CLASS TEST

S.No. : 01JP_CS_03042023

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COMPUTER SCIENCE & IT

Discrete Mathematics

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q. No. 1 to Q. No. 10 carry 1 mark each

Q.1 The number of ways of arranging the letters GGGGGAAATTTECCS in a row when no two T's are together is

(a) $\frac{12!}{5!3!2!} \times {}^{13}P_3$ (b) $\frac{12!}{5!3!2!} \times \frac{{}^{13}P_3}{3!}$

(c) $\frac{15!}{5!3!2!} - \frac{13!}{5!3!2!}$ (d) $\frac{15!}{5!3!3!2!} - 3!$

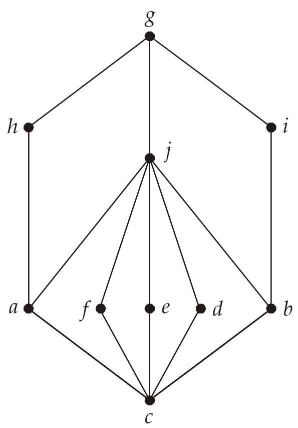
Q.2 Let G be a planar graph such that every face is bordered by exactly 3 edges. Which of the following can never be the value for $\chi(G)$? [Where $\chi(G)$ is chromatic number of G]

(a) 2 (b) 3
(c) 4 (d) None of these

Q.3 The proposition
$[(p \wedge q) \rightarrow (p \vee q)] \vee \sim p \vee q$ is

(a) A tautology
(b) Satisfiable but not tautology
(c) A contradiction
(d) None of the above

Q.4 Consider the following lattice:



Find the number of complements for the element 'a'

(a) 1 (b) 3
(c) 5 (d) 7

Q.5 Consider a relation $R = \{(x, y) \mid x, y$ are positive integers ≤ 4 and $(x + y) \leq 5\}$. Which of the following is true?

(a) R is reflexive, symmetric and transitive
(b) R is symmetric and transitive
(c) R is antisymmetric and transitive
(d) R is symmetric

Q.6 How many solutions are there of $x + y + z = 17$ in positive integers?

(a) 120 (b) 171
(c) 180 (d) 221

Q.7 Identify the contrapositive for the following statement. "If a real number is greater than 2, then its square is greater than 4."

(a) $\forall x \in R$, if $x^2 > 4$ then $x < 2$
(b) $\forall x \in R$, if $x^2 \leq 4$ then $x \leq 2$
(c) $\forall x \in R$, if $x \leq 2$ then $x^2 \leq 4$
(d) None of these

Q.8 Find the number of perfect matchings in a cycle graph with $2n$ vertices is ________.

(a) 0 (b) 2
(c) n (d) $\frac{n}{2}$

Q.9 Let X = {{ }, {a}}. The power set of X is ________.

(a) {{ }, {a}}
(b) {{ }, {a}, {{ }}, {{ }, a}}
(c) {{ }, {{a}}, {{ }, a}}
(d) {{ }, {{a}}, {{ }}, {{ }, {a}}}

Q.10 A set of mn objects can be partitioned into 'm' sets of size 'n' in ________ different ways.

(a) $\frac{m!}{(n!)^m}$ (b) $\frac{(mn)!}{(n!)^m m!}$

(c) $\frac{(mn)!}{(n!)^m}$ (d) None of these

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 Consider the following predicate statements:

$P_1 : \neg\forall x \neg(P(x) \rightarrow \exists y\, Q(y))$

$P_2 : \exists x(\neg P(x) \vee \exists y\, Q(y))$

$P_3 : \exists x(\neg \exists y Q(y) \rightarrow \neg P(x))$

$P_4 : \neg\forall x(P(x) \wedge \neg \exists y\, Q(y))$

Which of the above predicates are equivalent to the predicate statement:

