S.No. : 04 SP_EC_W+Y_110819
Network Theory

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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

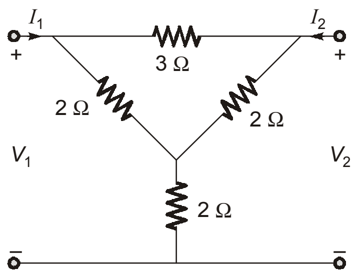
Date of Test : 11/08/2019

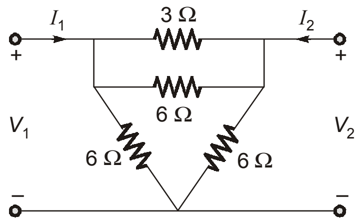
ANSWER KEY ➤ Network Theory

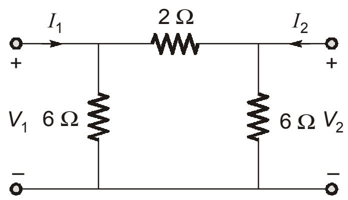
1. (b)	7. (d)	13. (a)	19. (b)	25. (d)
2. (c)	8. (c)	14. (c)	20. (b)	26. (a)
3. (b)	9. (a)	15. (b)	21. (b)	27. (b)
4. (b)	10. (b)	16. (c)	22. (c)	28. (b)
5. (a)	11. (c)	17. (a)	23. (d)	29. (b)
6. (b)	12. (d)	18. (c)	24. (b)	30. (d)

Detailed Explanations

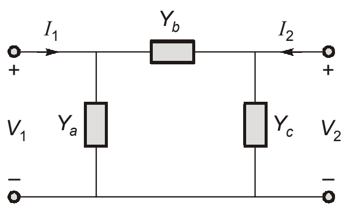
1.(b)







for Π-network



$$\because \qquad [Y] = \begin{bmatrix} Y_a + Y_b & -Y_b \\ -Y_b & Y_b + Y_c \end{bmatrix}$$

for the given problem,
$$Y_a = \frac{1}{6}\ \mho$$
$$Y_b = \frac{1}{2}\ \mho$$
$$Y_c = \frac{1}{6}\ \mho$$

$$\therefore \qquad [Y] = \begin{bmatrix} \frac{2}{3}\ \mho & -\frac{1}{2}\ \mho \\ -\frac{1}{2}\ \mho & \frac{2}{3}\ \mho \end{bmatrix}$$

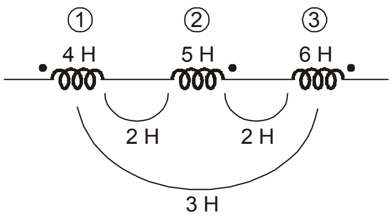
2.(c)

$$\text{Time period } (T) = \frac{2\pi}{\omega}$$

where
$$\omega = \frac{1}{\sqrt{LC}}$$

thus, $T = 2\pi\sqrt{LC}$

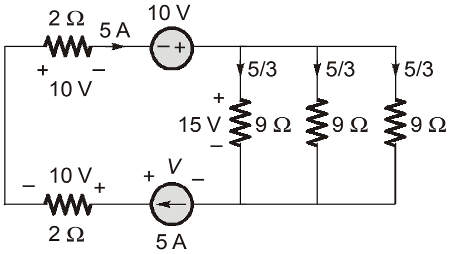
In figure



$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{13}$$
$$= 4 + 5 + 6 - 2(2) + 2(2) - 2(3) = 9\text{ H}$$
$$C = 1\text{ F}$$
$$T = 2\pi\sqrt{9} = 6\pi\text{ sec}$$

3.(b)

The circuit can be redrawn by short circuiting inductor and open circuiting capacitor as DC sources are used.



Applying KVL

$$V - 10 - 10 + 10 - 15 = 0$$
$$V = 25\text{ V}$$

4.(b)

$$\frac{R}{2}\sqrt{\frac{L}{C}} \Rightarrow \qquad \frac{R}{2}\sqrt{\frac{L\omega}{C\omega}} = \frac{R}{2}\sqrt{X_L X_C}$$

Unit of R_1 is Ω

Unit of 'X_L' and 'X_C' is Ω

Unit of $\frac{R}{2}\sqrt{\frac{L}{C}}$ is $\Omega \times \sqrt{\Omega \times \Omega} \quad \Rightarrow (\Omega)^2$

5.(a)

The average value of periodic signal can be calculated by considering one time period

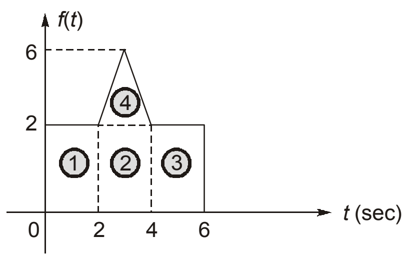
$$= \frac{\text{Total area under the graph for one period}}{T_0}$$

