CLASS TEST

S.No. : 01_ JP_CE_08052023

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CIVIL ENGINEERING

STRENGTH OF MATERIALS

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 Which of the following statement is correct regarding assumption in Euler's column theory?

1. Initially the column is perfectly straight and load applied is truly axial.
2. The failure of column occurs due to buckling alone.
3. The length of column is small as compared to its cross-section dimensions.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q.2 A closely-coiled helical spring of round steel wire 5 mm in diameter having 12 complete coils of 50 mm mean diameter is subject to an axial load of 100 N. What will be deflection of the spring. [Take modulus of rigidity (G) = 80 GPa.]

(a) 192 mm (b) 24 mm
(c) 42 mm (d) 252 mm

Q.3 A copper wire of 2 mm diameter is required to be wounded around a drum. What will be the minimum radius of drum, if the stress in the wire is not to exceed 80 MPa?
[Take modulus of elasticity for copper = 100 GPa]

(a) 1.5 m (b) 3 m
(c) 1.25 m (d) 2.5 m

Q.4 Which of the following statement is correct regarding conjugate beam?

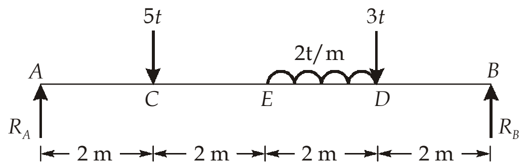
1. Slope at any section in given beam will be SF in conjugate beam at the corresponding section.
2. Deflection at any section in given beam will be BM in conjugate beam at the corresponding section.
3. Internal hinge in given beam will be internal hinge in conjugate beam also.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q.5 A section of shaft of 100 mm diameter is subjected to combined bending and twisting moments of 8 kNm and 6 kNm respectively. What will be the maximum shear stress induced on the section?

(a) 50.93 MPa
(b) 30.56 MPa
(c) 40.74 MPa
(d) 60.36 MPa

Q.6 The ratio of reactions R_A and R_B of the simply supported beam, as shown in figure below, will be



(a) 1

(b) $\frac{3}{2}$

(c) $\frac{1}{2}$

(d) $\frac{2}{3}$

Q.7 For a closed coiled helical spring, choose the incorrect statements.

1. Resultant shear stress at outer surface of the spring is more compared to inner surface.
2. If spring of stiffness k is divided into m equal parts then stiffness of each resultant spring is mk.

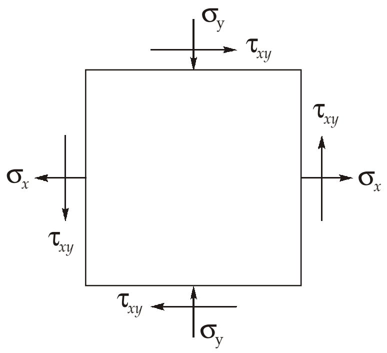
(a) 1 and 2
(b) 1 only
(c) 2 only
(d) None of above

Q.8 A rectangular beam 300 mm deep is simply supported over a span of 4 metres. What uniformly distributed load the beam may carry, if the bending stress is not to exceed 120 MPa?
[Take $I = 225 \times 10^6$ mm^4]

(a) 180 kN/m
(b) 45 kN/m
(c) 90 kN/m
(d) 360 kN/m

Q.9 In a flitched beam, one section is reinforced with another section. The purpose of such a beam is to improve
(a) shearing resistance of the section
(b) moment of resistance of the section
(c) appearance of the section
(d) all of these

Q.10 The state of stress on an element is shown in the given figure. The value of stresses are σ_x = 32 MPa; σ_y = 10 MPa and major principle stress σ_1 = 40 MPa. The minor principle stress σ_2 is



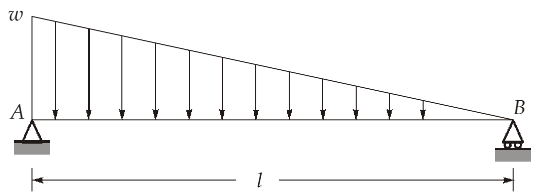
(a) 2 MPa
(b) –12 MPa
(c) 8 MPa
(d) –18 MPa

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 A 30 mm × 30mm square bar of 100 mm length is subjected to an axial compressive load of 90 kN. What will be the change in length if lateral strains are prevented by the application of uniform and equal lateral external pressure of suitable intensity.
[Take E = 100 GPa and ν = 0.25]
(a) 0.083 mm
(b) 0.042 mm
(c) 0.166 mm
(d) 0.053 mm

Q.12 A hollow steel shaft is having external and internal diameter as 80 mm and 60 mm respectively. Volume of shaft is 10^6 mm^3. What will be maximum strain energy stored in the shaft if maximum allowable shear stress is 80 MPa?
[Take shear modulus (G) = 100 GPa]
(a) 25 N-m
(b) 50 N-m
(c) 20 N-m
(d) 40 N-m

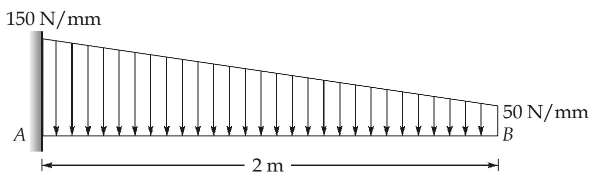
Q.13 A simply supported beam AB of length l with a gradually varying load zero at B and w per unit length at A is as shown. What will be the maximum value of bending moment in beam AB?