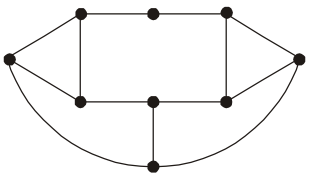
$\exists x(P(x) \rightarrow \exists y\, Q(y))$

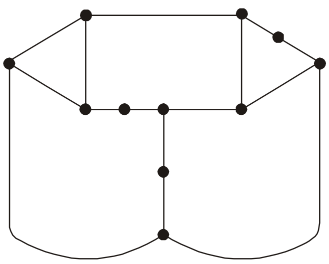
(a) P_1, P_2, P_3 (b) P_1, P_3, P_2
(c) P_2, P_3, P_4 (d) All of these

Q.12 Which one of the first order predicate calculus statements given below correctly expresses the following English statement? (Domain consists of all people; $F(x)$: x is a female, $M(x, y)$: x is mother of y, $P(x)$: x is a parent).
"If a person is female and is a parent, then this person is someone's mother"

(a) $\exists x\big((F(x) \wedge P(x)) \rightarrow \forall y\, M(x, y)\big)$
(b) $\forall x \exists y\big((F(x) \wedge P(x)) \rightarrow M(x, y)\big)$
(c) $\forall x\big((F(x) \vee P(x)) \rightarrow \exists y\, M(x, y)\big)$
(d) $\forall x \exists y\big((F(x) \vee P(x)) \rightarrow M(x, y)\big)$

Q.13 Consider the following graphs:

G_1 :


G_2 :


Identify the correct statement?
(a) Only G_1 is Hamiltonian.
(b) Only G_2 is Hamiltonian.
(c) Both G_1 and G_2 are Hamiltonian.
(d) Neither G_1 nor G_2 is Hamiltonian.

Linked Answer Q.14 and Q.15:

Let G be a graph with $V(G) = \{i \mid 1 \le i \le 4n, n \ge 1\}$ where $V(G)$ is the set of vertices of G. Such that two numbers x and y in $V(G)$ are adjacent if and only if $(x + y)$ is a multiple of 4.

Q.14 Find the number of components in graph G.
(a) 2 (b) 4
(c) n (d) None of these

Q.15 From the above k components, assume that each component C_k has m_k vertices, then what is the maximum value of m_k in the graph G?
(a) n (b) $2n$
(c) $3n$ (d) None of these

Q.16 Assume that connected simple graph 'G' has n vertices ($n \ge 3$) and e edges. Which of the following statement is invalid?
(a) If G is planar then $e \le (3n - 6)$.
(b) If $e \le (3n - 6)$ then G is a planar.
(c) If $e > (3n - 6)$ then G is not planar.
(d) If G is not planar then $e > (3n - 6)$.

Q.17 Solve the following recurrence relation.

$$T(n) = 3T(n - 1) + 2^n,\ n > 0 \text{ and } T(0) = 1$$

(a) $3^{n+1} - 2^{n+1}$ (b) $3^n - 2^n$
(c) $3^{n+1} - 2^n$ (d) $3^{n+1} + 2^n$

Q.18 Find the predicate logic for the following statement.
There are atmost two apples.
(a) $\forall x\ \forall y$ ((Apple $(x) \wedge$ Apple $(y)) \rightarrow (x = y \vee y = x)$)
(b) $\exists x\ \exists y$ (Apple $(x) \wedge$ Apple $(y) \wedge x \ne y \wedge \forall z$ (Apple $(z) \rightarrow (z = x \vee z = y)$))
(c) $\forall x\ \forall y\ \forall z$ ((Apple $(x) \wedge$ Apple $(y) \wedge$ Apple $(z)) \rightarrow (x = y \vee x \vee y = z)$)
(d) None of these

Q.19 Consider the following statements:
Let G be a connected graph of order n where $n \ge 3$.

S_1 : If $d(V) \ge \frac{n}{2}$ for each vertex V in G, then G is Hamiltonian. [$d(V)$ is the degree of a vertex V]
S_2 : If every vertex in G has even degree then G is eulerian.