Total area under the graph for one period = Area 1 + Area 2 + Area 3 + Area 4

here Area 1 = Area 2 = Area 3 = Area 4 = 4

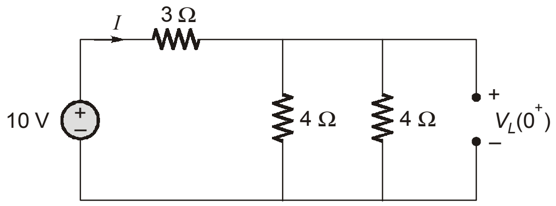
and T_0 = 8 sec

$$\text{Average value} = \frac{4+4+4+4}{8} = \frac{16}{8} = 2$$



6.(b)

Before closing the switch, the circuit was not energized, therefore, current through inductor and voltage across capacitor are zero.
After closing the switch, at $t = 0^+$ inductor acts as open-circuit and capacitor acts as short-circuit.
Equivalent circuit at $t = 0^+$

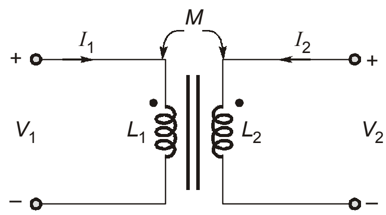


$$I = \frac{10}{3+4||4} = 2\text{ A}$$

$$V_L(0^+) = I \times (4||4)$$

$$= 2 \times 2 = 4\text{ V}$$

7.(d)



Applying KVL in both the loops we get

$$V_1 = (j\omega L_1)I_1 + (j\omega M)I_2$$

$$V_2 = (j\omega L_2)I_2 + (j\omega M)I_1$$

$$\frac{V_2}{V_1} = \frac{L_2I_2 + MI_1}{L_1I_1 + MI_2}$$

also, $M = K\sqrt{L_1L_2} = \sqrt{L_1L_2}$;

$\because K = 1$ for ideal transformer

$$\frac{V_2}{V_1} = \frac{L_2I_2 + \sqrt{L_1L_2}I_1}{L_1I_1 + \sqrt{L_1L_2}I_2}$$

$\therefore$
$$\frac{V_2}{V_1} = \frac{\sqrt{L_2}}{\sqrt{L_1}}$$

$\Rightarrow$
$$\frac{V_2}{V_1} = \sqrt{\frac{5}{25}} = \frac{1}{\sqrt{5}}$$

8.(c)

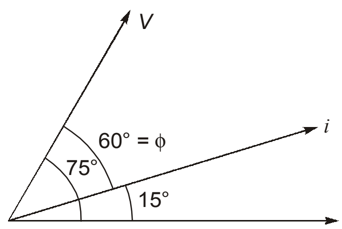
As we know,

$$\text{Real power} = V_{rms} \cdot I_{rms}\cos\phi \quad \text{...(i)}$$

$$\text{Reactive power} = V_{rms} \cdot I_{rms}\sin\phi \quad \text{...(ii)}$$

$$\text{Apparent power} = V_{rms} \cdot I_{rms} \quad \text{...(iii)}$$

Given that
$$v(t) = 10\cos(2t + 75°)$$
$$i(t) = 2\cos(2t + 15°)$$



$\Rightarrow \phi = 60°$

from equation (i)

$$\text{Real power} = \frac{10}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \frac{1}{2} = 5\text{ Watts}$$

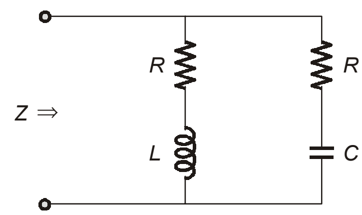
from equation (ii)

$$\text{Reactive power} = \frac{10}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = 5\sqrt{3} = 8.66 \text{ VAR}$$

from equation (iii)

$$\text{Apparent power} = \frac{10}{\sqrt{2}} \times \frac{2}{\sqrt{2}} = 10 \text{ VA}$$

9.(a)



$$Y = Y_1 + Y_2$$

$$Y = \frac{1}{R + jX_L} + \frac{1}{R - jX_C}$$

$$Y = \frac{R - jX_L}{(R^2 + X_L^2)} + \frac{(R + jX_C)}{(R^2 + X_C^2)}$$

$$\text{Im}\,(Y) = \frac{-X_L(R^2 + X_C^2) + X_C(R^2 + X_L^2)}{(R^2 + X_L^2)\,(R^2 + X_C^2)}$$

For 'Z' to purely resistive

also $\text{Im}(Y) = 0$

$$\Rightarrow \quad X_L(R^2 + X_C^2) = X_C(R^2 + X_L^2)$$

$$R^2X_L + X_C^2X_L = R^2X_C + X_L^2X_C$$

$$R^2(X_L - X_C) = X_LX_C(X_L - X_C)$$

$$R^2 = X_LX_C = \omega L \times \frac{1}{\omega C} = \frac{L}{C}$$

10.(b)

