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Communication Systems

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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

Date of Test : 08/08/2019

ANSWER KEY ➤ Communication Systems

1.	(d)	7.	(d)	13.	(b)	19.	(c)	25.	(b)
2.	(b)	8.	(c)	14.	(b)	20.	(b)	26.	(b)
3.	(d)	9.	(c)	15.	(c)	21.	(c)	27.	(c)
4.	(d)	10.	(d)	16.	(a)	22.	(c)	28.	(b)
5.	(c)	11.	(c)	17.	(c)	23.	(b)	29.	(d)
6.	(d)	12.	(b)	18.	(c)	24.	(c)	30.	(b)

Detailed Explanations

1. (d)

The maximum and minimum values of the envelope of an AM modulated signal can be given as,

$$E_{max} = A_c(1 + \mu)$$

$$E_{min} = A_c(1 - \mu)$$

Given that, $A_c = 10$ V and $\mu = 0.4$

So, $E_{max} = 10(1 + 0.4)$ V $= 14$ V

$E_{min} = 10(1 - 0.4)$ V $= 6$ V

2. (b)

By writing the given AM signal in standard form, we get,

$$s(t) = 12\left[1+\frac{1}{12}m(t)\right]\cos(2\pi f_c t) \text{ V}$$

The modulation index of the AM signal can be given as,

$$\mu = \frac{1}{12}|m(t)|_{max} = \frac{m_p}{12}; \quad m_p = \text{peak value of } m(t)$$

Given that, $\mu = 0.75$

So, $\frac{m_p}{12} = 0.75$

$$m_p = 12 \times 0.75 = 9 \text{ V}$$

3. (d)

- Hilbert transform does not alter the magnitude spectrum of a signal. It alters only the phase spectrum of the signal.
- As the magnitude spectrum is same for both $m(t)$ and $\hat{m}(t)$, the average power of both the signals is also same.

4. (d)

The modulation index of an FM signal can be given as,

$$\beta_{FM} = \frac{(\Delta f)_{max}}{f_m} = \frac{A_m k_f}{f_m} \quad \text{...(i)}$$

The modulation index of a PM signal can be given as,

$$\beta_{PM} = (\Delta\phi)_{max} = A_m k_p \quad \text{...(ii)}$$

From equations (i) and (ii), option (d) can be selected as the correct one.

5. (c)

Given that, IF = 450 kHz

range of f_c = (550 to 1650) kHz

$f_{LO} > f_c$

For proper reception of signals,

$$f_{LO} - f_c = \text{IF}$$

$$f_{LO\,(min)} = f_{c\,(min)} + \text{IF} = 550 + 450 = 1000 \text{ kHz}$$

$$f_{LO\,(max)} = f_{c\,(max)} + \text{IF} = 1650 + 450 = 2100 \text{ kHz}$$

So, the range of f_{LO} is (1000 to 2100) kHz.

6. (d)

The variance of a random variable "X" can be given as,

$$\sigma_X^2 = E[X^2] - (E[X])^2$$

For any real random variable, variance can't be negative.

i.e., $\sigma_X^2 \geq 0$

$$E[X^2] - (E[X])^2 \geq 0$$

So, $E[X^2] \geq (E[X])^2$

Hence, option (d) is correct.

7. (d)

The variance of a random variable "X" can be given as,

$$\sigma_X^2 = E[X^2] - (E[X])^2$$

$$E[X] = \sum_{i=0}^{1} x_i P(x_i) = 1(p) + 0(q) = p$$

$$E[X^2] = \sum_{i=0}^{1} x_i^2 P(x_i) = (1)^2(p) + (0)^2(q) = p$$

So the variance, $\sigma_X^2 = p - (p)^2 = p(1-p) = pq$ $\quad \because p + q = 1$

Hence, option (d) is correct.

8. (c)

From the basic properties of a PDF,

$$\int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$$

$$\int_{y=0}^{1} \int_{x=0}^{y} kxy\,dx\,dy = 1$$

$$k \int_{y=0}^{1} y\left[\frac{x^2}{2}\right]_0^y dy = 1$$

$$\frac{k}{2} \int_{y=0}^{1} y^3\,dy = 1$$

$$\frac{k}{2}\left[\frac{y^4}{4}\right]_0^1 = 1$$

$$\frac{k}{8} = 1$$

$$k = 8$$

9. (c)