Which of the above statements are correct?
(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

Q.20 Which of the following is/are tautology?
(a) $(p \to q) \to (q \to p)$
(b) $\sim(p \to q) \to \sim q$
(c) $(p \to q) \leftrightarrow (\sim q \to \sim p)$
(d) Both (b) and (c)

Q.21 Which of the following is a function from R to R? (Where R is the set of real numbers)
(a) $f(x) = \frac{1}{x}$ (b) $f(x) = \sqrt{x}$
(c) $f(x) = \pm\sqrt{x^2+1}$ (d) $f(x) = |x|$

Q.22 Consider the following statements:
S_1 : Bipartite graph cannot have an odd length cycle
S_2 : Any graph with only even cycles is bipartite graph
(a) Only S_1 is true.
(b) Only S_2 is true.
(c) S_1 and S_2 both are true.
(d) None of these

Q.23 The function $f(x, y) = x + y$ from $R \times R \to R$ is
(a) Injective and subjective
(b) Injective but not subjective
(c) Subjective but not injective
(d) Neither injective nor subjective

Q.24 Which of the following is false?
(a) The degree sequence of every simple graph must have at least one repetition of degree.
(b) A graph which is bipartite cannot contain any triangle.
(c) The number of vertices of even degree in any undirected graph is necessarily odd.
(d) For any graph either G or $\bar{G}$ is always connected.

Q.25 How many 6 letter passwords of lower case English letters contain exactly 1 vowel.
(a) $6 \times 5 \times 21^5$ (b) $5C_1 \times 21C_5 \times 6!$
(c) $26^6 - 21^6$ (d) None of these

Q.26 Consider a domain $s = \{1, 2, 3, 4\}$
$P(x, y): x * y \geq 2$
Which of the following is true?
1. $\forall x \forall y\, P(x, y)$ **2.** $\forall x \exists y\, P(x, y)$
3. $\exists x \forall y\, P(x, y)$ **4.** $\exists x \exists y\, P(x, y)$
(a) Only 2 (b) 2, 3, 4
(c) 1, 2, 3 (d) All of these

Q.27 The number of vertices in a 4-regular graph with 32 edges?
(a) 8 (b) 16
(c) 24 (d) 4

Q.28 Let Z be the set of integers and
Let $*$ be a binary operator defined as $a * b = ab + b$
The structure $(Z, *)$ is
(a) A groupoid and not a semigroup.
(b) A semigroup but not a monoid.
(c) A monoid but not a group.
(d) A group.

Q.29 Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is duck", "x is annoying", and "x is dancer", respectively. Then which of following is/are true when statement are expressed using quantifiers, logic connective and $P(x)$, $Q(x)$ and $R(x)$.
I. All ducks are annoying can be expressed as $\forall x(P(x) \wedge Q(x))$.
II. Some dancers are not annoying can be expressed as $\exists x\, (R(x) \to \neg Q(x))$.
(a) I is true II is false
(b) II is true I is false
(c) Both I and II are true
(d) Both I and II are false

Q.30 Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male". If $F(v, w)$ equals
$M(v) \wedge \exists x \exists y\, (P(x, y)\ P(x, v)\ (y \neq v) \wedge P(y, w))$.
What is the meaning of the expression $F(v, w)$?
(a) v is a brother of w.
(b) v is a nephew of w.
(c) v is an uncle of w.
(d) v is a grandfather of w.

■■■■

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Date of Test : 03/04/2023

ANSWER KEY

1.	(b)	7.	(b)	13.	(a)	19.	(c)	25.	(a)
2.	(a)	8.	(b)	14.	(d)	20.	(d)	26.	(b)
3.	(a)	9.	(d)	15.	(b)	21.	(d)	27.	(b)
4.	(a)	10.	(b)	16.	(b)	22.	(c)	28.	(a)
5.	(d)	11.	(d)	17.	(a)	23.	(c)	29.	(d)
6.	(a)	12.	(b)	18.	(c)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. **(b)**

Total number of letters = 15

Number of T's = 3

First place 12 letters other than T's at dot places.