For parallel resonant circuit

$$Q_0 = R\sqrt{\frac{C}{L}}$$

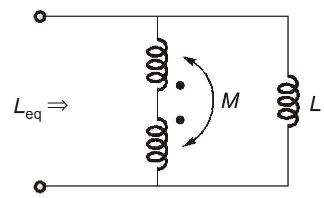
$$Q_0 = 2000\sqrt{\frac{54 \times 10^{-6}}{240 \times 10^{-3}}}$$

$$Q_0 = 2000\sqrt{\frac{9}{4} \times 10^{-4}}$$

$$Q_0 = \frac{2000}{100} \times \frac{3}{2}$$

$$Q_0 = 30$$

11.(c)



$$L_{eq} = (L + L - 2M)\ ||\ L$$

Also

$$M = k\sqrt{L_1L_2} = M = k\sqrt{L^2} = kL$$

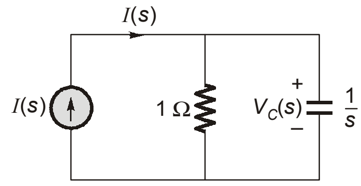
$$L_{eq} = (L + L - 2kL)\ ||\ L$$

$$\frac{L}{3} = \frac{(2L - 2kL) \times L}{2L - 2kL + L}$$

on solving, we get

$$k = 0.75$$

12.(d)



$$V_C(s) = I(s) \times \frac{1}{1+\frac{1}{s}} \times \frac{1}{s} = I(s) \times \frac{1}{s+1}$$

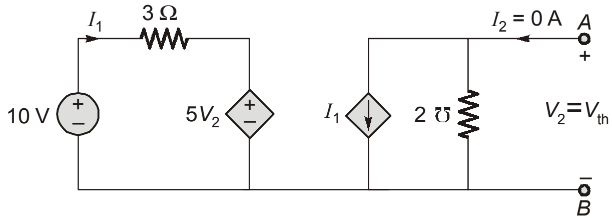
$$I(s) = \frac{2(s+1)}{(s+1)^2+1}$$

$$V_C(s) = \frac{2(s+1)}{(s+1)^2+1} \times \frac{1}{1+s} = \frac{2}{(s+1)^2+1}$$

$$v_C(t) = 2e^{-t} \sin t \, u(t) \text{ V}$$

13.(a)

to determine V_{th} :



$$I_1 = \frac{10-5V_2}{3} = \frac{10-5V_{th}}{3}$$

$$V_{th} = -\frac{I_1}{2} = \frac{5V_{th}-10}{6}$$

$$6V_{th} = 5V_{th} - 10$$

$$V_{th} = -10 \text{ V}$$

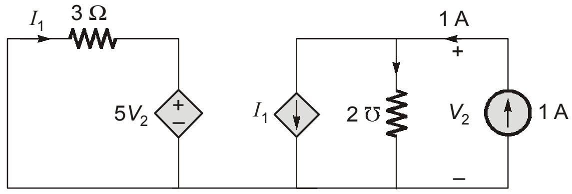
to determine R_{th}:

$$I_2 = 2V_2 + I_1$$

$$1 \text{ A} = 2V_2 + I_1$$

$$0 = 3I_1 + 5V_2$$

$$I_1 = -\frac{5}{3} V_2$$



$$1 \text{ A} = 2V_2 - \frac{5}{3} V_2$$

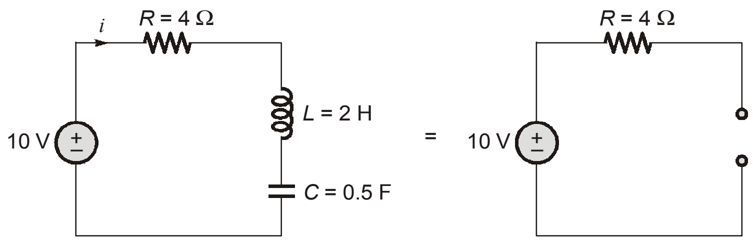
$$V_2 = 3 \text{ V}$$

$$R_{th} = \frac{V_2}{1 \text{ A}} = 3 \, \Omega$$

$$P_{L\,max} = \frac{V_{th}^2}{4R_{th}} = \frac{100}{12} \text{ W} = 8.33 \text{ W}$$

14.(c)

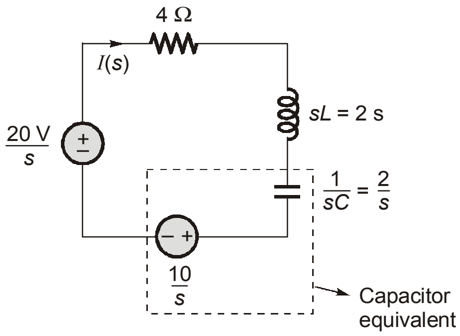
At ($t = 0^-$)



$$V_C(0^-) = 10 \text{ V}$$

$$i_L(0^-) = 0 \text{ A}$$

At ($t = 0^+$)



$$I(s) = \frac{10/s}{4+2s+\frac{2}{s}} = \frac{10/s}{\frac{4s+2s^2+2}{s}} = \frac{10}{2(s^2+2s+1)}$$

$$I(s) = \frac{5}{(s+1)^2}$$

$$i(t) = 5te^{-t}\,u(t) \text{ A}$$

15.(b)

