CLASS TEST

Sl. 01 IG_CE_A+C_27042023

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STRUCTURAL ANALYSIS

CIVIL ENGINEERING

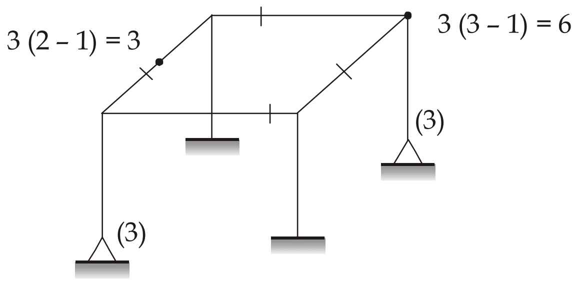
Date of Test : 27/04/2023

ANSWER KEY

1.	(c)	7.	(d)	13.	(a)	19.	(c)	25.	(c)
2.	(b)	8.	(d)	14.	(d)	20.	(b)	26.	(b)
3.	(b)	9.	(c)	15.	(b)	21.	(c)	27.	(a)
4.	(c)	10.	(d)	16.	(a)	22.	(b)	28.	(b)
5.	(a)	11.	(a)	17.	(c)	23.	(c)	29.	(a)
6.	(a)	12.	(b)	18.	(d)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. (c)



No. of cuts, $C = 4$

No. of restraints, $R = 15$

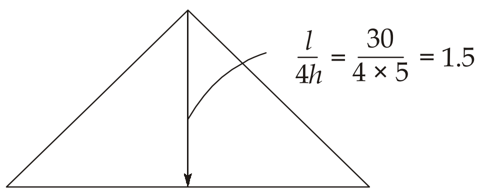
Now, $D_s = 6C - R$

$= 6 \times 4 - 15$

$= 24 - 15 = 9$

2. (b)

ILD for horizonal thrust

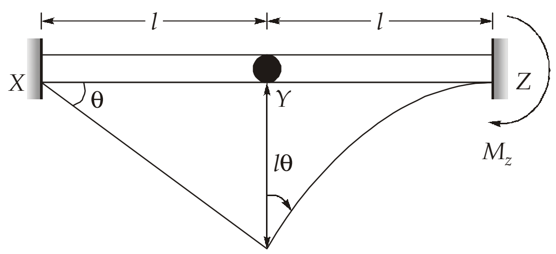


So max $H = 1.5 \times 30 = 45$ kN

3. (b)

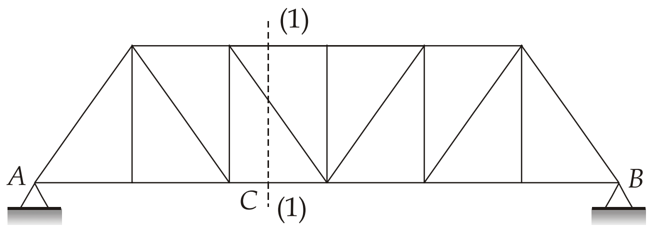
Stiffness of CD is less than AB so frame will sway towards right.

4. (c)



$$M_z = \frac{3EI\delta}{l^2} = \frac{3EI\theta l}{l^2} = \frac{3EI\theta}{l}$$

5. (a)



Pass a section (1)-(1) through the truss as shown.

When the load is on left side of section the diagonal member is under compression.

When the load is on right side of section, the diagonal member is in tension.

6. (a)

By Maxwell's reciprocal theorem

$$f_1\Delta_1 = f_2\Delta_2$$

$$\Rightarrow \quad 150 \times 0.005 = 750 \times \Delta_2$$

$$\Rightarrow \quad \Delta_2 = 0.001 \text{ radian}$$

7. (d)

Stiffness matrix coefficient,

$$k_{11} = \frac{AE}{l} = \frac{300 \times 200 \times 10^3 \times 10^3}{3000} = 2 \times 10^7 \text{ N/m}$$

8. (d)