X. X. X. X. X. X. X. X. X. X. X. X. X

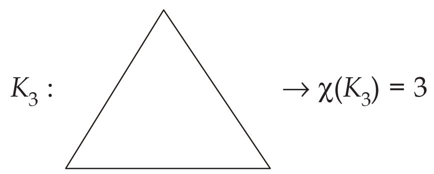
$$\text{The number of ways} = \frac{12!}{5!3!2!}$$

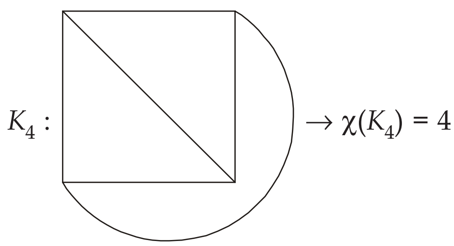
Since no two T's are together, thus place T's at cross places whose number = 13

$$\text{Their arrangements are } \frac{^{13}P_3}{3!}$$

$$\text{Total number of ways} = \frac{12!}{5!3!2!} \times \frac{^{13}P_3}{3!}$$

2. **(a)**





Any planar graph $G \Rightarrow \chi(G) \le 4$

Every face is bordered by exactly 3 edges $\Rightarrow \chi(G) \ge 3$

$\therefore \quad \chi(G) = 3 \text{ or } 4$

So, $\chi(G)$ can never have the value 2.

3. **(a)**

$$[(p \wedge q) \rightarrow (p \vee q)] \vee \sim p \vee q \equiv \sim(p \wedge q) \vee (p \vee q) \vee \sim p \vee q$$
$$\equiv (\sim p \vee \sim q) \vee (p \vee q) \vee \sim p \vee q$$
$$\equiv (\sim p \vee p) \vee (\sim q \vee q) \vee \sim p \vee q$$
$$\equiv T \vee T \vee \sim p \vee q \equiv T$$

4. **(a)**

LUB = g, GLB = c

$a \vee i = g$ and $a \wedge i = c$

$\therefore$ a has only 1 complement [complement of $a = i$]

[**Note:** x and y are complement to each other iff $x \vee y$ = LUB and $x \wedge y$ = GLB]

5. (d)

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$$

R is not reflexive: $(3, 3) \notin R$

R is symmetric: if $(x + y) \leq 5 \Rightarrow (y + x) \leq 5$

R is not antisymmetric: (1, 2) and (2, 1) in R

R is not transitive: (3, 1) and (1, 3) in R, but $(3, 3) \notin R$

$\therefore$ R is symmetric.

6. (a)

$$x + y + z = 17$$

$$x \geq 1, y \geq 1, z \geq 1$$

Put $x = 1 + u, y = 1 + v, z = 1 + w$

$\Rightarrow$ $u + v + w = 17 - 3 = 14$

Now number of solutions in non-negative integers

$$\binom{14+3-1}{14} = \binom{16}{14} = \binom{16}{2} = 120$$

7. (b)

Given: $\forall x \in R$, if $x > 2$ then $x^2 > 4$

Contrapositive is: $\forall x \in R$, if $x^2 \leq 4$ then $x \leq 2$

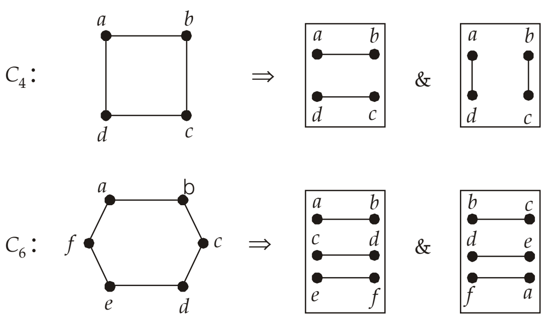
Converse is: $\forall x \in R$, if $x^2 > 4$ then $x > 2$

Inverse is: $\forall x \in R$, if $x \leq 2$ then $x^2 \leq 4$

$\therefore$ Option (b) is correct.