(a) $\dfrac{wl^2}{9\sqrt{3}}$
(b) $\dfrac{wl^2}{9}$
(c) $\dfrac{wl^2}{2\sqrt{3}}$
(d) $\dfrac{wl^2}{9\sqrt{2}}$

Q.14 A beam of triangular cross-section having each side of 100 mm is subjected to a shear force of 13 kN. What will be the maximum value of shear stress induced in the section?
(a) 3 MPa
(b) 4 MPa
(c) 4.5 MPa
(d) 2 MPa

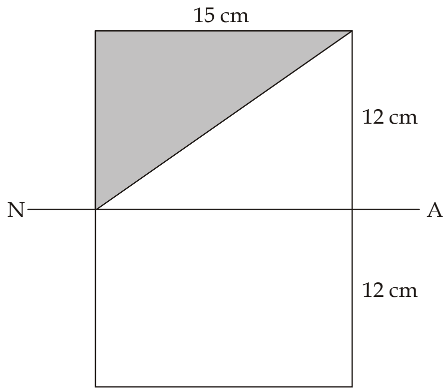
Q.15 A metallic cantilever beam with span 2 m carries a uniformly varying load of 50 N/mm at the free end to 150 N/mm at the fixed end as shown in the figure. What will be slope at free end?
[Take flexual rigidity of beam as 10^{13} N-mm^2]



(a) 0.005 rad
(b) 0.01 rad
(c) 0.02 rad
(d) 0.04 rad

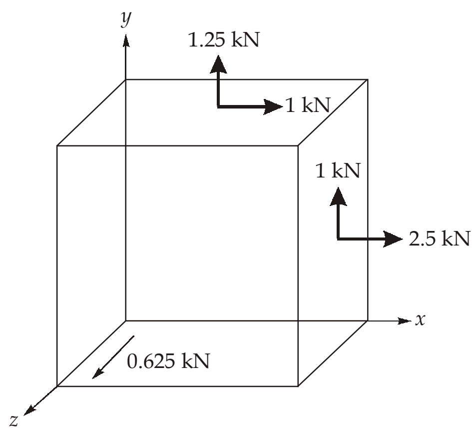
Q.16 A rod of steel is 20 m long at a temp of 20°C. Temperature of rod is then raised to 65°C. What will be thermal stress produced when the rod is permitted to expand only by 5.8 mm?
[Take α = 12 × 10^{-6}/°C and E = 200 GPa]
(a) 108 MPa
(b) 58 MPa
(c) 50 MPa
(d) 65 MPa

Q.17 The beam section as shown in figure is subjected to a maximum bending stress of 90 kg/cm². What will be the force on the shaded area?



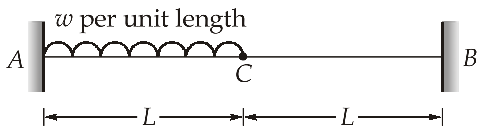
(a) 2700 kg (b) 10800 kg
(c) 5400 kg (d) 1350 kg

Q.18 A cube of 5 mm side is subjected to load system as shown. What will be FOS according to maximum shear stress theory if yield strength of the material is 75 MPa?



(a) 1.3 (b) 0.77
(c) 0.80 (d) 1.25

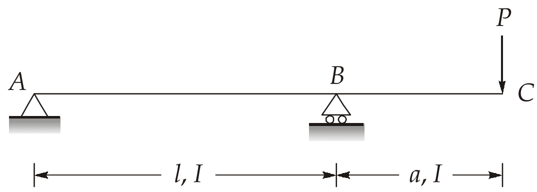
Q.19 What is the reaction on the pin C for the beam shown in the figure below.



(a) $\frac{3}{8}wL$ (b) $\frac{5}{8}wL$

(c) $\frac{3}{16}wL$ (d) $\frac{5}{16}wL$

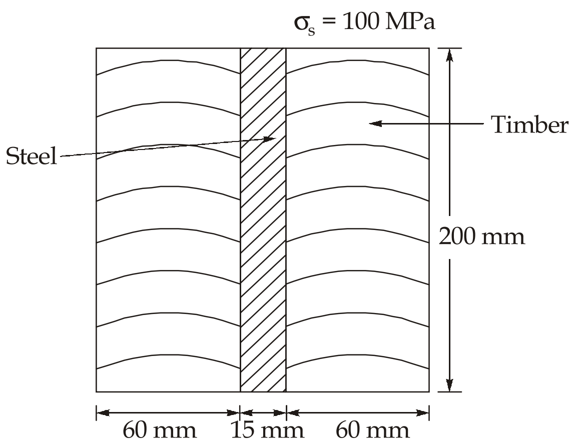
Q.20 What will be the deflection at C for the beam shown?



(a) $\frac{Pa^2l}{2EI}+\frac{Pa^3}{3EI}$ (b) $\frac{Pa^2l}{3EI}+\frac{Pa^3}{2EI}$

(c) $\frac{Pa^2l}{2EI}+\frac{Pa^3}{2EI}$ (d) $\frac{Pa^2l}{3EI}+\frac{Pa^3}{3EI}$

Q.21 A flitched timber beam made up of steel and timber has a section as shown in figure. What will be the moment of resistance of the beam? [Take σ_T = 5 MPa and σ_S = 100 MPa]



(a) 10 kNm (b) 7 kNm
(c) 14 kNm (d) 17 kNm

Q.22 In a strained material, the principal stresses in the x and y directions are 120 N/mm² (tensile) and 80 N/mm² (compression) respectively. On an inclined plane, the normal to which makes an angle of 30° to the x-axis, what is the tangential stress (in N/mm²)

(a) $25\sqrt{3}$ (b) 50
(c) $50\sqrt{3}$ (d) 100

Q.23 A cantilever beam of length 'L' carries a concentrated load W at the mid-span. If the free end is supported on a rigid prop. Which of the following option(s) is incorrect?

