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EC + EE

SIGNALS AND SYSTEMS**Duration : 1:00 hr.****Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A, B, C, D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

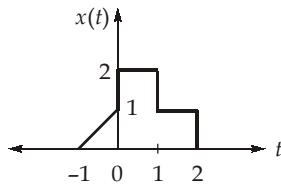
DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

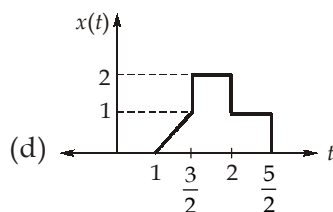
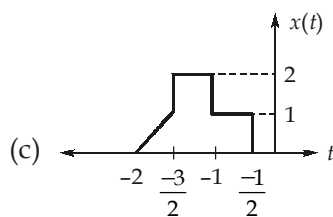
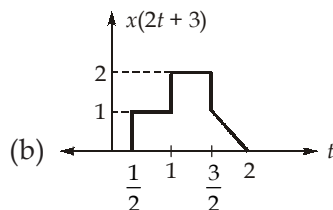
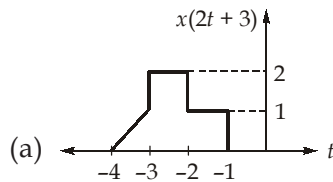
Q.1 Consider a discrete time signal $x(n]$, the energy of even and odd parts of the signal $x(n]$ is 6, 8 respectively. Then the energy of signal $x(n]$ is

- (a) 7 (b) 10
(c) 2 (d) 14

Q.2 For the signal $x(t)$ shown in figure below:



Which of the following signal represents $x(2t + 3)$.



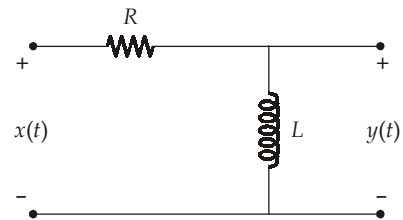
Q.3 Find the Laplace transform of $u(-2t - 1)$ where $u(t)$ is unit step function.

- (a) $\frac{e^{-s/2}}{s}$ (b) $\frac{e^{s/2}}{s}$
(c) $\frac{-e^{-s/2}}{s}$ (d) $\frac{-e^{s/2}}{s}$

Q.4 If the z -transform of a sequence $X(n] = \{1, 1, -1, -1\}$ is $X(z)$, then the value of $X\left(\frac{1}{2}\right)$ is

- (a) -9 (b) 1.875
(c) -1.125 (d) 15

Q.5 An electrical network is shown in the figure below:



Which one of the following statement is true regarding the system considering a causal system?

- (a) The system is BIBO stable.
(b) The system is BIBO unstable.
(c) The system stability depends on values of R and L .
(d) None of these.

Q.6 Let $x(n] = \{3, 4, 5, 6\}$, then $h(n] = x(0.5n - 1)$

will be

- (a) $h(n] = \{4, 6\}$
(b) $h(n] = \{3, 3, 4, 4, 5, 5, 6, 6\}$
(c) $h(n] = \{3, 0, 4, 0, 5, 0, 6, 0\}$
(d) $h(n] = \{4, 4, 6, 6\}$

Q.7 A continuous time signal is defined as, $x(t)$

$$= 4\cos\left(\frac{2\pi}{3}t + 40^\circ\right) + 3\sin\left(\frac{4\pi}{5}t + 20^\circ\right).$$

The fundamental time period of $x(t)$ is

- (a) 30π sec (b) 15π sec
(c) 15 sec (d) 30 sec

Q.8 The Fourier transform of the signal $e^{-(2t-2)}u(t-1)$ is

- (a) $\frac{e^{-j2\omega}}{2+j\omega}$ (b) $\frac{e^{-j\omega}}{2+j\omega}$
(c) $\frac{e^{-j2\omega}}{2+j2\omega}$ (d) $\frac{e^{j\omega}}{2-j\omega}$

Q.9 Two LTI systems with impulse responses $h_1(t) = \delta(t+1)$ and $h_2(t) = e^{-t}u(t)$ are cascaded. The overall impulse response of the system is

- (a) $e^{-t}u(t+1)$ (b) $e^{t+1}u(t+1)$
(c) $e^{-(t+1)}u(t+1)$ (d) $e^{-(t-1)}u(t-1)$

Q.10 The z-transform of the discrete signal $u[-n+2]$ is

- (a) $\frac{z^{-2}}{1+z}, |z| < 1$ (b) $\frac{z^{-2}}{1-z}, |z| > 1$
(c) $\frac{z}{z-z^2}, |z| < 1$ (d) $\frac{z^{-1}}{z-z^2}, |z| < 1$