For a series RL circuit with DC excitation,

$$i(t) = \frac{V_s}{R}\left(1-e^{-\frac{Rt}{L}}\right)u(t) \text{ A}$$

$$v(t) = V_s\left(e^{-\frac{Rt}{L}}\right)u(t) \text{ A}$$

$$A = \frac{V_s}{R}$$

$$B = \frac{R}{L}$$

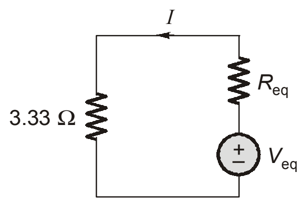
$$C = V_s$$

$$\frac{AB}{C} = \frac{\frac{V_s}{R}\times\frac{R}{L}}{V_s} = \frac{1}{L}$$

$$= \frac{1}{5\times10^{-3}} = 200$$

16.(c)

Applying Millman's Theorem



$$\frac{1}{R_{eq}} = \frac{1}{10}+\frac{1}{30}+\frac{1}{90}+\ldots..$$

$$\frac{1}{R_{eq}} = \frac{1}{10}\left(1+\frac{1}{3}+\frac{1}{9}+\ldots\right)$$

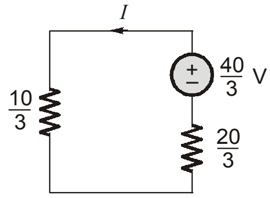
$$\frac{1}{R_{eq}} = \frac{1}{10}\left(\frac{1}{1-\frac{1}{3}}\right) = \frac{3}{10\times2} = \frac{3}{20}$$

$$R_{eq} = \frac{20}{3}\ \Omega$$

$$V_{eq} = \frac{\dfrac{E_1}{R_1} + \dfrac{E_2}{R_2} + \dfrac{E_3}{R_3} + \ldots}{\dfrac{1}{R_{eq}}}$$

$$= \frac{1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots}{\dfrac{3}{20}} = \frac{\dfrac{1}{1-\dfrac{1}{2}}}{\dfrac{3}{20}} = \frac{20}{3} \times 2$$

$$V_{eq} = \frac{40}{3}\ \text{V}$$



$$I = \frac{\dfrac{40}{3}}{\dfrac{20}{3} + \dfrac{10}{3}} = \frac{\dfrac{40}{3}}{\dfrac{30}{3}} = \frac{4}{3}\ \text{A}$$

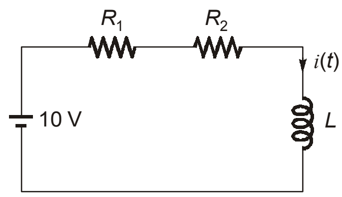
$$I = 1.33\ \text{A}$$

17.(a)

At ($t = 0^-$), both the switches are opened.

L is initially uncharged $i_L(0^-) = 0$

At ($t = 0^+$)



$$i(t) = i(\infty) + (i(0^+) - i(\infty))\ e^{-\frac{R_{eq} t}{L_{eq}}}$$

$$R_{eq} = 5\ \Omega$$

$$L_{eq} = 1\ \text{H}$$

$$i(0^+) = 0\ \text{A}$$

$$i(t) = 2 + (0-2)e^{-\frac{5t}{1}}\ \text{A}\ ;\ \text{for } t > 0$$

At ($t = 2^-$)

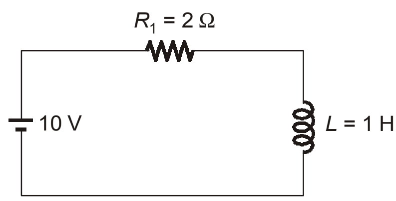
$$i(2^-) = 2 - 2e^{-\frac{10}{1}}\ \text{A}$$

$$i(2^-) \approx 2\ \text{A}$$

At ($t = 2^+$)

$$i(2^-) = i(2^+) = 2\ \text{A}$$

for $t > 2$ sec



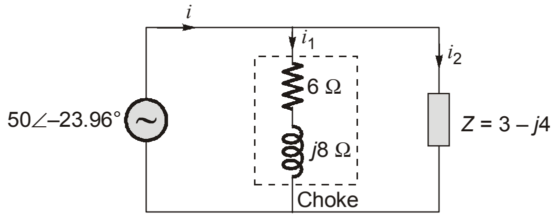
$$i(t) = i(\infty) + (i(2^+) - i(\infty))\ e^{-\frac{R_1(t-2)}{L}}\ ;\ \text{for } t > 2$$

$$i(t)\big|_{t=3s} = 5 + (2-5)e^{-\frac{2}{1}(3-2)} = 5 - 3e^{-2}$$

$$= 4.594\ \text{A}$$

18.(c)