(a) The prop reaction is $\frac{5W}{16}$.

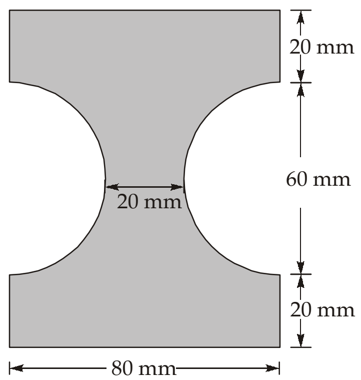
(b) The point of contraflexure is at $\frac{3L}{11}$ from fixed support.

(c) Magnitude of bending moment at the prop is $\left(\frac{3WL}{16}\right)$.

(d) Magnitude of bending moment under the concentrated load is $\left(\frac{5WL}{32}\right)$.

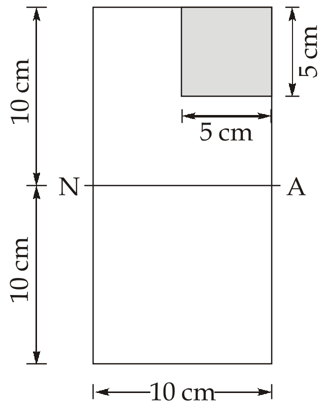
Q.24 A cantilever beam 2.5 m long is loaded with a uniformly distributed load of 10 kN/m over a length of 1.5 m from the fixed end. What will be the deflection at free end of the cantilever?
[Take $EI = 1.9 \times 10^{12}$ Nmm^2]
(a) 6.3 mm (b) 3.3 mm
(c) 4.3 mm (d) 5.3 mm

Q.25 A steel section as shown in figure is subjected to a shear force of 20 kN. What will be shear stress at the centre of section?



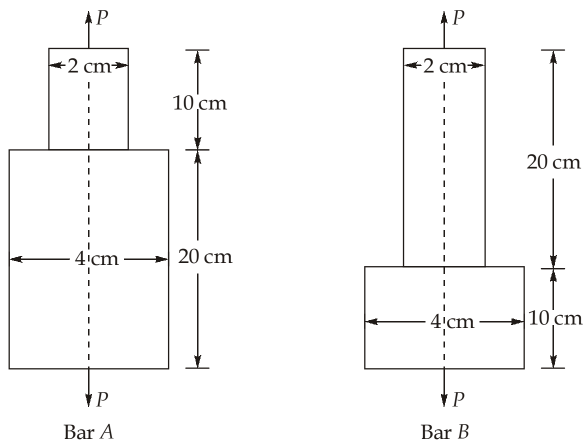
(a) 20.4 MPa (b) 13.6 MPa
(c) 16.6 MPa (d) 11.4 MPa

Q.26 Figure shows a rectangular beam section 10 cm wide and 20 cm deep. If the maximum flexural stress is 80 kg/cm^2. What will be total force on area shaded and moment of this force about the neutral axis respectively.



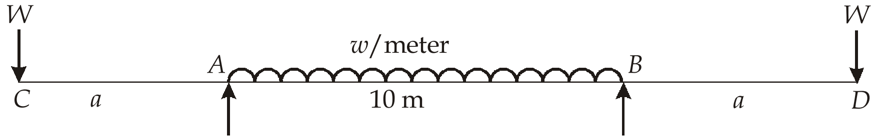
(a) 1500 kg, 11250 kg cm
(b) 1000 kg, 11666.67 kg cm
(c) 1500 kg, 11666.67 kg cm
(d) 1000 kg, 11250 kg cm

Q.27 Two similar round bars A and B are each 30 cm long and loaded as shown in the given figure. The ratio of the energies stored by the bars A and B, $\frac{U_B}{U_A}$ is ________. (Ignore self weight of bars).



(a) 0.5 (b) 1
(c) 1.5 (d) 2

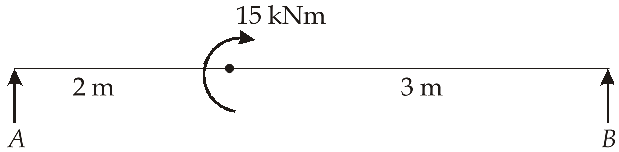
Q.28 A simply supported beam with over-hanging ends carries transverse loads as shown in the figure.



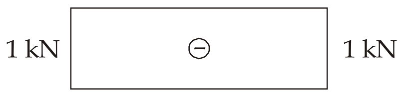
If $W = 10w$, what will be the overhanging length on each side, such that the bending moment at the middle of the beam is zero?

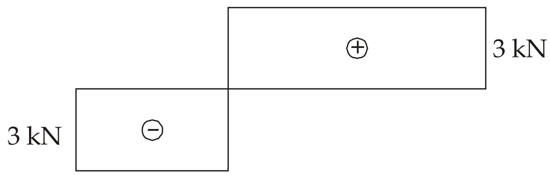
(a) 2 m (b) 1.25 m
(c) 2.5 m (d) 1 m

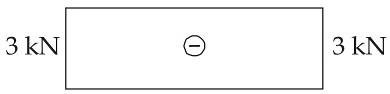
Q.29 A simply supported beam of 5 m span is subjected to a clockwise moment of 15 kN-m at a distance of 2 m from left end as shown in the figure.

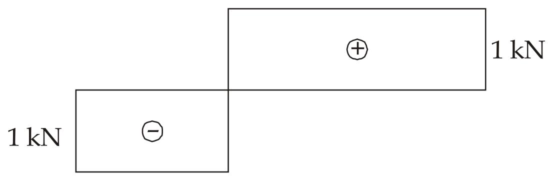


Which of the following options show correct SFD of beam *AB*?