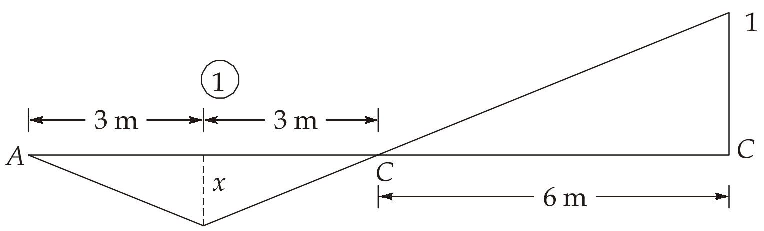
$$\text{Cyclic frequency} = \frac{w_d}{2\pi}$$

$$= \frac{1}{2\pi}\left[\sqrt{\frac{k}{m}}\left(\sqrt{1-\varepsilon^2}\right)\right]$$

$$= \frac{1}{2\pi}\left\{\sqrt{\frac{15 \times 10^3}{20}}\left(\sqrt{1-0.025^2}\right)\right\}$$

$$= 13.69/\pi$$

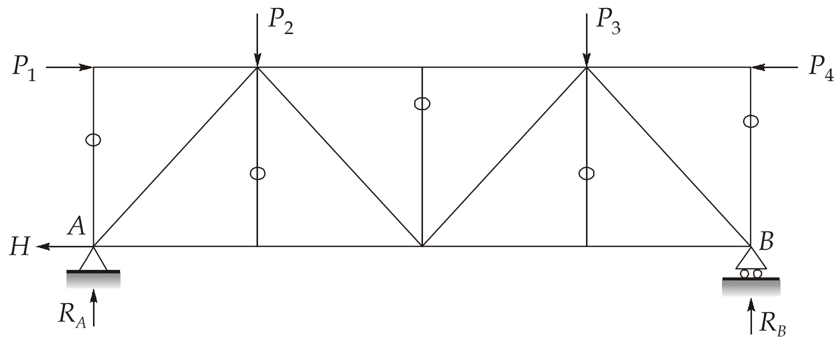
9. (c)



$$\frac{1}{6} = \frac{x}{3}$$

$$x = 0.5$$

10. (d)



11. (a)

$$\text{Vertical reaction; } V = \frac{wl}{2} = \frac{15 \times 150}{2} = 1125 \text{ kN}$$

$$\text{Horizontal reaction; } H = \frac{wl^2}{8h} = \frac{15 \times 150^2}{8 \times 10} = 4218.75 \text{ kN}$$

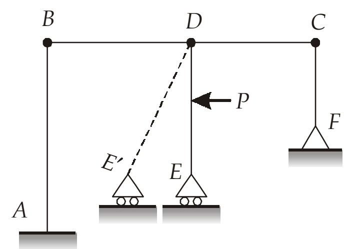
$$\therefore \quad \text{Maximum tension} = T_{max} = \sqrt{V^2 + H^2} = \sqrt{1125^2 + 4218.75^2} = 4366.17 \text{ kN}$$

$$\text{Minimum tension} = T_{min} = \text{H} = 4218.75 \text{ kN}$$

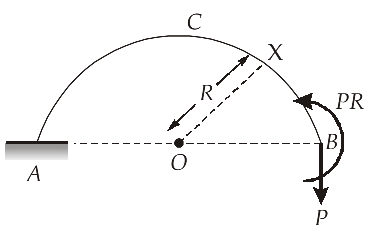
$$\therefore \quad T_{max} - T_{min} = 147.42 \text{ kN}$$

12. (b)

No rigid body motion is possible in figure (1) but in figure (2), rigid body motion is possible as shown below.



13. (a)



For member *ACB*,

Moment at *X*, $\quad M_x = PR - PR(1 - \cos\theta) = PR\cos\theta$

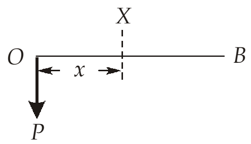
$$\text{Strain energy stored, } W_i = \int \frac{M_x^2 ds}{2EI} = \int_0^{\pi} \frac{(PR\cos\theta)^2 R d\theta}{2EI}$$

$$\Rightarrow \quad W_i = \frac{P^2R^3}{2EI} \times 2\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \frac{P^2R^3}{EI} \times \frac{\pi}{4}$$

$\therefore$ Vertical deflection at O due to member ACB,

$$\delta = \frac{\partial W_i}{\partial P} = \frac{\pi}{2}\frac{PR^3}{EI}$$

For member OB,



Strain energy stored, $$W_i = \int \frac{M_x^2 ds}{2EI} = \int_0^R \frac{(Px)^2 dx}{2EI}$$

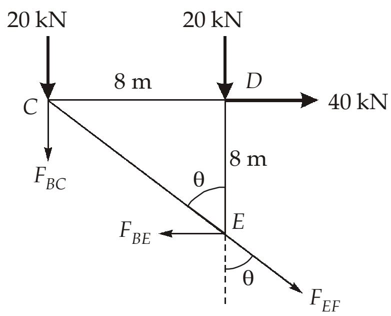
$$W_i = \frac{P^2R^3}{6EI}$$

$\therefore$ Vertical deflection at O due to member OB,

$$\delta = \frac{\partial W_i}{\partial P} = \frac{1}{3}\frac{PR^3}{EI}$$

$$\text{Total deflection} = \left(\frac{\pi}{2}+\frac{1}{3}\right)\frac{PR^3}{EI} = 1.904\frac{PR^3}{EI}$$