Here, applying KCL



$$i(t) = i_1(t) + i_2(t)$$

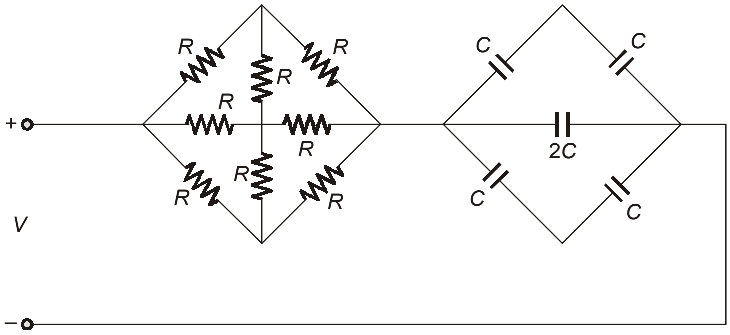
$$= \frac{V_1}{6+j8} + \frac{V_1}{3-j4}$$

$$\Rightarrow \quad V_1\left(\frac{3-j4+6+j8}{2(3+j4)(3-j4)}\right) = \frac{V_1(9+j4)}{2\times 25}$$

$$\Rightarrow \quad \frac{50\angle -23.96^\circ \times \sqrt{97}\angle 23.96^\circ}{50}\text{ A} = \sqrt{97}\text{ A}$$

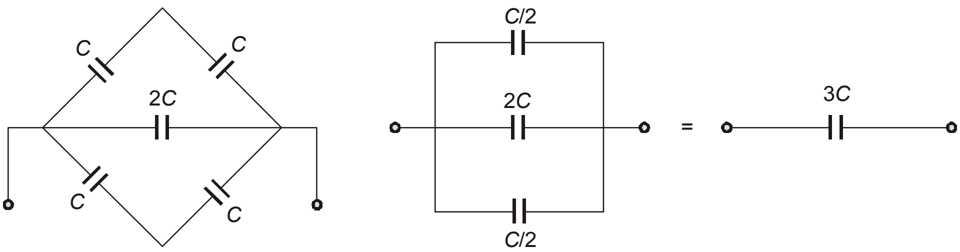
$$= 9.85\text{ A}$$

19.(b)

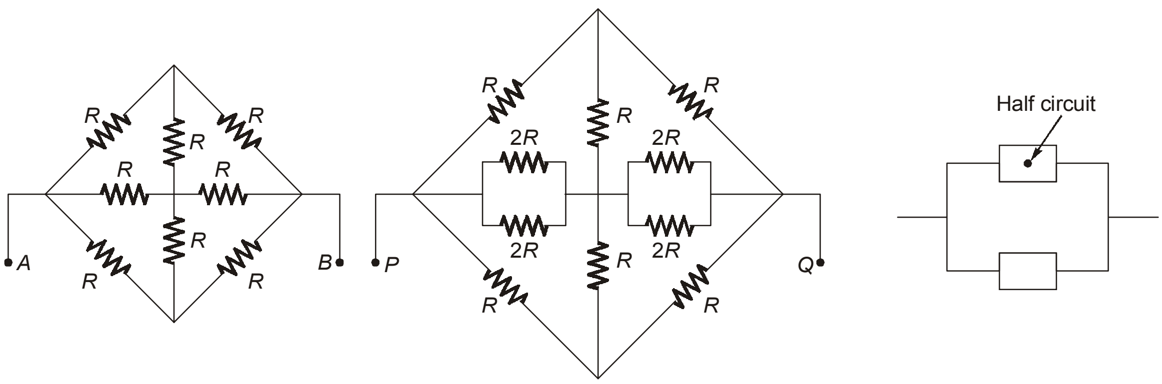


The time constant of an RC circuit is $\tau = R_{eq}C_{eq}$

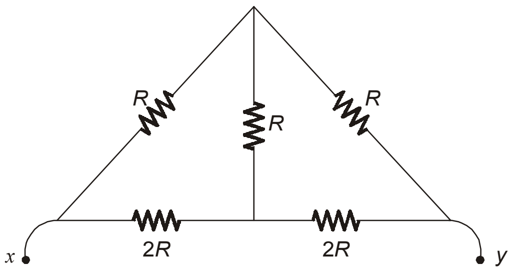
Calculation of C_{eq}

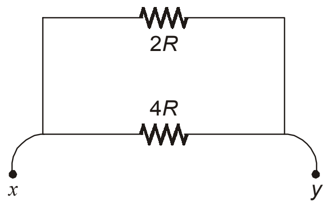


Calculation of R_{eq}



It is Wheatstone bridge.





$$(R_{eq})_{\text{Half circuit}} = \frac{4R \times 2R}{6R} = \frac{4R}{3}$$

$$R_{eq} = (R_{eq})_{\text{Half circuit}} \,||\, (R_{eq})_{\text{Half circuit}}$$