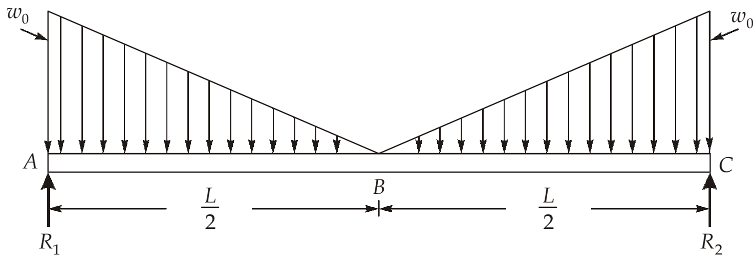
(a) 

(b) 

(c) 

(d) 

Q.30 A beam is loaded as shown in figure below:



The moment at B (M_B) will be

(a) 0 (b) $\frac{1}{24}w_0L^2$
(c) $\frac{1}{12}w_0L^2$ (d) $\frac{1}{18}w_0L^2$

■■■■

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CIVIL ENGINEERING

Date of Test : 08/05/2023

ANSWER KEY

1.	(a)	7.	(b)	13.	(a)	19.	(c)	25.	(b)
2.	(b)	8.	(c)	14.	(c)	20.	(d)	26.	(c)
3.	(c)	9.	(b)	15.	(b)	21.	(c)	27.	(c)
4.	(a)	10.	(d)	16.	(c)	22.	(c)	28.	(b)
5.	(a)	11.	(a)	17.	(c)	23.	(c)	29.	(c)
6.	(a)	12.	(a)	18.	(b)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

2. (b)

We know deflection of spring,

$$\delta = \frac{64WR^3n}{Gd^4}$$

where, $W = 100$ N, $R = 25$ mm, $n = 12$, $G = 80$ GPa, $d = 5$ mm

So, $$\delta = \frac{64\times100\times(25)^3\times12}{80\times10^3\times5^4} = 24 \text{ mm}$$

3. (c)

$$d = 2 \text{ mm}$$

$$\sigma_{b(\max)} = 80 \text{ N/mm}^2$$

$$E = 100 \times 10^3 \text{ N/mm}^2$$

Distance between the neutral axis of wire and its extreme fibre

$$y = \frac{2}{2} = 1 \text{ mm}$$

So, minimum radius of the drum

$$R = \frac{y}{\sigma_{b(\max)}}E \qquad \left(\because \frac{f}{y} = \frac{E}{R}\right)$$

$$= \frac{1}{80}\times100\times10^3$$

$$= 1.25 \times 10^3 \text{ mm} = 1.25 \text{ m}$$

4. (a)

Internal hinge in given beam will become hinged support in conjugate beam.

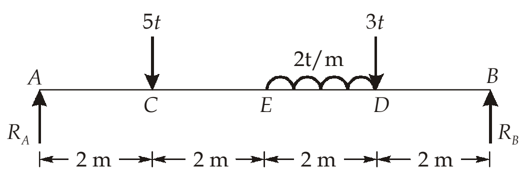
5. (a)

$$\tau_{\max} = \frac{16}{\pi D^3}\sqrt{M^2+T^2}$$

$$= \left[\frac{16}{\pi(100)^3}\sqrt{(8)^2+(6)^2}\right]\times10^6$$

$$= \frac{16}{\pi}\times\frac{10\times10^6}{10^6} = 50.93 \text{ MPa}$$

6. (a)



Taking moments about B,

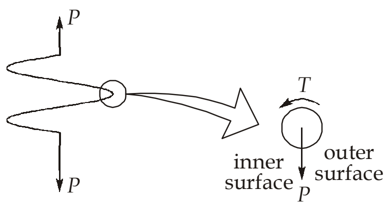
$$R_A = \left(\frac{6}{8}\right)\times 5+\frac{3}{8}\times 4+3\times\frac{2}{8}$$
$$= 3.75 + 1.5 + 0.75 = 6t$$

Taking moment about A,

$$R_B = (5+3+2\times 2)-R_A$$
$$= 12t - 6t = 6t$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{6t}{6t}=1$$

7. **(b)**



Due to combined effect of torque and shear force, inner surface will have more shear stress compared to outer surface.

On dividing a spring into m parts, the no. of turns on each spring will be m times less. Since stiffness is inversely proportional to no. of turns, the stiffness will become mk.

8. **(c)**

$$Z = \frac{I}{y}=\frac{225\times 10^6}{\frac{300}{2}}=1.5\times 10^6 \text{ mm}^3$$

$$M = \sigma_{max} \times Z = 120 \times 1.5 \times 10^6$$
$$= 180 \times 10^6 \text{ N-mm}$$

$$M_{max} = M = \frac{wl^2}{8}$$

So
$$\frac{wl^2}{8} = 180 \times 10^6 = \frac{w(4\times 10^3)^2}{8}$$

$$\Rightarrow \quad w = \frac{180}{2}=90 \text{ N/mm} = 90 \text{ kN/m}$$

9. **(b)**

Flitched beam has a composite section made of two or more materials joined together in such a manner that they behave as a unit piece and each material bends to the same radius of curvature. The total moment of resistance of a flitched beam is equal to the sum of the moments of resistance of individual sections.