14. (d)



Cut a section through BC, BE and EF

$$\Sigma F_y = 0; \Rightarrow 20 + 20 + F_{EF}\cos\theta + F_{BC} = 0 \quad \text{...(1)}$$

$$\Sigma M_E = 0; \Rightarrow F_{BC} \times 8 + 20 \times 8 = 40 \times 8$$

$$\Rightarrow F_{BC} = 20 \text{ kN}$$

From (1); $$F_{EF} = -60\sqrt{2} \text{ kN} \quad \text{(–ve i.e. compression)} \quad \left(\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}\right)$$

$\therefore$ Magnitude of $F_{EF} = 60\sqrt{2}$ kN

15. (b)

Since stiffness matrix is inverse of flexibility matrix.

$\therefore$ If $$[A] = k\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}adj(A)$$

then $$[A]^{-1} = \frac{1}{k|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\therefore$ Stiffness matrix, $$[k] = \frac{6EI}{7L^3}\begin{bmatrix} 16 & -5 \\ -5 & 2 \end{bmatrix}$$

16. (a)

Calculation of fixed end moment,

$$\overline{M}_{AB} = \frac{-75 \times 3 \times 2^2}{5^2} = -36 \text{ kNm}$$

$$\overline{M}_{BA} = \frac{75 \times 2 \times 3^2}{25} = 54 \text{ kNm}$$

Calculation of distribution factors,

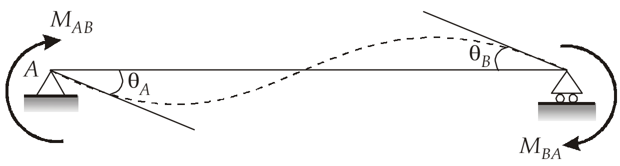
Joint	Member	k	Σk	Deflection factor
B	BA	$\frac{4EI}{5}$	$\frac{9EI}{5}$	$\frac{4}{9}$
	BC	$\frac{3EI}{3}$		$\frac{5}{9}$

Moment distribution method table (All moments in kNm)

Joint	A	B		C
Member	AB	BA	BC	CB
DF	0	4/9	5/9	1
FEM	–36	54	0	0
Balance		–24	–30	
C.O.	–12			
Final moment	–48	+30	–30	0

$\therefore$ Magnitude of moment at fixed support A = 48 kNm

17. (c)



Given: $$\theta_A = 3\theta_B$$

$\therefore$ $$\frac{M_{AB}}{M_{BA}} = \frac{2\theta_A + \theta_B}{2\theta_B + \theta_A}$$ [From slope-deflection equation]

$\Rightarrow$ $$\frac{M_{AB}}{M_{BA}} = \frac{7}{5}$$

18. (d)

Sway moment in column AB,

$$M_{AB} = \frac{6EI\Delta}{3^2} = \frac{6EI\Delta}{9} = \frac{2}{3}EI\Delta$$

Sway moment in column CD,

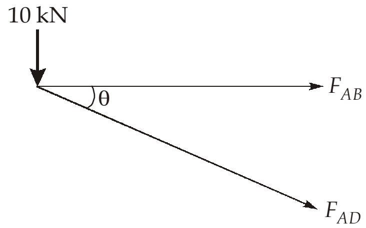
$$M_{CD} = \frac{6E(2I)\Delta}{4^2} = \frac{12EI\Delta}{16}$$

$$= \frac{3EI\Delta}{4}$$

$$\therefore \qquad \frac{M_{AB}}{M_{CD}} = \frac{\frac{2}{3}EI\Delta}{\frac{3}{4}EI\Delta} = \frac{8}{9}$$

19. (c)

Applying the method of joints consider joint A,



$$\Sigma F_y = 0$$

$$\Rightarrow \qquad F_{AD}\sin\theta = -10$$

$$\Rightarrow \qquad F_{AD} = \frac{-10}{\sin\theta}$$

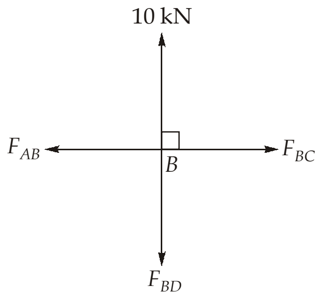
$$\Rightarrow \qquad F_{AD} = -\frac{50}{3}\text{kN} = \frac{50}{3}\text{kN (C)}$$

$$\Sigma F_x = 0$$

$$\Rightarrow \qquad F_{AD}\cos\theta = F_{AB}$$

$$\Rightarrow \qquad F_{AB} = \frac{50}{3} \times \frac{4}{5} = \frac{40}{3}(T)$$