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 Consider a causal LTI system with frequency response $H(j\omega) = \frac{1}{3+j\omega}$ for a

particular input $x(t)$, this system is observed to produce the output as $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$. Then the input $x(t)$ is

- (a) $e^{-3t}u(t)$ (b) $e^{-t}u(t)$
(c) $e^{-4t}u(t)$ (d) $e^{4t}u(t)$

Q.12 Consider $y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)$, if $y(t) = Ae^{-t}$ for $0 \leq t < 2$, then value of A is

- (a) $\frac{1}{1-e}$ (b) $\frac{1}{1-e^{-2}}$
(c) $\frac{1}{1-e^{-3}}$ (d) $\frac{1-e}{1-e^{-2}}$

Q.13 Consider a continuous time system with input $x(t)$ and output $y(t)$ are related by,

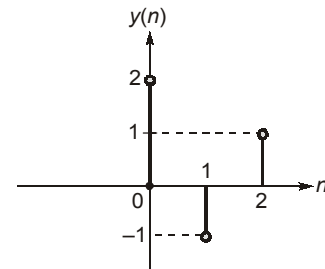
$$y(t) = 3x(\sin t)$$

The system is

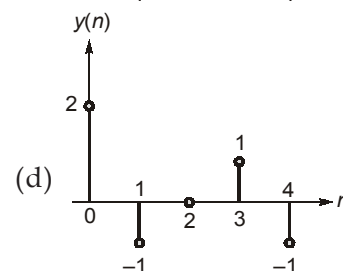
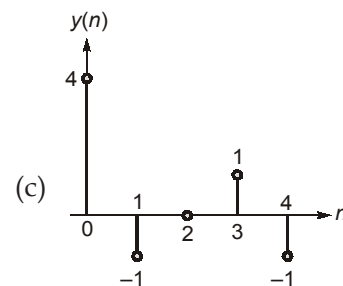
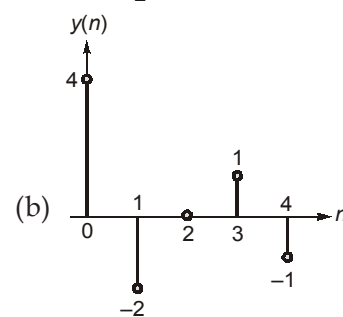
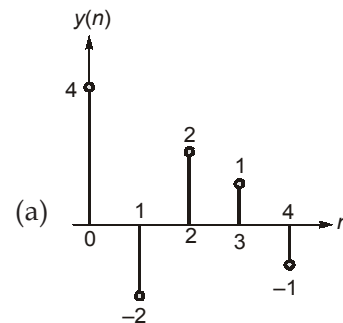
- (a) causal and linear

- (b) non-causal and linear
(c) causal and non-linear
(d) non-causal and non-linear

Q.14 Consider a discrete-time system which is both linear and time invariant. For an input $x(n) = \delta(n)$, the output $y(n)$ is given by,



If $x[n] = 2\delta[n] - \delta[n-2]$, then the output $y(n)$ can be given by



Q.15 Consider the z -transform $X(z) = \frac{10 - 8z^{-1}}{2 - 5z^{-1} + 2z^{-2}}$. If ROC includes unit circle then the value of $x(1)$ is

- (a) 0.5 (b) 1
(c) 1.5 (d) 2

Q.16 Consider a continuous-time system with input-output relation:

$$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau$$

Select the correct option given below,

- (a) Linear, time variant, causal
(b) Non-linear, time invariant, causal
(c) Linear, time invariant, non casual
(d) Linear, time invariant, causal

Q.17 Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by $y(t) = x_1(t - 2) * x_2(-t + 3)$

Where $x_1(t) = e^{-2t} \cdot u(t)$ and $x_2(t) = e^{-3t} u(t)$,

Then the Laplace transform of $Y(s)$ is

- (a) $\frac{e^s}{(s+2)(s+3)}$ (b) $\frac{e^{-s}}{(s+2)(s+3)}$
(c) $\frac{e^{-5s}}{(s+2)(3-s)}$ (d) $\frac{e^{-5s}}{(s+2)(s-3)}$

Q.18 Consider a system described by following differential equations:

$$\frac{d^2 y(t)}{dt^2} + \frac{2dy(t)}{dt} - 3y(t) = x(t)$$

The impulse response of the system, when the system is stable is

- (a) $\frac{1}{4} e^{3t} u(t) - \frac{1}{4} e^{-2t} u(t)$
(b) $-\frac{1}{4} e^{-3t} u(t) - \frac{1}{4} e^t u(-t)$
(c) $\frac{1}{2} e^{2t} u(t) - \frac{1}{4} e^{-3t} u(t)$
(d) $-\frac{1}{4} e^{-3t} u(t) + \frac{1}{4} e^{-t} u(-t)$