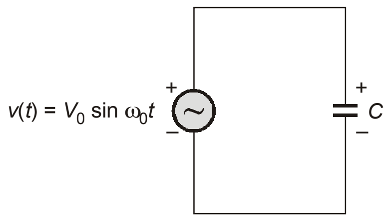
$$= \left(\frac{4R}{3}\right) \Big|\Big| \left(\frac{4R}{3}\right) = \left(\frac{2R}{3}\right)$$

$$\therefore \quad \tau = R_{eq} C_{eq}$$

$$= \frac{2R}{3} \times 3C = 2RC$$

20.(b)

$$W(t) = \frac{1}{2} C v^2(t)$$



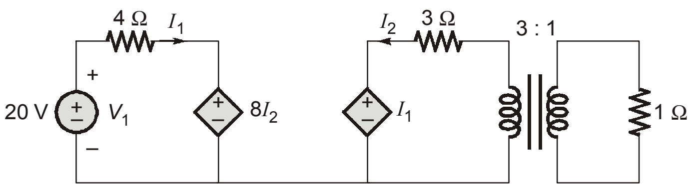
$$= \frac{1}{2} C V_0^2 \sin^2 \omega_0 t$$

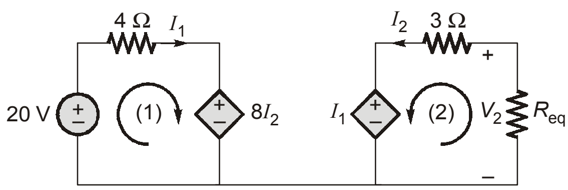
$$= \frac{1}{4} C V_0^2 (1 - \cos 2\omega_0 t)$$

thus only option (b) satisfies this condition.

21.(b)

Network 'N' can be replaced as





$$R_{eq} = (1) \times \left(\frac{3}{1}\right)^2 = 9\,\Omega$$

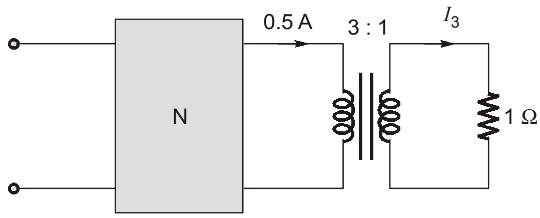
Applying KVL at loop (1)

$$20 = 4I_1 + 8I_2$$

also $\quad V_2 = -9I_2 = I_1 + 3I_2$

$$\Rightarrow \quad I_1 = -12I_2$$
$$20 = 4(-12I_2) + 8I_2$$
$$20 = -40I_2$$
$$\Rightarrow \quad I_2 = -0.5 \text{ A}$$
$$I_1 = -12(-0.5) = 6 \text{ A}$$

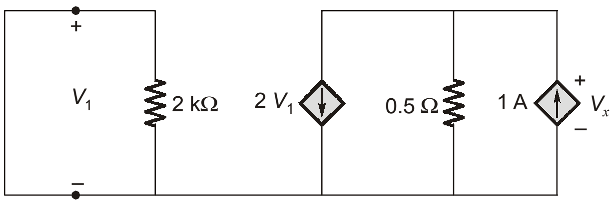
Now,



$$\frac{I_{primary}}{I_{secondary}} = \frac{1}{3}$$
$$\Rightarrow \quad 3I_{primary} = I_{secondary}$$
$$3(-I_2) = I_3$$
$$\Rightarrow \quad I_3 = 1.5 \text{ A}$$
$$\text{Power delivered to } 1\ \Omega = (I_3)^2 \times R_L = (1.5)^2 \times 1$$
$$= 2.25 \text{ Watts}$$

22.(c)

Calculating R_{th}

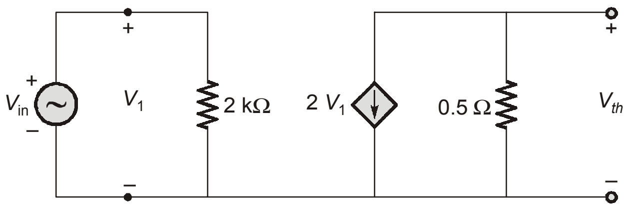


$$1 \text{ A} = \frac{V_x}{0.5} + 2V_1$$

$\because \quad V_1 = 0$ (As independent voltage source is short circuited)

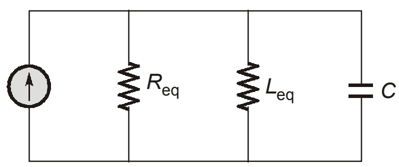
$$\Rightarrow \quad V_x = 0.5 \text{ V}$$
$$R_{th} = 0.5\ \Omega = 500\ \text{m}\Omega$$

Calculating V_{th}



$$V_{th} = -2V_1 \times 0.5$$
$$V_1 = V_{in} = 5\angle 0° \text{ V}$$
$$V_{th} = -2 \times 5 \times 0.5 = -5\angle 0°$$
$$= 5\angle 180° \text{ V}$$

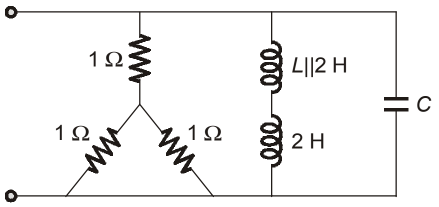
23.(d)