10. **(d)**

By stress invariant law, $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

$\Rightarrow \quad 32 + (-10) = 40 + \sigma_2$

$\Rightarrow \quad \sigma_2 = -18 \text{ MPa}$

11. (a)

Direct longitudinal stress,

$$\sigma_x = \frac{90\times10^3}{30\times30} = 100 \text{ MPa} \qquad \text{(Compressive)}$$

$$\in_x = \frac{1}{E}\left[-\sigma_x + \nu\left(\sigma_y + \sigma_z\right)\right] \qquad ...(i)$$

$$\in_y = \in_z = \frac{1}{E}\left[-\sigma_y + \nu\left(\sigma_x + \sigma_z\right)\right] = 0 \qquad ...(ii)$$

As we know $\sigma_y = \sigma_z$

Also on solving equation (ii)

$$\sigma_y = \frac{\nu}{1-\nu}\sigma_x = \frac{0.25}{1-0.25}\sigma_x = \frac{\sigma_x}{3}$$

So,

$$\in_x = \frac{1}{E}\left[-\sigma_x + \nu\left(\sigma_y + \sigma_z\right)\right] = \frac{1}{E}\left[-\sigma_x + \nu\times2\sigma_y\right]$$

$$= \frac{1}{E}\left[-\sigma_x + 0.25\times2\times\frac{\sigma_x}{3}\right] = \frac{1}{E}\left[-\sigma_x + \frac{0.5\sigma_x}{3}\right]$$

$$= \frac{1}{100\times10^3}\left[-100 + \frac{0.5\times100}{3}\right] = \frac{1}{100\times10^3}\left[-\frac{250}{3}\right]$$

$$\delta l = l\in_x = \frac{1}{100\times10^3}\left[-\frac{250}{3}\right]\times100$$

$$= 0.083 \text{ mm} \qquad \text{(Reduction in length)}$$

12. (a)

Strain energy stored in hollow shaft, $U = \frac{\tau_{max}^2}{4G}\left[\frac{D^2+d^2}{D^2}\right]V$

$$= \frac{80^2}{4\times100\times10^3}\left[\frac{80^2+60^2}{80^2}\right]\times10^6 = \frac{10000\times10^6}{4\times10^5} = 2.5\times10^4 \text{ N-mm}$$

$$= 25 \text{ N-m}$$

13. (a)

Bending moment at A and B is zero. It increases in the form of cubic curve.

Maximum value of bending moment in beam AB occurs where shear force changes sign.

Now we know

$$R_A = \frac{wl}{3}$$

$$R_B = \frac{wl}{6}$$

Let at point x from B, shear force will be zero

$$V = R_B - \frac{1}{2}\times\frac{w}{l}x\cdot x$$

$$\Rightarrow \quad \frac{wl}{6} - \frac{wx^2}{2l} = 0$$

$$\Rightarrow \quad x = \frac{l}{\sqrt{3}}$$

So, bending moment at $x = \dfrac{l}{\sqrt{3}}$ from B is

$$M_{max} = R_B \cdot x - \frac{1}{2}\times\frac{w}{l}x\cdot x\times\frac{x}{3} = \frac{wl}{6}\times\frac{l}{\sqrt{3}} - \frac{1}{2}\times\frac{w}{l}\times\frac{1}{3}\times\frac{l^3}{(\sqrt{3})^3}$$

$$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{6\times 3\sqrt{3}} = \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{18\sqrt{3}} = \frac{2wl^2}{18\sqrt{3}}$$

$$= \frac{wl^2}{9\sqrt{3}}$$

14. (c)

We know that area of triangular beam section

$$A = \frac{\sqrt{3}}{4}a^2 \text{ for equilateral triangle}$$

$$= \frac{\sqrt{3}}{4}(100)^2 = 2500\sqrt{3}\text{ mm}^2$$

Average shear stress across the section

$$\tau_{avg} = \frac{F}{A} = \frac{13\times 10^3}{2500\sqrt{3}} = 3\text{ MPa}$$

So maximum shear stress for triangular section

$$\tau_{max} = 1.5\,\tau_{avg}$$

$$= 1.5 \times 3 = 4.5\text{ MPa}$$

15. (b)

Let us split up the trapezoidal load into a uniformly distributed load (w_1) of 50 N/mm and a triangular load (w_2) of 100 N/mm at A to zero at B.

Now slope at free end

$$\theta_B = \frac{w_1 l^3}{6EI} + \frac{w_2 l^3}{24EI} = \left[\frac{50\times(2\times 10^3)^3}{6\times 10^{13}} + \frac{100\times(2\times 10^3)^3}{24\times 10^{13}}\right]\text{rad}$$

$$= 0.0067 + 0.0033 = 0.01\text{ rad}$$

16. (c)

Free expansion of rod $= \delta l = \alpha l \Delta t = 12\times 10^{-6}\times 20\times 10^3\,(65-20) = 10.8$ mm

When the rod is permitted to expand by 5.8 mm in this case, expansion prevented = 10.8 – 5.8

= 5 mm

$\therefore$ Strain prevented $= \dfrac{\text{Expansion Prevented}}{\text{Original length}}$

$$= \frac{5}{20\times 10^3} = \frac{1}{4000}$$

$\therefore$ Thermal stress = Strain prevented × E

$$= \frac{1}{4000}\times 200\times 10^3 = 50\text{ MPa}$$

17. (c)

From bending equation, $\frac{f}{y} = \frac{f_{max}}{y_{\max}}$

$$\therefore \quad f = \frac{f_{max}}{y_{\max}} \times y$$

$$\therefore \quad \text{Force on shaded area} = \frac{f_{\max}}{y_{\max}} \times \Sigma Ay$$

$$= \frac{f_{max}}{y_{\max}} (A\bar{y})$$

[where A is shaded area, $\bar{y}$ = distance of centroid of shaded area from N.A.]