Q.19 For an input $x[n] = 3\delta[n - 2]$, output $y[n]$ of a LTI system is given as,

$$y[n] = 2\left(-\frac{1}{2}\right)^n + 8\left(\frac{1}{4}\right)^n, \quad n \geq 2$$

The impulse response of system is

- (a) $h[n] = \frac{1}{6} \left(\left(-\frac{1}{2}\right)^{n-2} + \left(\frac{1}{4}\right)^{n-2} \right) u[n]$
(b) $h[n] = 6 \left(\left(-\frac{1}{2}\right)^{n+2} + \left(\frac{1}{4}\right)^{n+2} \right) u[n]$
(c) $h[n] = \frac{1}{6} \left(\left(-\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n \right) u[n]$
(d) none of these

Q.20 The impulse response $h[n]$ of a discrete time system is given by,

$$h[n] = 2n \left(\frac{1}{2} \right)^n u[n]$$

The system is

- (a) causal and stable
(b) causal and unstable
(c) non causal and stable
(d) non causal and unstable

Q.21 The Laplace transform of a signal $x(t)$ is

$$X(s) = s^{-1} \frac{d}{ds} \left(\frac{e^{-3s}}{s} \right), \quad \text{ROC : } \text{Re}(s) >$$

0

The signal $x(t)$ is

- (a) $-0.1 [t - 4.5] u(t - 3)$
(b) $-0.5 [t^2 - 9] u(t - 3)$
(c) $0.25 [t^3 - 9] u(t + 3)$
(d) $0.5 [t - 2] u(t - 3)$

Q.22 Consider a discrete time signal $x[n]$ and its z -transform, $X(z)$ given as

$$X(z) = \frac{z^2 + 5z}{z^2 - 2z - 3}$$

If ROC of $X(z)$ is $|z| < 1$, then signal $x[n]$ would be

- (a) $[-2(3)^n + (-1)^n] u[-n - 1]$
(b) $[2(3)^n - (-1)^n] u[n]$
(c) $-2(3)^n u[-n - 1] - (-1)^n u[n]$
(d) $[2(3)^n + 1] + u[n]$

Q.23 A continuous time signal $x(t)$ is given by,

$$x(t) = \left(\frac{\sin 10000\pi t}{\pi t} \right) * \left(\frac{\sin 20000\pi t}{\pi t} \right); \text{ where } '*'$$

denotes convolution

The Nyquist sampling rate of $x(t)$ is

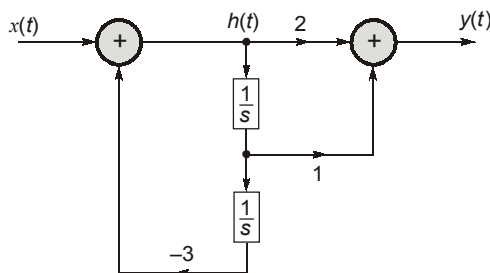
- (a) 30 kHz (b) 15 kHz
(c) 60 kHz (d) 10 kHz

Q.24 If the Laplace transform of a signal $x(t)$ is $X(s)$, then the Laplace transform of

$$x\left(\frac{t-2}{3}\right) \text{ will be}$$

- (a) $3e^{-2s}X(3s)$ (b) $3e^{-\frac{2}{3}s}X(3s)$
(c) $\frac{1}{3}e^{-2s}X(3s)$ (d) $3e^{-2s}X\left(\frac{s}{3}\right)$

Q.25 The block diagram representation of a system is shown in figure below.



The differential equation description of this system can be given as

- (a) $\frac{d^2 y(t)}{dt^2} = \frac{dx(t)}{dt} - \frac{2d^2 x(t)}{dt^2} - 3y(t)$
(b) $\frac{d^2 y(t)}{dt^2} = \frac{dx(t)}{dt} + \frac{2d^2 x(t)}{dt^2} - 3y(t)$
(c) $\frac{d^2 y(t)}{dt^2} = \frac{dx(t)}{dt} - \frac{2d^2 x(t)}{dt^2} + 3y(t)$
(d) $\frac{d^2 y(t)}{dt^2} = \frac{dx(t)}{dt} + \frac{2d^2 x(t)}{dt^2} + 3y(t)$

Q.26 The input $x(t) = 5\sin\left(2t + \frac{\pi}{4}\right)$ is given to a system with impulse response $h(t) = te^{-|t|}$. The output $y(t)$ will be