Entropy, $H(X) = -\sum_{i=0}^{3} P(x_i)\log_2 P(x_i)$

$$= -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right)\right] \text{bits/symbol}$$

$$= \frac{1}{2}\log_2(2) + \frac{1}{4}\log_2(4) + \frac{2}{8}\log_2(8) \text{ bits/symbol}$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{6}{8} \text{ bits/symbol}$$

$$= 1.75 \text{ bits/symbol}$$

10. (d)

Sampling rate, $f_s = 44.1$ kHz

Bits per sample, $n = 16$

Number of bits for piece of music with a duration of 1 minute is,

$$R = n \times f_s \times (60 \text{ sec}) \qquad \because 1 \text{ minute} = 60 \text{ seconds}$$

$$= 16 \times 44.1 \times 10^3 \times 60 \text{ bits} = 42.336 \times 10^6 \text{ bits}$$

11. (c)

The angle of the modulated signal $s(t)$ can be given as,

$$\theta(t) = 2\pi f_c t + 4\sin(3000\pi t) + 3\cos(3000\pi t)$$

The instantaneous frequency of the modulated signal can be given as,

$$f_i = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$= f_c + \frac{1}{2\pi}[12000\pi\cos(3000\pi t) - 9000\pi\sin(3000\pi t)]$$

$$= f_c + [6000\cos(3000\pi t) - 4500\sin(3000\pi t)] \text{ Hz}$$

$$= f_c + 1500[4\cos(3000\pi t) - 3\sin(3000\pi t)] \text{ Hz}$$

$$= f_c + 1500[5\cos(3000\pi t + \alpha)] \text{ Hz}; \qquad \text{Where, } \alpha = \tan^{-1}\left(\frac{3}{4}\right)$$

$$f_i = f_c + 7500\cos(3000\pi t + \alpha) \text{ Hz}$$

Maximum frequency deviation of the signal $s(t)$ is,

$$(\Delta f)_{max} = 7500 \text{ Hz} = 7.5 \text{ kHz}$$

12. (b)

$$\beta_1 = \frac{\Delta f_1}{f_{m_1}} = \frac{A_{m_1}k_f}{f_1} = 10 \qquad \text{...(i)}$$

$$\beta_2 = \frac{\Delta f_2}{f_{m_2}} = \frac{A_{m_2}k_f}{f_2} = 20 \qquad \text{...(ii)}$$

From equations (i) and (ii), it is clear that,

$$\frac{A_{m_2}k_f}{f_2} = 2 \times \frac{A_{m_1}k_f}{f_1}$$

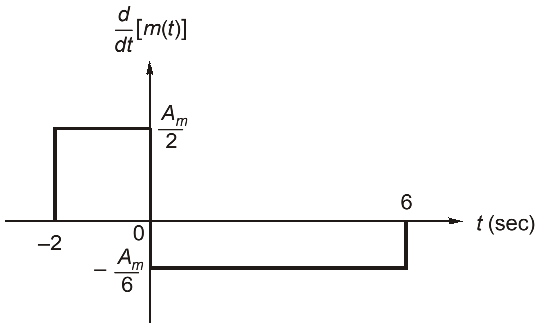
$$\frac{f_2}{f_1} = \frac{A_{m_2}}{2A_{m_1}} = \frac{2}{2\times 1} = 1 \qquad \because \text{Given that, } A_{m_1} = 1 \text{ V and } A_{m_2} = 2 \text{ V}$$

So, $f_2 = f_1$

13. (b)

$$(\Delta f_{max})_{FM} = A_m k_f$$

$$(\Delta f_{max})_{PM} = \frac{k_p}{2\pi}\left|\frac{dm(t)}{dt}\right|_{max}$$



So, $$(\Delta f_{max})_{PM} = \frac{k_p}{2\pi}\left(\frac{A_m}{2}\right) = \frac{k_p A_m}{4\pi}$$

If, $$(\Delta f_{max})_{PM} = (\Delta f_{max})_{FM}$$

Then, $$\frac{k_p A_m}{4\pi} = k_f A_m$$

$$\frac{k_p}{k_f} = 4\pi \text{ rad/Hz}$$

14. (b)

The average power of the random process $X(t)$ can be given as,

$$P_X = \int_{-\infty}^{\infty} S_X(f)\,df = \int_{-\infty}^{\infty} 4e^{-2|f|}\,df \text{ W}$$

$$= (4\times 2)\int_0^{\infty} e^{-2f}\,df \text{ W} = 8\left[-\frac{e^{-2f}}{2}\right]_0^{\infty} \text{ W}$$

$$= \frac{8}{2}\text{ W} = 4\text{ W}$$

15. (c)

