{\partial P}\right) R d \theta}{E I} \quad(\text { where } U=\text { strain energy }) \\
& =\frac{P R^{3}}{E I} \int_{0}^{\pi / 2} \cos ^{2} \theta d \theta \\
\delta_{v} & =\frac{\pi P R^{3}}{4 E I}
\end{aligned}
$$

25. (d)

From equilibrium, $\quad \Sigma V=0$,
$\Rightarrow$

$$
R_{A}+R_{B}=2+2 \times 4=10 \mathrm{kN}
$$

$$
\Sigma M=0
$$

$\Rightarrow \quad 2 \times 4 \times 2+2 \times 6=R_{B} \times 4$
So, $\quad R_{B}=\frac{16+12}{4}=\frac{28}{4}=7 \mathrm{kN}$
and
$R_{A}=3 \mathrm{kN}$
Bending moment at a distance ' $x$ ' from end $A$ will be

$$
(\mathrm{BM})_{x}=3 x-2 \times x \times \frac{x}{2}=3 x-x^{2}
$$

Now, the BMD changes sign in section $A B$, so the point of contraflexure is where the BM is zero.
So,

$$
\begin{aligned}
3 x-x^{2} & =0 \\
x & =3 \mathrm{~m}
\end{aligned}
$$

26. (b)

$$
\begin{aligned}
& I=\frac{\pi R^{4}}{4}-2\left[\frac{\pi}{4}\left(\frac{R}{3}\right)^{4}+\pi\left(\frac{R}{3}\right)^{2}\left(\frac{R}{2}\right)^{2}\right] \\
& I=0.592 R^{4}
\end{aligned}
$$

Bending stress will be maximum at point $A$ and $B$.

$$
\begin{array}{ll}
\text { So, } & \sigma_{\max }=\frac{P \times 1.5 \times 0.1}{0.592 \times(0.1)^{4}} \\
\therefore & P=236.8 \mathrm{kN}
\end{array}
$$

27. (c)

$$
\begin{aligned}
\sigma_{1} & =\left(\frac{\sigma_{x}+\sigma_{y}}{2}\right)+\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\left(\tau_{x y}\right)^{2}} \\
& =\frac{200+120}{2}+\sqrt{\left(\frac{200-120}{2}\right)^{2}+30^{2}} \\
& =210 \mathrm{MPa}
\end{aligned}
$$

28. (b)

$$
\begin{aligned}
\text { Column I } & =P_{c r}=\frac{2 \pi^{2} E I}{h_{1}^{2}} \\
\text { Column II } & =P_{c r}=\frac{4 \pi^{2}}{3} \frac{E I}{h_{2}^{2}} \\
\frac{2 \pi^{2} E I}{h_{1}^{2}} & =\frac{4 \pi^{2} E I}{3 h_{2}^{2}} \\
\frac{h_{2}}{h_{1}} & =\sqrt{\frac{2}{3}}=0.82
\end{aligned}
$$

29. (d)

$$
\begin{aligned}
\gamma_{\max } & =r_{\max } \frac{\theta}{l} \\
\gamma & =35 \times \frac{22 \times 1.8}{7 \times 180} \times \frac{1}{1500} \\
& =7.33 \times 10^{-4}
\end{aligned}
$$

30. (d)

$$
\begin{array}{rlrl}
E I \frac{d^{4} y}{d x^{4}} & =\sqrt{\frac{x}{L}} q \\
V_{x} & =E I \frac{d^{3} y}{d x^{3}}=\frac{2 x^{3 / 2}}{3 \sqrt{L}} q+C_{1} \\
x=0 & M_{x} & =E I \frac{d^{2} y}{d x^{2}}=\frac{2}{3} \times \frac{2}{5} \times \frac{x^{5 / 2}}{\sqrt{L}} q+C_{1} x+C_{2} \\
\Rightarrow & V_{x} & =0 \\
x=0 & C_{1} & =0 \\
\Rightarrow & M_{x} & =0 \\
\text { At } x=l & C_{2} & =0 \\
& M_{x} & =\frac{4}{15} q \frac{x^{5 / 2}}{\sqrt{l}} \\
& M_{x} & =M_{\max } \\
& M_{\max } & =\frac{4}{15} q l^{2}
\end{array}
$$