- (a) $1.31(\cos 2t - \sin 2t)$
(b) $1.31(\sin 2t - \cos 2t)$
(c) $1.13(\cos 2t - \sin 2t)$
(d) $1.13(\sin 2t - \cos 2t)$

Q.27 If $f(t)$ is a real even continuous time signal, then its Fourier transform will be

- (a) $\int_0^\infty f(t)\cos(2\omega t)dt$
(b) $2\int_0^\infty f(t)\cos(\omega t)dt$
(c) $\int_0^\infty f(t)\sin(\omega t)dt$
(d) $2\int_0^\infty f(t)\sin(\omega t)dt$

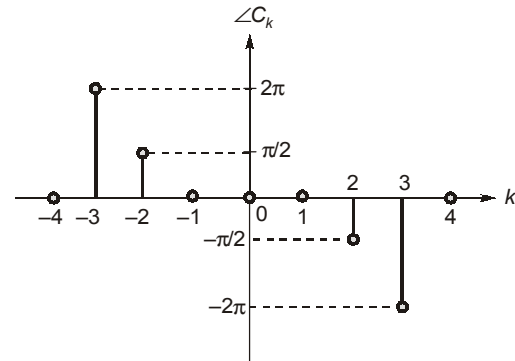
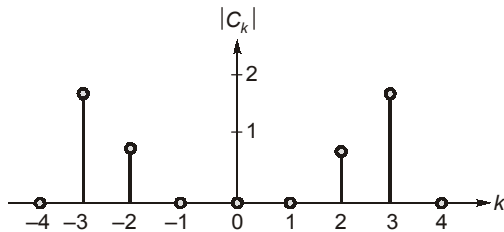
Q.28 Laplace transform of double differentiation of unit impulse signal is

- (a) 1 (b) s
(c) s^2 (d) $\frac{1}{s^2}$

Q.29 The inverse Laplace transform of the signal $X(s) = \log(s+2) - \log(s+3)$. Then $x(t)$ is equal to
(Assume all initial conditions are zero).

- (a) $\left(\frac{e^{3t} - e^{-2t}}{t}\right)u(t)$
(b) $\left(\frac{e^{-3t} - e^{-2t}}{t}\right)u(t)$
(c) $\left(\frac{e^{-3t} - e^{2t}}{t}\right)u(t)$
(d) $\left(\frac{e^{3t} - e^{2t}}{t}\right)u(t)$

Q.30 A time domain signal $x(t)$ is represented by the following exponential Fourier series spectra (magnitude and phase).



The corresponding signal $x(t)$ can be given by

(Assume that, $\omega_0 = \pi$)

- (a) $4\cos 3\pi t - 2\sin 2\pi t$
- (b) $4\sin 3\pi t + 2\cos 2\pi t$
- (c) $4\sin 3\pi t - 2\cos 2\pi t$
- (d) $4\cos 3\pi t + 2\sin 2\pi t$



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SIGNALS AND SYSTEMS

EC + EE

Date of Test : 08/05/2023

ANSWER KEY ➤

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (d) | 7. (c) | 13. (b) | 19. (c) | 25. (b) |
| 2. (c) | 8. (b) | 14. (b) | 20. (a) | 26. (d) |
| 3. (d) | 9. (c) | 15. (a) | 21. (b) | 27. (b) |
| 4. (a) | 10. (d) | 16. (c) | 22. (a) | 28. (c) |
| 5. (a) | 11. (c) | 17. (c) | 23. (d) | 29. (b) |
| 6. (c) | 12. (b) | 18. (b) | 24. (a) | 30. (d) |

DETAILED EXPLANATIONS

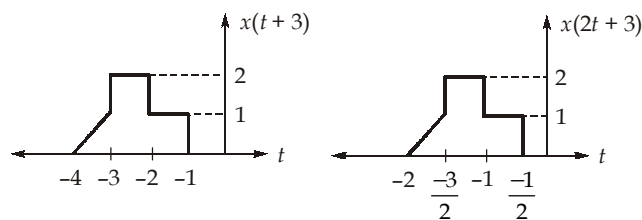
1. (d)

Energy of signal $x(n)$ = Energy in even part of signal $x(n)$ + Energy in odd part of signal $x(n)$

$$\text{Energy of signal } x(n) = 6 + 8 = 14$$

2. (c)

The signal $x(2t + 3)$ can be obtained by first shifting $x(t)$ to the left by 3 units and then scaling by 2 units.