When a stationary random process is applied to an LTI system, the output process of the system is also stationary and the mean value of the output process is related to the mean value of the input process as,

$$E[Y(t)] = H(0)\,E[X(t)]$$

Here, $$H(0) = H(f)\big|_{f=0}$$

$$H(f) = \text{Fourier transform of } h(t)$$

$$= \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt$$

$$H(0) = \int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} 5e^{-2t}u(t)\,dt$$

$$= 5\int_0^{\infty} e^{-2t}\,dt = \frac{5}{2}\left[-e^{-2t}\right]_0^{\infty} = 2.5$$

So, $$E[Y(t)] = 2.5\,E[X(t)]$$

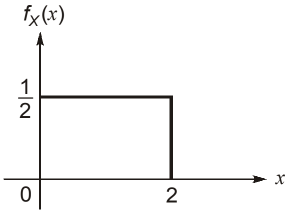
$$= 2.5(4) = 10 \qquad \because \text{Given that, } E[X(t)] = 4$$

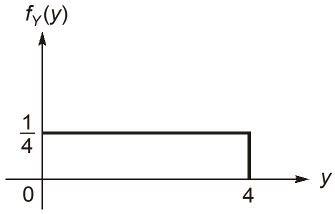
16. (a)

It is given that, X and Y are statistically independent.

So,
$$E[Z^2] = E[(X+Y)^2]$$
$$= E[(X^2 + Y^2 + 2XY]$$
$$= E[X^2] + E[Y^2] + 2E[X]E[Y]$$

By considering the PDFs of the random variables X and Y,





$$E[X] = \frac{0+2}{2} = 1 \;; \qquad E[Y] = \frac{0+4}{2} = 2$$

$$\sigma_X^2 = \frac{(2)^2}{12} = \frac{1}{3} \;; \qquad \sigma_Y^2 = \frac{(4)^2}{12} = \frac{4}{3}$$

$$E[X^2] = (1)^2 + \frac{1}{3} = \frac{4}{3} \;; \qquad E[Y^2] = (2)^2 + \frac{4}{3} = \frac{16}{3}$$

So,
$$E[Z^2] = \frac{4}{3} + \frac{16}{3} + 2(1)(2)$$
$$= \frac{20}{3} + 4 = \frac{32}{3}$$

17. (c)

The PSD of the output signal can be given as,

$$S_Y(f) = S_X(f)\,|H(f)|^2$$

$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{1+sRC} = \frac{1}{1+s\tau} \;; \qquad \text{Where, } \tau = RC$$

$$H(f) = \frac{1}{1 + j2\pi f\tau}$$

$$|H(f)|^2 = H(f)H^*(f) = \frac{1}{1+4\pi^2 f^2\tau^2}$$

Output noise power,
$$P_Y = \int_{-\infty}^{\infty} S_Y(f)\,df$$

$$= \frac{N_0}{2}\int_{-\infty}^{\infty} \frac{1}{1+4\pi f^2\tau^2}\,df$$

$$= \frac{N_0}{4\pi\tau}\int_{-\infty}^{\infty} \frac{2\pi\tau}{1+4\pi f^2\tau^2}\,df$$

$$= \frac{N_0}{4\pi\tau}\left[\tan^{-1}(2\pi f\tau)\right]_{-\infty}^{\infty} = \frac{N_0}{4\pi\tau}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right]$$

$$= \frac{N_0}{4\pi\tau}(\pi) = \frac{N_0}{4\tau} = \frac{N_0}{4RC}$$

18. (c)

The mutual information between two random variables X and Y can be expressed in the following ways:

$$\begin{aligned} I(X;Y) &= H(X) - H(X|Y) \\ &= H(Y) - H(Y|X) \\ &= H(X) + H(Y) - H(X;Y) \\ &= H(X;Y) - H(X|Y) - H(Y|X) \end{aligned}$$

19. (c)

The condition required to produce uniquely decodable code words is that, no code word of a symbol can be the prefix of the code word of another symbol.

For Coding Scheme - *A* :

Code word of S_0(00) is the prefix of the code word of S_4(001).
Code word of S_1(10) is the prefix of the code word of S_3(101).

For Coding Scheme - *B* :

Code word of S_0(11) is the prefix of the code word of S_2(111).

For Coding Scheme - *C* :

No code word of a symbol is the prefix of the code word of another symbol.
So, only Coding Scheme - C can produce uniquely decodable code words for the given discrete source.