8. (b)

Example: $C_{2n} \Rightarrow n = 2 \Rightarrow C_4$



A cycle graph with even vertices has 2 perfect matchings.

9. (d)

$$X = \{\{ \}, \{a\}\}$$

$$X = \{p, q\} \text{ [Assume } p = \{ \}, q = \{a\}]$$

$$P(X) = \{\{ \}, \{p\}, \{q\}, \{p, q\}\}$$

$$= \{\{ \}, \{\{ \}\}, \{\{a\}\}, \{\{ \}, \{a\}\}\}$$

10. (b)

Example: In how many ways can the pack of 52 cards be partitioned into 4 sets of size 13.

$$\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} = \frac{(52)!}{(13!)^4}$$

All partitions are not distinct. Each distinct partition arises in 4! ways.

Therefore number of ways $= \dfrac{(52)!}{(13!)^4 \cdot 4!}$

Similarly, $\dfrac{\prod_{i=0}^{m-1}\binom{mn-in}{n}}{m!} = \dfrac{(mn)!}{(n!)^m \cdot m!}$

So, option (b) is correct.

11. (d)

$$\begin{aligned}\exists x\left(P(x)\rightarrow \exists y\ Q(y)\right) &\cong \neg\forall x\neg\left(P(x)\rightarrow \exists y\ Q(y)\right)\\ &\cong \exists x\left(\neg P(x)\vee \exists y\ Q(y)\right)\\ &\cong \exists x\left(\neg\ \exists y\ Q(y)\rightarrow \neg P(x)\right)\\ &\cong \neg\forall x\left(P(x)\wedge \neg\ \exists y\ Q(y)\right)\end{aligned}$$

So option (d) is correct.

12. (b)

For every person x, if person x is female and person x is a parent, then there exists a person y such that person x is the mother of person y.

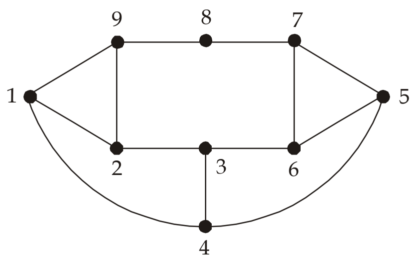
$F(x)$: x is female

$P(x)$: x is a parent.

$M(x, y)$: x is the mother of y

$$\forall x\left(\left(F(x)\wedge P(x)\right)\rightarrow \exists y\, M(x, y)\right) \cong \forall x\exists y\left(\left(F(x)\wedge P(x)\right)\rightarrow M(x, y)\right)$$

13. (a)

G_1 : 

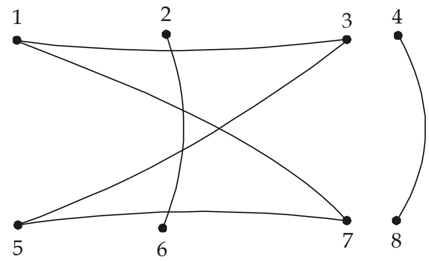
is Hamiltonian [∵ Hamiltonian cycle exists 1, 2, 9, 8, 7, 5, 6, 3, 4, 1]

G_2 **:** Not Hamiltonian graph [∵ Hamiltonian cycle does not exists in G_2]

∴ Only G_1 is Hamiltonian graph.