$\therefore$ Now consider joint B



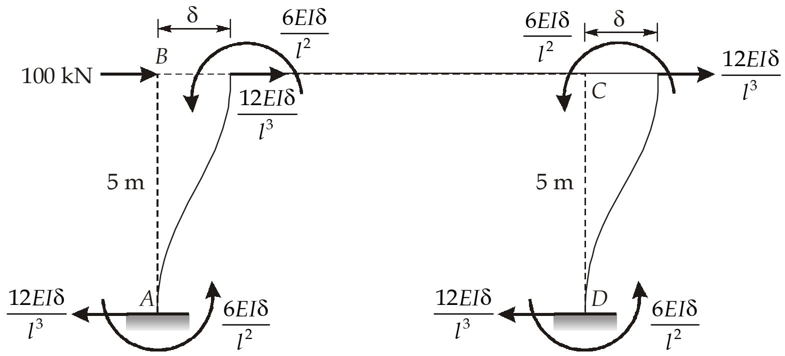
$\therefore$ Take $\Sigma F_x = 0$

$$\Rightarrow \qquad F_{AB} = F_{BC}$$

$$\Rightarrow \qquad F_{BC} = \frac{40}{3}\text{kN (T)}$$

So, correct option is (c).

20. **(b)**



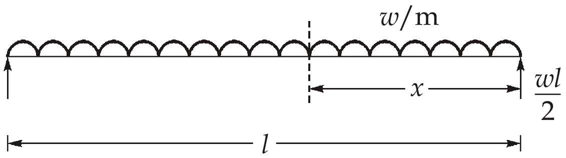
$$\frac{12EI\delta}{l^3} + \frac{12EI\delta}{l^3} = 100$$

$$\Rightarrow \quad \frac{24EI\delta}{l^3} = 100$$

$$M_A = \frac{6EI\delta}{l^2}$$

$$= \frac{100 \times l}{4} = \frac{100 \times 5}{4} = 125 \text{ kNm}$$

21. **(c)**



$$M_x = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\therefore \quad \text{Strain energy} = \int_0^L \frac{M_x^2 dx}{2EI} = \int_0^L \frac{\left(\frac{wlx}{2} - \frac{wx^2}{2}\right)^2 dx}{2EI}$$

$$= \frac{w^2 l^5}{240EI}$$

22. **(b)**

Using slope deflection method

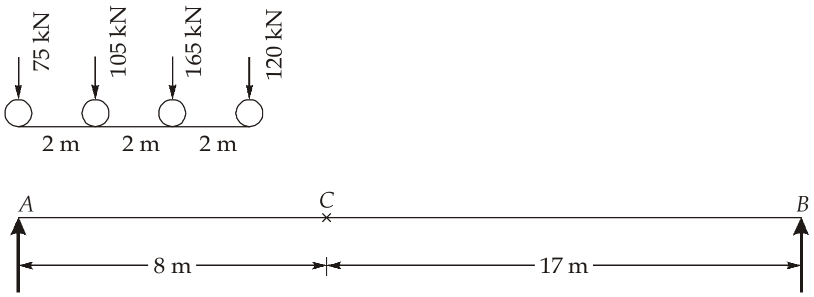
Equilibrium equation at joint B is:

$$M_{BA} + M_{BC} = 10 \times 3 + 12 \times 3 \times 1.5$$

$$\Rightarrow \quad \frac{2EI}{4}(2\theta_B) + \frac{2EI}{4}(2\theta_B) = 84$$

$$\Rightarrow \quad \theta_B = \frac{42}{EI}$$

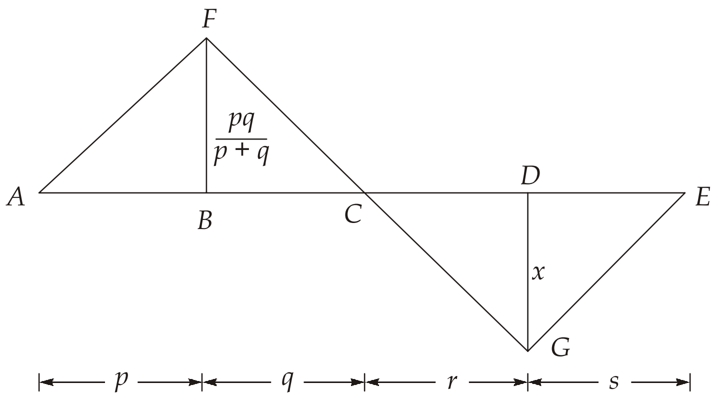
23. (c)