20. (b)

$P(m_1|r_0)$ = Probability of finding the transmitted symbol as m_1 when r_0 is received

$$P(m_1|r_0) = \frac{P(m_1 \cap r_0)}{P(r_0)}$$

$$\begin{aligned} P(r_0) &= P(r_0|m_0)\,P(m_0) + P(r_0|m_1)\,P(m_1) \\ &= (0.6)(0.4) + (0.4)(0.6) = 0.48 \end{aligned}$$

$$\begin{aligned} P(m_1 \cap r_0) &= P(r_0|m_1)\,P(m_1) \\ &= (0.4)(0.6) = 0.24 \end{aligned}$$

So, $$P(m_1|r_0) = \frac{0.24}{0.48} = 0.50$$

21. (c)

From the Shannon's channel capacity theorem,

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

Where,
C = Capacity of the channel
B = Bandwidth of the channel
S = Average signal power in the channel
N = Average noise power in the channel
$N = N_0B$

When the channel is ideal with infinite bandwidth,

$$\begin{aligned} C_\infty &= \lim_{B\to\infty} B\log_2\left(1+\frac{S}{N_0B}\right) = \lim_{B\to\infty}\log_2\left(1+\frac{S}{N_0B}\right)^B \\ &= \lim_{B\to\infty} K\log_2\left(1+\frac{K}{B}\right)^{B/K}; \qquad \text{Where, } K = \frac{S}{N_0} \\ &= K\log_2\left[\lim_{B\to\infty}\left(1+\frac{K}{B}\right)^{B/K}\right] \end{aligned}$$

$$= K\log_2\left[\lim_{\alpha\to\infty}\left(1+\frac{1}{\alpha}\right)^{\alpha}\right]; \qquad \text{Where, } \alpha = \frac{B}{K}$$

$$C_\infty = K\log_2(e) = \frac{S}{N_0}\log_2 e$$

For error free transmission,

$$R_b \le C\ ; \qquad \text{Where, } R_b = \text{bit rate}$$

So, $$R_b \le \frac{S}{N_0}\log_2 e$$

The average signal power S can be related to E_b as,

$$E_b = ST_b = \frac{S}{R_b}$$

So, $$R_b \le \frac{E_b R_b}{N_0}\log_2 e$$

$$\frac{E_b}{N_0} \ge \frac{1}{\log_2 e}$$

$$\frac{E_b}{N_0} \ge \log_e 2$$

22. (c)

For a baseband binary transmission,

$$\frac{R_b}{2} \le (\text{Channel BW})$$

$$R_b \le 2\,(20\text{ kHz}) = 40\text{ kHz}$$

$$nf_s \le 40\text{ kHz}$$

For a delta modulator based system, $n = 1$

So, $$f_s \le 40\text{ kHz}$$

$$f_{s(\max)} = 40\text{ kHz}$$

For proper reconstruction of the sampled message signal,

$$f_s \ge 2\,f_{m(\max)}$$

$$2\,f_{m(\max)} = f_{s(\max)} = 40\text{ kHz}$$

$$f_{m(\max)} = 20\text{ kHz}$$

23. (b)

Given that, $f_{m(\max)} = 20$ kHz

Bits per sample, $n = 3$

Sampling rate, $f_s = 2\,f_{m(\max)} = 40$ kHz

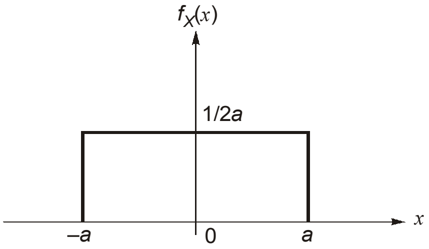
Bit rate, $R_b = nf_s = 120$ kbps

Minimum channel bandwidth required is,

$$(\text{BW})_{\min} = \frac{R_b}{2} = 60\text{ kHz}$$

24. (c)

Given that,



Mean value, $E[X(t)] = \bar{X} = 0$ $\quad\because f_X(x)$ is symmetric about $x = 0$

Variance, $\sigma_X^2 = \dfrac{(2a)^2}{12} = \dfrac{a^2}{3}$

Signal power, $S = \sigma_X^2 + (\bar{X})^2 = \dfrac{a^2}{3}$

Quantization noise power, $N_Q = \dfrac{\Delta^2}{12}$

Step size, $\Delta = \dfrac{(2a)}{2^n} = \dfrac{2a}{16} = \dfrac{a}{8}$ $\quad\because$ Given that, $n = 4$