$$= \frac{90}{12} \times \left[\frac{15}{2} \times 12\right] \times \frac{2}{3} \times 12 = 5400 \text{ kg}$$

18. (b)

$$\sigma_x = \frac{2.5 \times 10^3}{25} = 100 \text{ MPa (T)}$$

$$\sigma_y = \frac{1250}{25} = 50 \text{ MPa (T)}$$

$$\sigma_z = \frac{625}{25} = 25 \text{ MPa (T)}$$

$$\tau_{xy} = \frac{1000}{25} = 40 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{100 + 50}{2} \pm \frac{1}{2}\sqrt{(100 - 50)^2 + 4(40)^2}$$

$$\sigma_1 = 122.17 \text{ MPa}$$

$$\sigma_2 = 27.83 \text{ MPa}$$

$$\sigma_3 = 25 \text{ MPa}$$

Now according to maximum shear stress theory

$$(\tau_{abs})_{\max} \leq \frac{\sigma_y}{2(\text{FOS})}$$

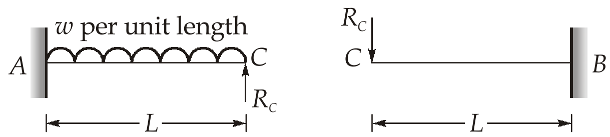
$$\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right\}_{\max} = \frac{\sigma_y}{2(\text{FOS})}$$

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_y}{2(\text{FOS})}$$

$$\text{FOS} = \frac{75}{122.17 - 25} = 0.77$$

19. (c)

We know that at internal hinge, deflection will be same at just left and right of hinge. So,



$$(\delta_c)_{\text{left}}\downarrow = \frac{wL^4}{8EI} - \frac{R_cL^3}{3EI} \quad \text{...(i)}$$

$$(\delta_c)_{\text{right}}\downarrow = \frac{R_cL^3}{3EI} \quad \text{...(ii)}$$

So from eq. (i) and (ii)

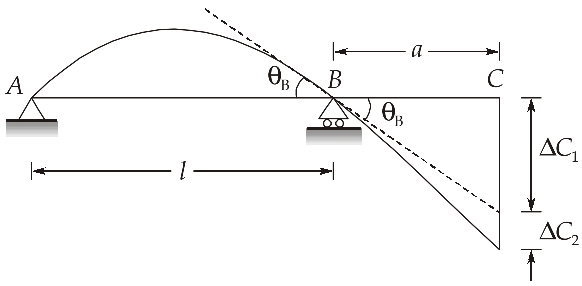
$$\frac{wL^4}{8EI} - \frac{R_cL^3}{3EI} = \frac{R_cL^3}{3EI}$$

$$\Rightarrow \quad \frac{2R_cL^3}{3EI} = \frac{wL^4}{8EI}$$

$$\Rightarrow \quad R_c = \frac{3}{16}wL$$

20. (d)

The deformation of the beam will be as shown below.



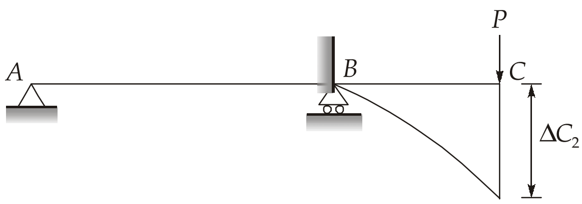
Now ΔC_1 is produced due to deflection of C as caused due to deformation of AB,

$$\Delta C_1 = \theta_B\,(BC) = \theta_B a$$

$$\theta_B = \frac{M_{BA}l}{3EI} = \frac{Pal}{3EI}$$

$$\therefore \quad \Delta C_1 = \frac{Pala}{3EI} = \frac{Pa^2l}{3EI}$$

ΔC_2 is produced due to deformation of BC



$$\Delta C_2 = \frac{Pa^3}{3EI}$$

So total deflection at $C, \Delta C = \Delta C_1 + \Delta C_2$

$$= \frac{Pa^2 l}{3EI} + \frac{Pa^3}{3EI}$$

21. (c)

Modulus of section for both timber sections

$$Z_T = 2\left[\frac{60 \times (200)^2}{6}\right] = 8 \times 10^5 \text{ mm}^3$$

Similarly modulus of section for the steel section

$$Z_S = \frac{15 \times (200)^2}{6} = 10^5 \text{ mm}^3$$

Now, moment of resistance of timber section,

$$M_T = Z_T \cdot \sigma_T$$
$$= 8 \times 10^5 \times 5$$
$$= 4 \times 10^6 \text{ N-mm} = 4 \text{ kN-m}$$

Similarly,

Moment of resistance of steel section,

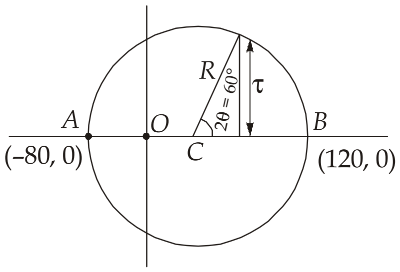
$$M_S = Z_S \cdot \sigma_S$$
$$= 10^5 \times 100$$
$$= 10 \times 10^6 \text{ N-mm}$$
$$= 10 \text{ kNm}$$

Total moment of resistance of the beam

$$M = M_S + M_T = 10 + 4 = 14 \text{ kNm}$$

22. (c)