3. (d)

$$u(t) \xrightarrow{\text{L.T.}} \frac{1}{s}$$

$$u(t-1) \xrightarrow{\text{L.T.}} \frac{e^{-s}}{s}$$

$$u(2t-1) \xrightarrow{\text{L.T.}} \frac{1}{2} \cdot \frac{e^{-s/2}}{s/2}$$

$$u(-2t-1) \xrightarrow{\text{L.T.}} -\frac{1}{2} \cdot \frac{e^{s/2}}{s/2} = \frac{-e^{s/2}}{s}$$

4. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} = 1 + z^{-1} - z^{-2} - z^{-3}$$

$$X\left(\frac{1}{2}\right) = 1 + 2 - 4 - 8 = -9$$

5. (a)

System transfer function using laplace transform would be,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{Ls}{R + Ls} = 1 - \frac{R/L}{(R/L) + s}$$

taking inverse laplace transform

$$h(t) = \delta(t) - \frac{R}{L} e^{-(R/L)t}$$

for stability of any system,

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

$$\begin{aligned}\int_{-\infty}^{\infty} |h(t)| dt &= \int_0^{\infty} \left| \delta(t) - \frac{R}{L} e^{-(R/L)t} \right| dt = 1 - \left[\frac{R}{L} \frac{e^{-(R/L)t}}{(-R/L)} \right]_0^{\infty} \\ &= 1 - (0 + 1) = 0\end{aligned}$$

so $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

so system is BIBO stable.

6. (c)

We first apply time-shifting operation to find,

$$y(n) = x(n-1) = \{3, \underset{\uparrow}{4}, 5, 6\}$$

$$y\left(\frac{n}{2}\right) = x(0.5n-1) = \{3, 0, \underset{\uparrow}{4}, 0, 5, 0, 6, 0\}$$

7. (c)

$$x(t) = \underbrace{4 \cos\left(\frac{2\pi}{3}t + 40^\circ\right)}_{x_1(t)} + \underbrace{3 \sin\left(\frac{4\pi}{5}t + 20^\circ\right)}_{x_2(t)}$$

$$\omega_1 = \frac{2\pi}{3} \Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/3} = 3$$

$$\omega_1 = \frac{4\pi}{5} \Rightarrow T_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$$

$$T = \text{LCM of } (T_1, T_2)$$

$$\frac{T_1}{T_2} = \frac{3}{5/2} = \frac{6}{5}$$

$$\Rightarrow T = 3 \times 5 \text{ or } 6 \times \frac{5}{2} = 15 \text{ sec}$$

8. (b)

$$e^{-(2t-2)} u(t-1) = e^{-2(t-1)} u(t-1)$$

Now, $e^{-2t} u(t) \leftrightarrow \frac{1}{2+j\omega}$

$$e^{-2(t-1)} u(t-1) \leftrightarrow \frac{e^{-j\omega}}{2+j\omega}$$

9. (c)

Overall impulse response is,

$$\begin{aligned}h_3(t) &= h_1(t) * h_2(t) \\ &= \delta(t+1) * e^{-t} u(t) \\ &= e^{-(t+1)} u(t+1)\end{aligned}$$

10. (d)

$$u[n] \leftrightarrow \frac{1}{1-z^{-1}}, |z| > 1$$

$$u[n+2] \leftrightarrow \frac{z^2}{1-z^{-1}}, |z| > 1$$

$$u[-n+2] \leftrightarrow \frac{z^{-2}}{1-z}, |z| < 1 = \frac{z^{-1}}{z-z^2}, |z| < 1$$

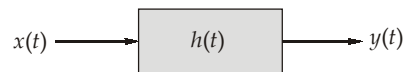
11. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output,

$$y(t) = e^{-3t} u(t) - e^{-4t} u(t)$$



We know that,

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$$\therefore X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t} u(t)$$

12. (b)

$$\text{Given, } y(t) = e^{-t} u(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)$$

$$= e^{-t} u(t) * (\dots + \delta(t+4) + \delta(t+2) + \delta(t) + \delta(t-2) + \delta(t-4) + \dots)$$

Using convolution property of impulse response,

$$\text{i.e., } x(t) * \delta(t-t_0) = x(t-t_0)$$

$$y(t) = \dots + e^{-(t+4)} u(t+4) + e^{-(t+2)} u(t+2) + e^{-t} u(t) + e^{-(t-2)} u(t-2) + e^{-(t-4)} u(t-4)$$

+ ...