For a parallel resonant circuit

the damping ratio $$\xi = \frac{1}{2Q} = \frac{1}{2R_{eq}}\sqrt{\frac{L_{eq}}{C}} = \frac{1}{\sqrt{2}} \quad ...(i)$$

given



$$R_{eq} = 1\,||\,1 + 1 = 1.5\ \Omega$$

$$L_{eq} = 2 + \frac{2L}{L+2}$$

$$C_{eq} = \frac{4}{9}$$

From equation (i)

$$\frac{L_{eq}}{C} = \frac{4R_{eq}^2}{2} = 2R_{eq}^2$$

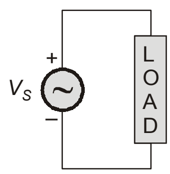
or $$L_{eq} = 2R_{eq}^2 C = 2 \times (1.5)^2 \times \frac{4}{9} = 2\text{ H}$$

$\because$ $$L_{eq} = 2H + \frac{2L}{L+2}$$

$$2 + \frac{2L}{L+2} = 2\text{ H}$$

$\Rightarrow$ $$L = 0\text{ H}$$

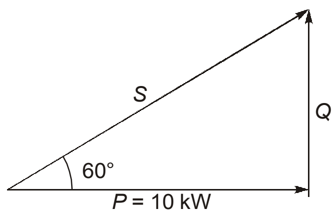
24.(b)



$$pf = 0.50\text{ lagging}$$

$$\cos\phi = 0.5$$

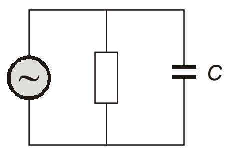
$$\phi = 60^\circ$$

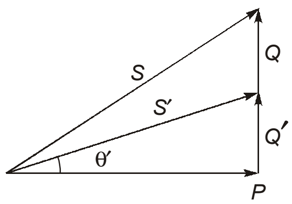


$\Rightarrow$ $$Q = P\tan 60^\circ = 10\sqrt{3}\text{ kVAR}$$

$$S = \sqrt{P^2 + Q^2} = 20\text{ kVA}$$

Now,





As per question $$S' = 14.14 = 10\sqrt{2}\text{ kVA}$$

$$P' = 10\text{ kW}$$

$$\cos\theta' = \frac{p}{s} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta' = 45^\circ$$

$$Q' = 10\text{ kVAR}$$

Reduction in reactive power

$$= (10\sqrt{3} - 10)\text{ kVAR}$$

$$= 10(\sqrt{3} - 1) = 10(0.732)$$

$$= 7.32\text{ kVAR}$$

25. (d)

Let $v_{in}(t)$ be the input voltage while v_{out} (t) be the output voltage

$$h(t) = (e^{-2t} + e^{-3t})\, u(t)\text{ V}$$

$$\therefore \quad H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)}$$

if

$$v_{out}(t) = te^{-2t}\, u(t)\text{ V}$$

$$V_{out}(s) = \frac{1}{(s+2)^2}$$

However,

$$H(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{V_{out}(s)}{V_{in}(s)}$$

$$\therefore \quad V_{in}(s) = \frac{V_{out}(s)}{H(s)}$$

or

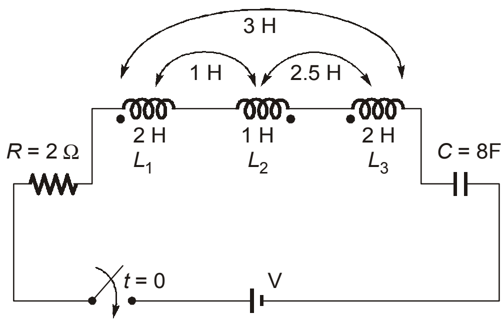
$$V_{in}(s) = \frac{1}{(s+2)^2} \times \frac{(s+2)(s+3)}{(2s+5)} = \frac{1}{2}\left[\frac{2}{s+2} - \frac{1}{s+2.5}\right]$$

Taking inverse of Laplace

$$v_{in}(t) = \left(e^{-2t} - \frac{1}{2}e^{-2.5t}\right)u(t)\,\text{volts}$$

26.(a)

For the circuit



$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$L_1 = 2\text{ H}$$

$$L_2 = 1\text{ H}$$

$$L_3 = 2\text{ H}$$

$$M_{12} = 1\text{ H}$$

$$M_{23} = 2.5\text{ H}$$

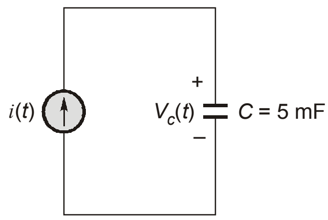
$$M_{13} = 3\text{ H}$$
$$L_{eq} = 2 + 1 + 2 - 2 - 5 + 6 = 11 - 7 = 4\text{ H}$$
$$C = 8\text{ F}$$
$$R = 2\ \Omega$$

Note : M_{12}, M_{23} is negative, because both L_1, L_2 and L_2, L_3 opposes the flux of respective loops.

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{2}{2}\sqrt{\frac{8}{4}} = \sqrt{2} = 1.414$$

Thus, the given circuit is overdamped.