So, $N_Q = \dfrac{a^2}{64\times 12}$

Signal to quantization noise ratio,

$$(\text{SQNR}) = \frac{S}{N_Q} = \frac{a^2}{3}\times\frac{64\times 12}{a^2} = 256$$

In decilogs, $[\text{SQNR}] = 10\log_{10}(\text{SQNR}) = 10\log_{10}(256)\text{ dB} = 24.08\text{ dB}$

25. (b)

Number of voice signals, $N = 4$

Maximum frequency of each signal, $f_{m(\max)} = 4$ kHz

Bits per sample, $n = 8$

Sampling rate of each voice signal, $f_s = 2f_{m(\max)} = 8$ kHz

Sampling rate of the multiplexed signal, $f_{s(\text{overall})} = Nf_s = 32$ kHz

Overall transmission rate, $R_b = nf_{s(\text{overall})} = 256$ kbps

Theoretical minimum channel bandwidth required is,

$$(\text{BW})_{\min} = \frac{R_b}{2} = 128\text{ kHz}$$

26. (b)

To eliminate slope-overload distortion,

$$\frac{\Delta}{T_s} \geq \left|\frac{dx(t)}{dt}\right|_{\max}$$

$$\left|\frac{dx(t)}{dt}\right|_{\max} = \left|\frac{10-0}{1-(1.5)}\right| = \frac{10}{0.5} = 20\text{ V/sec}$$

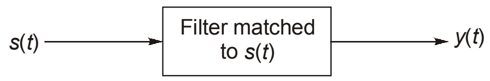
$$T_s = \frac{1}{f_s} = 10^{-4}\text{ sec}$$ $\quad\because$ Given that, $f_s = 10$ kHz

So, $$\Delta \geq T_s \left|\frac{dx(t)}{dt}\right|_{max}$$

$$\Delta \geq (10^{-4})\,(20)\text{ V}$$

$$\Delta_{min} = 2\text{ mV}$$

27. (c)



For a matched filter,

Peak value of the output signal = Energy of the input signal

Energy of the signal $s(t)$, $E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt$

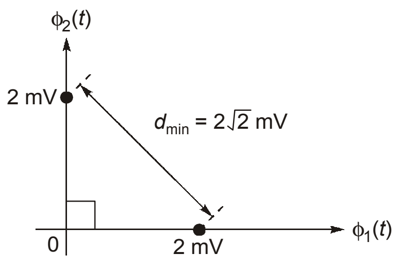
$$s(t) = \begin{cases} \frac{3}{2}t \text{ V}; & 0 < t < 2 \\ 0 \ ; & \text{otherwise} \end{cases}$$

So, $$E_s = \int_0^2 \left(\frac{3}{2}t\right)^2 dt = \frac{9}{4}\int_0^2 t^2 dt$$

$$= \frac{9}{4}\left(\frac{t^3}{3}\right)_0^2 = \frac{9}{4}\left(\frac{8}{3}\right) = 6 \text{ V}$$

Hence, the peak value of $y(t)$ = 6 V.

28. (b)



$$\text{BER} = Q\left[\sqrt{\frac{d_{min}^2}{2N_0}}\right]$$

Given that, $$\frac{N_0}{2} = 2 \times 10^{-8} \text{ W/Hz}$$

$$N_0 = 4 \times 10^{-8} \text{ W/Hz}$$

From the constellation diagram,

$$d_{min} = 2\sqrt{2} \text{ mV}$$

So, $$\text{BER} = Q\left[\sqrt{\frac{4 \times 2 \times 10^{-6}}{2 \times 4 \times 10^{-8}}}\right] = Q\left[\sqrt{100}\right] = Q(10)$$

29. (d)

The condition for an error correcting code which can detect upto e_d bit errors and simultaneously correct upto e_c bit errors ($e_d \geq e_c$) is,

$$d_{min} \geq (e_d + e_c + 1)$$

For the given question, $e_d = 4$ and $e_c = 3$.

So,
$$d_{min} \geq (4 + 3 + 1)$$
$$d_{min} \geq 8$$

30. (b)

If the received code vector is r and the parity-check matrix is H, then the syndrome vector can be can be given as,

$$s = rH^T$$

$$= [001110]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [10\,0]$$

Note : In the problems related to linear block codes, use Ex-OR operation, whenever there is an addition operation of bits.

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