In the range $0 \leq t < 2$, we may write $y(t)$ as,

$$y(t) = [\dots + e^{-(t+4)} u(t+4) + e^{-(t+2)} u(t+2) + e^{-t} u(t) + e^{-(t-2)} u(t-2) + e^{-(t-4)} u(t-4) + \dots] (u(t) - u(t-2))$$

$$= (e^{-t} + e^{-(t+2)} + e^{-(t+4)} + \dots) ; 0 \leq t < 2$$

$$= e^{-t} (1 + e^{-2} + e^{-4} + \dots) ; 0 \leq t < 2$$

$$= e^{-t} \left[\frac{1}{1-e^{-2}} \right] ; 0 \leq t < 2$$

$$\therefore y(t) = Ae^{-t} \text{ for } 0 \leq t < 2$$

$$\therefore A = \frac{1}{1-e^{-2}}$$

13. (b)

Linearity :

$$x_1(t) \rightarrow 3x_1(\sin t) = y_1(t)$$

$$x_2(t) \rightarrow 3x_2(\sin t) = y_2(t)$$

$$x_1(t) + x_2(t) \rightarrow 3[x_1(\sin t) + x_2(\sin t)] = y_1(t) + y_2(t) \Rightarrow \text{System is linear.}$$

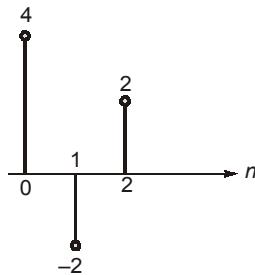
Causality :

At $t = -\pi$,

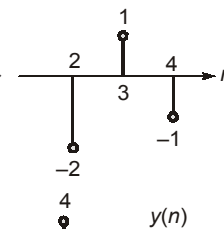
$$y(-\pi) = 3x(0) \Rightarrow \text{Non-causal.}$$

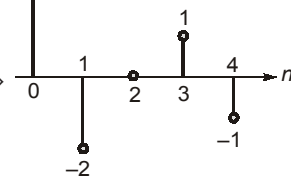
14. (b)

For $x_1[n] = 2\delta[n]$,

$$y_1(n) = 2y(n) \Rightarrow$$


For $x_2[n] = -\delta[n - 2]$,

$$y_2(n) = -y(n - 2) \Rightarrow$$


$$y(n) = y_1(n) + y_2(n) \Rightarrow$$


15. (a)

Given,

$$X(z) = \frac{10 - 8z^{-1}}{2 - 5z^{-1} + 2z^{-2}} = \frac{2}{(2 - z^{-1})} + \frac{4}{(1 - 2z^{-1})}$$

$$X(z) = \frac{2z}{2z - 1} + \frac{4z}{z - 2}$$

$$X(z) = \frac{z}{\left(z - \frac{1}{2}\right)} + \frac{4z}{(z - 2)}$$

Since, ROC includes unit circle,

$$\therefore \text{ROC of } X(z) \text{ is } \frac{1}{2} < |z| < 2$$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n) u[-n - 1]$$

$$\therefore x(1) = \frac{1}{2} = 0.5$$

16. (c)

Integration is linear system

Time-variant (or) time invariant system:Delay input by t_0 units

$$= \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau - t_0) d\tau \quad \dots(i)$$

Delay output by t_0 units (or) substitute $(t - t_0)$ in the place of t .

$$y(t - t_0) = \frac{1}{T} \int_{t-t_0-T/2}^{t-t_0+T/2} x(\tau) d\tau \quad \dots(ii)$$

From equation (i) and (ii), we can say equation (i) = equation (ii),

 \therefore The given system is time invariant.**Causal (or) Non-causal system:**

$$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau$$

Let,

$$T = 4$$

$$y(0) = \frac{1}{4} \int_{-2}^2 x(\tau) d\tau$$

here, $y(0)$ depends on future value $x(2)$. \therefore The given system is non-causal system.

17. (c)

$$x_1(t) \xrightarrow{\text{L.T.}} \frac{1}{s+2}$$

$$x_2(t) \xrightarrow{\text{L.T.}} \frac{1}{s+3}$$

$$x_1(t-2) \xrightarrow{\text{L.T.}} \frac{e^{-2s}}{s+2}$$

$$x_2(t+3) \xrightarrow{\text{L.T.}} \frac{e^{3s}}{s+3}$$

$$x_2(-t+3) \xrightarrow{\text{L.T.}} \frac{e^{-3s}}{3-s}$$

$$\therefore y(t) \xrightarrow{\text{L.T.}} \frac{e^{-2s}}{(s+2)} \cdot \frac{e^{-3s}}{(3-s)}$$

$$y(t) \xrightarrow{\text{L.T.}} \frac{e^{-5s}}{(s+2)(3-s)}$$

18. (b)