Radius, $R = \dfrac{120 - (-80)}{2} = 100$



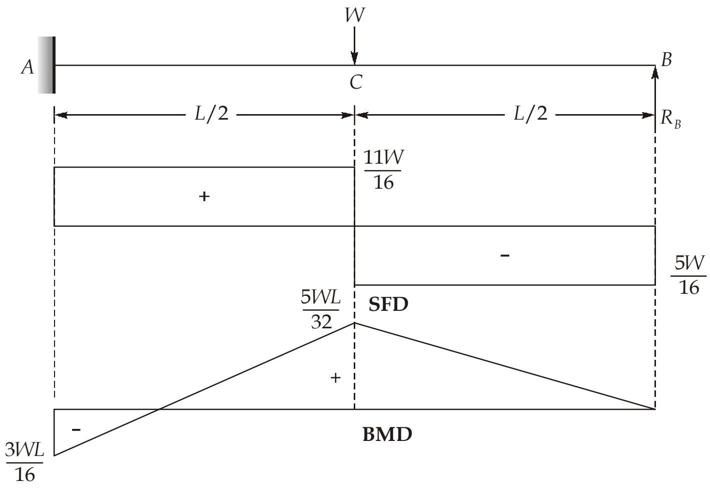
Tangential stress, $\tau = R \sin 2\theta$

$$= 100 \sin 60°$$

$$= 100 \times \frac{\sqrt{3}}{2}$$

$$= 50\sqrt{3}$$

23. (c)



Since end B is propped

Net deflection at B is zero.

$$\Rightarrow \qquad (\delta_B\downarrow)_W = (\delta_B\uparrow)_{R_B}$$

$$\frac{W\left(\frac{L}{2}\right)^3}{3EI} + \frac{W\left(\frac{L}{2}\right)^2}{2EI}\times\frac{L}{2} = \frac{R_B L^3}{3EI}$$

$$\frac{WL^3}{24EI} + \frac{WL^3}{16EI} = \frac{R_B L^3}{3EI}$$

$$\frac{2WL^3 + 3WL^3}{48EI} = \frac{R_B L^3}{3EI}$$

$$R_B = \frac{5W}{16}$$

$$\therefore \qquad R_A = W - R_B = W - \frac{5W}{16}$$

$$= \frac{11W}{16}$$

Bending moment diagram:

For BC: $\quad M_x = R_B x = \frac{5Wx}{16}$

At $x = 0$; $\quad M_B = 0 \Rightarrow$ Moment at propped end is zero.

At $x = \frac{L}{2}$; $\quad M_C = \frac{5WL}{32}$

For CA: $\quad M_x = \frac{5Wx}{16} - W\left(x - \frac{L}{2}\right)$

At $x = \frac{L}{2}$; $\quad M_C = \frac{5WL}{32}$

At $x = L$; $\quad M_A = -\dfrac{3WL}{16}$

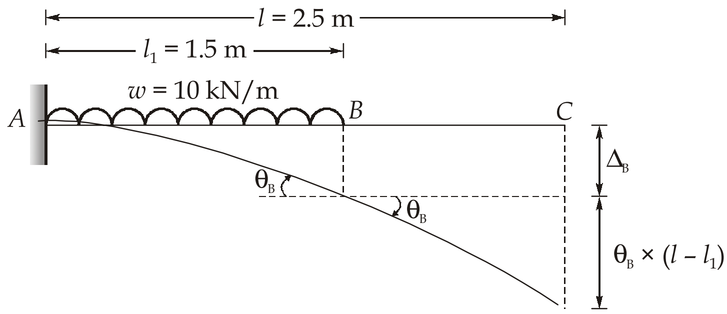
$BM_{xx} = 0$

$$\frac{5Wx}{16} = Wx - \frac{WL}{2}$$

$$x = \frac{8L}{11} \qquad \text{(From prop end)}$$

$\therefore$ The point of contraflexure is at $\left(L - \dfrac{8L}{11}\right) = \dfrac{3}{11}$ from fixed end.

24. (a)



We know deflection at free end for the case given in question,

$$y = \frac{wl_1^4}{8EI} + \frac{wl_1^3}{6EI}(l - l_1)$$

where l = 2.5 m, l_1 = 1.5 m, w = 10 kN/m = 10 N/mm

$$y = \frac{10\left[1.5\times10^3\right]^4}{8\times1.9\times10^{12}} + \frac{10\left[1.5\times10^3\right]^3}{6\times1.9\times10^{12}}\times[2.5-1.5]\times10^3$$

$$= 6.29 \text{ mm} \simeq 6.3 \text{ mm}$$

25. (b)

Since section is symmetric about x-x and y-y, therefore centre of section will lie on the geometrical centroid of section.

The semi-circular grooves may be assumed together and consider one circle of diameter 60 mm.

So, $\quad I_{xx} = \dfrac{80\times(100)^3}{12} - \dfrac{\pi}{64}(60)^4$

$$= 6.03 \times 10^6 \text{ mm}^4$$

Now for shear stress at neutral axis, consider the area above the neutral axis,

$$A\bar{y} = [80\times50\times25] - \frac{\pi}{2}(30)^2\times\frac{4\times30}{3\pi}$$

$$= 100000 - 18000 = 82000 \text{ mm}^3$$

$$b = 20 \text{ mm}$$

So, $$\tau = \frac{VA\bar{y}}{Ib} = \frac{20\times10^3\times82000}{6.03\times10^6\times20}$$
$$= 13.60 \text{ MPa}$$

26. (c)