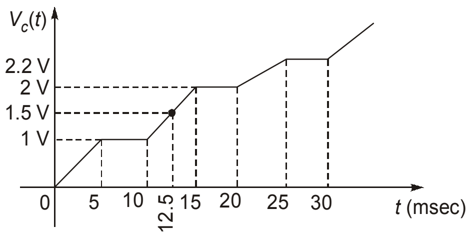
27.(b)



$$i = C\frac{dv}{dt}$$

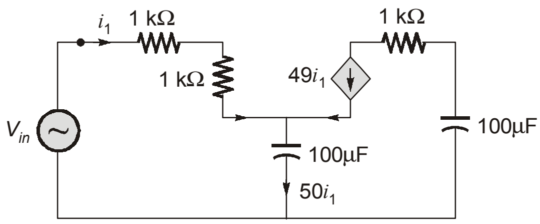
$$V_C = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

For $0 < t < 5$; Unit step current is applied ; voltage will increase linearly.
For $5 < t < 10$; No current is applied, hence open circuit, the capacitor will hold the charge.
For $10 < t < 15$; again capacitor's voltage increases linearly.
From above analysis,



$$V_C(t)\big|_{12.5\text{ msec}} = 1.5\text{ V}$$

28.(b)



Applying KVL,

$$V_{in} - i_1(1 + 1) - 50\,i_1(-jX_C) = 0$$

$$\Rightarrow \quad V_{in} = i_1[2 - j50\,X_C]$$

$$\text{Input impedance} = \frac{V_{in}}{i_1} = 2 - j50X_C$$

As imaginary part is negative, input impedance has equivalent capacitive reactance $X_{C\,eq.}$

$$X_{C\,eq.} = 50\,X_C$$

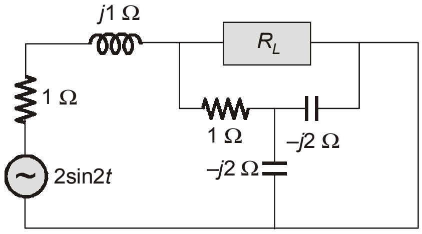
$$\frac{1}{\omega C_{eq.}} = \frac{50}{\omega C}$$

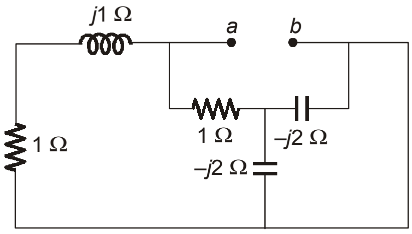
$$C_{eq} = \frac{C}{50} = \frac{100}{50}\ \mu\text{F}$$

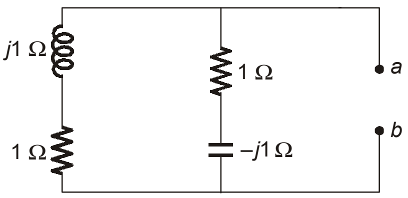
$$C_{eq.} = 2\ \mu\text{F}$$

29.(b)

Since (ω) = 2 rad/sec, the network is drawn as







$$Z_{th} = \frac{(1+j1)(1-j1)}{2} = 1\,\Omega$$

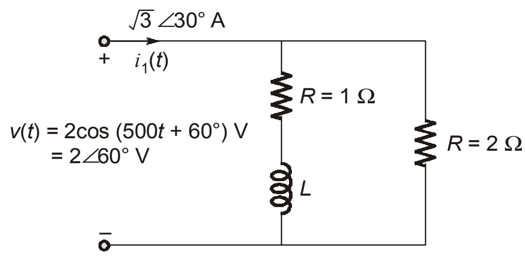
$\therefore$ $$R_L = |Z_{th}| = 1\,\Omega$$

Hence, for maximum power to R_L, it should be 1 Ω.

30. (d)

$$v(t) = 2\cos(500t + 60°)\text{ V}$$
$$= 2\angle 60°\text{ V}$$

Using AC phasor



$$i(t) = -\sqrt{3}\cos(500t + 30°)\text{ A}$$
$$v(t) = 2\angle 60°\text{ V}$$
$$i_1(t) = -i(t) = \sqrt{3}\angle 30°\text{ A}$$
$$\sqrt{3}\angle 30° = \frac{2\angle 60°}{1 + jX_L} + \frac{2\angle 60°}{2}$$
$$\sqrt{3}\angle 30° = \frac{2\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)}{(1 + jX_L)} + \frac{1}{2} + j\frac{\sqrt{3}}{2}$$

By equating real parts on both sides,

$$\frac{3}{2} = \frac{1}{1 + X_L^2} + \frac{1}{2} + \frac{X_L\sqrt{3}}{1 + X_L^2}$$
$$= \frac{1 + X_L\sqrt{3}}{1 + X_L^2} = \frac{3}{2} - \frac{1}{2} = 1 = 1 + X_L\sqrt{3} = 1 + X_L^2$$
$$X_L = \omega L = \sqrt{3}\ \Omega$$

or $$L = \frac{\sqrt{3}}{500} = 3.46\text{ mH}$$

■■■■