14. (b)

Let $n = 2 \Rightarrow$ Number of vertices = 8 $[\because$ Number of vertices in $G = 4n]$



$\Rightarrow$ 3 components [**Note:** For any n, the #components in G = 3]

$\therefore$ Option (d) is correct for Q.14

15. (b)

$$\left.\begin{aligned} V(C_1) &= \{1, 3, 5, 7\} \Rightarrow m_1 = 4 \\ V(C_2) &= \{2, 6\} \Rightarrow m_2 = 2 \\ V(C_3) &= \{4, 8\} \Rightarrow m_3 = 2 \end{aligned}\right\} \max = 4$$

In general, 'G' with $4n$ vertices has 3 components

$$V(C_1) = \{1, 3, 5, 7, 9, 11, 13, 15, ...\} \Rightarrow m_1 = 2n$$
$$V(C_2) = \{2, 6, 10, 14, 18, ...\} \Rightarrow m_2 = n$$
$$V(C_3) = \{4, 8, 12, 16, 20, ...\} \Rightarrow m_3 = n$$

$\therefore$ $\text{Max}(m_1, m_2, m_3) = 2n$

$\therefore$ Option (b) is correct for Q. 15

16. (b)

If G is planar then $e \le 3n - 6$ but converse need not be true.

If G is not planar then $e > 3n - 6$ and converse holds.

$\therefore$ If $e \le (3n - 6)$ then G need not be planar.

17. (a)

General solution: $T(n) = C_1 . 3^n + C_2 . 2^n$

$$\begin{bmatrix} \text{Where Homogeneous part: } (x-3) = 0 \Rightarrow x = 3 \\ \text{Particular solution: } C_2 .2^n \end{bmatrix}$$

$$T(0) = 1, T(1) = 3.T(0) + 2' = 3 + 2 = 5$$
$$T(n) = C_1.3^n + C_2.2^n$$
$$T(0) = C_1.3^0 + C_2.2^0 \Rightarrow 1 = C_1 + C_2$$
$$T(1) = C_1.3' + C_2.2' \Rightarrow 5 = 3C_1 + 2C_2$$
$$\overline{C_1 = 3, C_2 = -2}$$

$\therefore \quad T(n) = 3.3^n - 2.2^n$
$= 3^{n+1} - 2^{n+1}$

18. **(c)**
$\forall x \; \forall y \; \forall z \; ((\text{Apple}(x) \wedge \text{Apple}(y) \wedge \text{Apple}(z)) \rightarrow (x = y \vee x = z \vee y = z))$

19. **(c)**
S_1 is true but converse of S_1 is not true.
S_2 is true and converse of S_2 is also true because G is connected graph.

20. **(d)**

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim q$	$\sim(p \rightarrow q) \rightarrow \sim q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

p	q	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

21. **(d)**
(a) If $x = 0$ then we don't get real number.
(b) If $x = -1$ we don't get real number.
(c) For $x = 1$ we get two images.
(d) For every value of x we get only one image and if we give real numbers, we get real numbers.

23. **(c)**
Since $\quad f(1, 2) = 3$
$f(2, 1) = 3$
It is not one-to-one i.e., not injective.
Since $\forall a \in R$
$f(a, 0) = 0$
So, every real number has a partner $(a, 0)$ in $R \times R$.
So it is onto i.e., subjective.
So the function is subjective but not injective.

25. **(a)**
The position of the one vowel can be chosen in $6C_1 = 6$ ways. Then the vowel can be chosen in $5C_1 = 5$ ways. Then the remaining 5 positions in the word can be filled in 21^5 ways by using consonants.
So answer is $6 \times 5 \times 21^5$.

26. **(b)**
If $x = 1$ and $y = 1$ it is not true so '1' is false.

For all x we can have atleast one y so that given equation is true. So 2 is true.
For atleast one x we have any y such that $p(x, y)$ is true. If $x >= 2$ it is true so 3 is true.
For atleast one x we have atleast one y such that $p(x, y)$ is true. So 4 is true.

27. **(b)**

Sum of degrees theorem. $4*V = 2 \times E = 64$

$V = 16$

29. **(d)**

(i) Correct expression is $\forall x(P(x) \rightarrow Q(x))$.

(ii) $\exists x(R(x) \wedge \neg Q(x))$ is the correct expression.

■■■■