(i) Force on shaded area $= \dfrac{f_{\max}}{y_{\max}}Ay$

where, A is area of shaded portion, y is distance of centroid of shaded area from NA

$$Ay = 5\times5\times(5+2.5) = 187.5 \text{ cm}^3$$

So, $$\text{Force} = \frac{80}{10}\times187.5 = 1500 \text{ kg}$$

(ii) Moment of this force about the neutral axis

$$M = \frac{f_{\max}}{y_{\max}}I_o$$

(I_o = Moment of inertia of shaded area about neutral axis)

$$I_o = \frac{5\times5^3}{12}+5\times5\times(7.5)^2 = \frac{4375}{3} \text{ cm}^4$$

So, $$M = \frac{80}{10}\times\frac{4375}{3} = 11666.67 \text{ kg cm}$$

27. (c)

$$U = \frac{P^2L}{2AE} \qquad \text{(For axially loaded bar)}$$

$L_1 = 10$ cm, $L_2 = 20$ cm, $d_1 = 2$ cm and $d_2 = 4$ cm

$$U_A = \frac{P^2L_1}{2A_1E}+\frac{P^2L_2}{2A_2E}$$
$$U_B = \frac{P^2L_2}{2A_1E}+\frac{P^2L_1}{2A_2E}$$

$$\therefore \quad \frac{U_B}{U_A} = \frac{\dfrac{L_2}{A_1}+\dfrac{L_1}{A_2}}{\dfrac{L_1}{A_1}+\dfrac{L_2}{A_2}}$$

$$= \frac{L_1d_1^2+L_2d_2^2}{L_1d_2^2+L_2d_1^2}$$

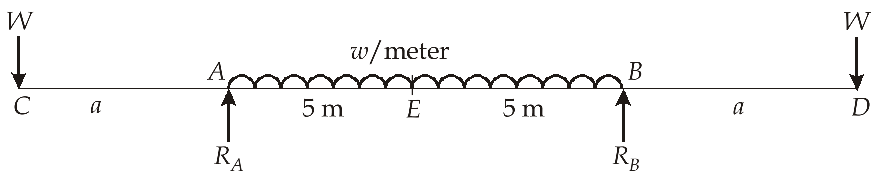
$$= \frac{10\times2^2+20\times4^2}{10\times4^2+20\times2^2} = \frac{3}{2} = 1.5$$

28. (b)

$$R_A = R_B = \frac{1}{2}(2W + 10w) = W + 5w$$

$$= 15w \qquad (W = 10\,w)$$

Now



$$M_E = R_A \times 5 - W(a + 5) - w \times 5 \times 2.5 = 0$$

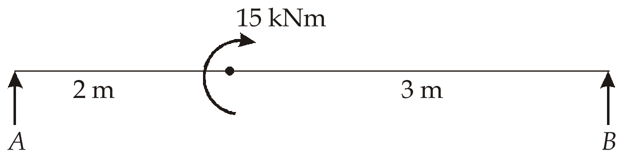
$\Rightarrow$ $$15w \times 5 - 10w\,(a + 5) - 12.5w = 0$$

$\Rightarrow$ $$75 - 10a - 50 - 12.5 = 0$$

$\Rightarrow$ $$12.5 = 10a$$

$\Rightarrow$ $$a = 1.25 \text{ m}$$

29. (c)



$$R_A + R_B = 0$$

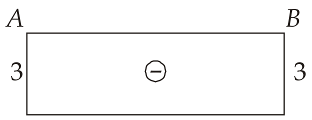
$$\Sigma M_A = 0$$

$\Rightarrow$ $$R_B \times 5 = 15$$

$\Rightarrow$ $$R_B = 3 \text{ kN}$$

$$R_A = -3 \text{ kN}$$

Now the for SFD will be as shon below.



30. (b)

$$\text{Total load} = 2\left[\frac{1}{2}\left(\frac{L}{2}\right)\times w_0\right] = \frac{w_0 L}{2}$$

By symmetry, $$R_1 = R_2 = \frac{1}{2}\times\text{Total load}$$

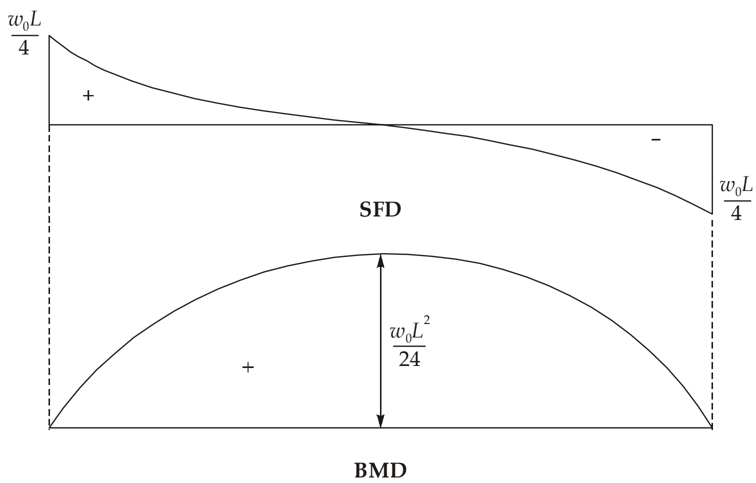
$$\Rightarrow \quad R_1 = R_2 = \frac{w_0 L}{4}$$

Bending moment at B,

$$(M_B) = R_1 \times \frac{L}{2} - \frac{1}{2} w_0 \times \frac{L}{2} \times \frac{2}{3}\left(\frac{L}{2}\right)$$

$$= \frac{w_0 L}{4} \times \frac{L}{2} - \frac{w_0}{2} \times \frac{L}{2} \times \frac{2}{3}\left(\frac{L}{2}\right)$$

$$= \frac{w_0 L^2}{8} - \frac{w_0 L^2}{12} = \frac{w_0 L^2}{24}$$



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