By applying Laplace transform on differential equation,

$$s^2 Y(s) + 2s Y(s) - 3 Y(s) = X(s)$$

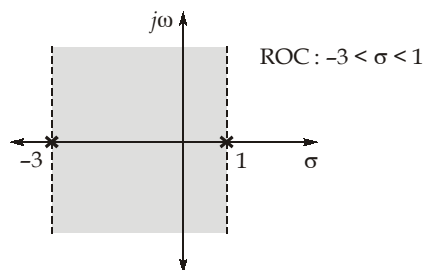
$$Y(s) [s^2 + 2s - 3] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + 2s - 3} = \frac{1}{(s-1)(s+3)}$$

$$= \frac{A}{s-1} + \frac{B}{s+3}$$

$$= \frac{1/4}{s-1} - \frac{1/4}{s+3}$$

Given system is stable:



$$h(t) = -\frac{1}{4}e^{-3t}u(t) - \frac{1}{4}e^t u(-t)$$

19. (c)

$$y[n] = h[n] * x[n]$$

$$= h[n] * 3\delta[n-2]$$

$$h[n] = \frac{1}{3}y[n+2]$$

$$y[n] = \left[\frac{1}{2} \left(\frac{-1}{2} \right)^{n-2} + \frac{1}{2} \left(\frac{1}{4} \right)^{n-2} \right] u[n-2]$$

$$h[n] = \frac{1}{3} \left[\left(\frac{1}{2} \right) \left(\frac{-1}{2} \right)^{n-2+2} + \frac{1}{2} \left(\frac{1}{4} \right)^{n-2+2} \right] u[n+2-2]$$

$$= \frac{1}{6} \left(\left(-\frac{1}{2} \right)^n + \left(\frac{1}{4} \right)^n \right) u[n]$$

20. (a)

• $h[n] = 0$ for $n < 0 \Rightarrow$ causal

• $\sum_{n=-\infty}^{\infty} |h[n]|$ is finite \Rightarrow stable

21. (b)

We have,

$$\frac{1}{s} \xleftrightarrow{L^{-1}} u(t)$$

$$\frac{e^{-3s}}{s} \xleftrightarrow{L^{-1}} u(t-3) \quad (\text{Time shifting})$$

$$\frac{d}{ds} \left(\frac{e^{-3s}}{s} \right) \longleftrightarrow -tu(t-3) \quad (\text{Differentiation in s-domain})$$

$$\frac{1}{s} \frac{d}{ds} \left(\frac{e^{-3s}}{s} \right) \xleftrightarrow{L^{-1}} \int_{-\infty}^t -\tau u(\tau-3) d\tau$$

$$x(t) = -\int_3^t \tau d\tau = -\left(\frac{t^2}{2}\right)_3^t; t > 3$$

$$x(t) = -\frac{1}{2}(t^2 - 9)u(t-3)$$

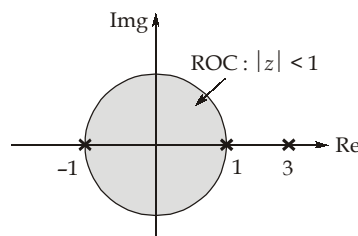
22. (a)

$$X(z) = \frac{z^2 + 5z}{z^2 - 2z - 3} = \frac{z(z+5)}{(z-3)(z+1)}$$

$$\frac{X(z)}{z} = \frac{z+5}{(z-3)(z+1)} = \frac{2}{z-3} - \frac{1}{z+1}$$

Thus,

$$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$



ROC : $|z| < 1$, which is not exterior of circle outside the outermost pole $z = 3$

So, $x[n]$ is anti-causal given as,

$$x[n] = [-2(3)^n + (-1)^n] u[-n-1]$$

23. (d)

$$\left(\frac{\sin 10^4 \pi t}{\pi t} \right) \Rightarrow f_{1\max} = \frac{10^4 \pi}{2\pi} = 5 \text{ kHz}$$

$$\left(\frac{\sin 2 \times 10^4 \pi t}{\pi t} \right) \Rightarrow f_{2\max} = 10 \text{ kHz}$$

$$f_s = 2[\min \text{ of } (f_{1\max}, f_{2\max})] \\ = 2 \times 5 = 10 \text{ kHz}$$

24. (a)

$$x\left(\frac{t-2}{3}\right) = x\left(\frac{t}{3} - \frac{2}{3}\right)$$

Now,

$$x\left(t - \frac{2}{3}\right) \leftrightarrow e^{\frac{2}{3}s} X(s)$$

$$x\left(\frac{t}{3} - \frac{2}{3}\right) \leftrightarrow \frac{1}{\left|\frac{1}{3}\right|} e^{-\frac{2}{3} \frac{s}{1/3}} X\left(\frac{s}{1/3}\right)$$

\Rightarrow

$$x\left(\frac{t-2}{3}\right) \leftrightarrow 3e^{-2s} X(3s)$$

25. (b)

$$X(s) - \frac{3H(s)}{s^2} = H(s)$$

\Rightarrow

$$X(s) = \left(1 + \frac{3}{s^2}\right) H(s)$$

•

$$2H(s) + \frac{H(s)}{s} = Y(s)$$

\Rightarrow

$$\left(2 + \frac{1}{s}\right) H(s) = Y(s)$$

\Rightarrow

$$\left(2 + \frac{1}{s}\right) \frac{X(s)}{\left(1 + \frac{3}{s^2}\right)} = Y(s)$$

\Rightarrow

$$\frac{Y(s)}{X(s)} = \frac{2 + \frac{1}{s}}{\frac{3}{s^2} + 1} = \frac{s + 2s^2}{3 + s^2}$$

\Rightarrow

$$\frac{d^2y(t)}{dt^2} + 3y(t) = \frac{dx(t)}{dt} + 2\frac{d^2x(t)}{dt^2}$$

26. (d)

$$5 \sin\left(2t + \frac{\pi}{4}\right) \xrightarrow{\quad} \boxed{h(t) = te^{-|t|}} \xrightarrow{\quad} 5|H(j\omega_0)| \sin\left(2t + \frac{\pi}{4} + \angle H(j\omega_0)\right)$$

$\omega_0 = 2 \text{ rad/sec}$

$$h(t) = te^{-|t|}$$

$$H(j\omega) = j \frac{d}{d\omega} \left(\frac{2}{1 + \omega^2} \right) = \frac{-4j\omega}{(1 + \omega^2)^2}$$

$$|H(j\omega_0)| = \left| \frac{-4j(2)}{(1 + 4)^2} \right| = \frac{8}{25} \quad (\omega_0 = 2 \text{ rad/sec})$$

$$\angle H(j\omega_0) = -90^\circ$$

$$\begin{aligned}
 \text{output} &= 5 \times \frac{8}{25} \sin\left(2t + \frac{\pi}{4} - \frac{\pi}{2}\right) = \frac{8}{5} \sin\left(2t - \frac{\pi}{4}\right) \\
 &= \frac{8}{5} \left(\frac{\sin 2t}{\sqrt{2}} - \frac{\cos 2t}{\sqrt{2}} \right) \\
 &= \frac{8}{5\sqrt{2}} (\sin 2t - \cos 2t) \\
 &= 1.13 (\sin 2t - \cos 2t)
 \end{aligned}$$

27. (b)

$$\begin{aligned}
 F(\omega) &= \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \\
 &= \int_{-\infty}^{\infty} [f(t) \cos \omega t - j f(t) \sin \omega t] dt \\
 &= \int_{-\infty}^{\infty} f(t) \cos \omega t dt - j \int_{-\infty}^{\infty} f(t) \sin \omega t dt
 \end{aligned}$$

 $f(t) \Rightarrow$ even signal $f(t) \cos \omega t \Rightarrow$ even signal $f(t) \sin \omega t \Rightarrow$ odd signal

$$\int_{-\infty}^{\infty} f(t) \sin \omega t dt = 0$$

$$\int_{-\infty}^{\infty} f(t) \cos \omega t dt = 2 \int_0^{\infty} f(t) \cos \omega t dt$$

$$\therefore F(\omega) = 2 \int_0^{\infty} f(t) \cos \omega t dt$$

28. (c)

We know that, unit impulse let $x(t)$,

$$x(t) = \delta(t)$$

$$\text{for } \delta(t) \xrightarrow{LT} 1$$

$$\text{for } \frac{d}{dt} x(t) \xrightarrow{LT} sX(s)$$

$$\frac{d}{dt} \delta(t) \xrightarrow{LT} s$$

$$\frac{d^2}{dt^2} \delta(t) \xrightarrow{LT} s^2$$

29. (b)

Given, $X(s) = \log(s+2) - \log(s+3)$

Differentiating both the sides with respect to s

$$\frac{d}{ds} X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \dots(i)$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or,

$$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t} \right] u(t)$$

30. (d)

$$C_k = j\delta(k+2) - j\delta(k-2) + 2\delta(k+3) + 2\delta(k-3)$$

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} C_k e^{jk\pi t} \\ &= je^{-j2\pi t} - je^{j2\pi t} + 2e^{-j3\pi t} + 2e^{j3\pi t} \\ &= 4\cos(3\pi t) + 2\sin(2\pi t) \end{aligned}$$

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