Load	Average load on AC (y_1) kN	Average load on CB (y_2) kN	Remark
All load on span AC	$\frac{75+105+165+120}{8} = 58.125$	$\frac{0}{17} = 0$	$y_1 > y_2$
120 kN load crosses C	$\frac{75+105+165}{8} = 43.125$	$\frac{120}{17} = 7.06$	$y_1 > y_2$
165 kN load crosses C	$\frac{75+105}{8} = 22.5$	$\frac{165+120}{17} = 16.76$	$y_1 > y_2$
105 kN load crosses C	$\frac{75}{8} = 9.375$	$\frac{105+165+120}{17} = 22.94$	$y_2 > y_1$

So, 105 kN load should be placed at the section.

24. (c)



ILD for BM at 'B'

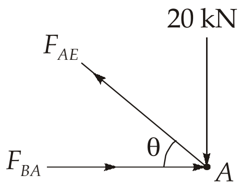
By similar triangles FBC and GDC $$= \frac{\left(\frac{pq}{p+q}\right)}{q} = \frac{x}{r}$$

$$x = \frac{pr}{p+q}$$

$\therefore$ Ratio of ordinates $= \frac{pq}{p+q} \times \frac{p+q}{pr} = \frac{q}{r}$

25. (c)

Consider joint A



$\Sigma F_x = 0$

$$F_{AE}\cos\theta - F_{BA} = 0$$

$$F_{BA} = \frac{F_{AE}}{\sqrt{2}} \qquad \text{...(i)}$$

$\Sigma F_y = 0$

$$F_{AE}\sin 45° - 20 = 0$$

$$F_{AE} = 20\sqrt{2}\ \text{kN}(\text{Tension})$$

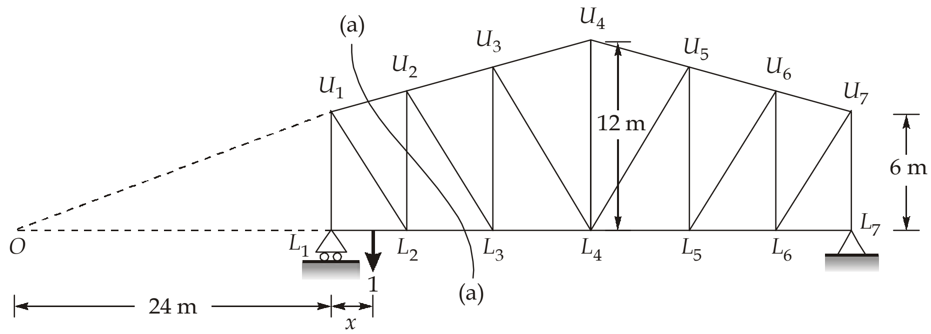
Substitute the value of F_{AE} in eq. (i)

$$F_{BA} = \frac{20\sqrt{2}}{\sqrt{2}} = 20\ \text{kN}\ (\text{Comp.})$$

$$\text{Required ratio} = \frac{F_{BA}}{F_{AE}} = \frac{20}{20\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Hence option (c) is correct.

26. (b)



From ΔOL_1U_1 and ΔOL_4U_4

$$\frac{OL_1}{OL_4} = \frac{6}{12}$$

$$\frac{OL_1}{OL_1 + 24} = \frac{6}{12}$$

$\Rightarrow$ $$OL_1 = 24\ \text{m}$$

Now, Case I : Consider the load is between L_1L_2

$\Sigma M_{L1} = 0$ $\qquad V_{L7} = \dfrac{x}{48}$

Consider structure to the right of section (a)-(a)

$\Sigma M_O = 0$

$$F_{U_2L_2} \times (24+8) = V_{L_7} \times (24+48)$$

$$F_{U_2L_2} = \frac{x}{48} \times \frac{72}{(32)}$$

At x = 8 m, i.e. at L_2

$$F_{U_2L_2} = \frac{8 \times 72}{48 \times 32} = 0.375 \text{ kN (Tensile)}$$

Now, Case II: Consider the load is between L_2L_7

$$V_{L_1} = \left(1 - \frac{x}{48}\right)$$

Consider structure to the left of section (a)-(a)

$\Sigma M_0 = 0$

$$V_{L_1} \times 24 = F_{U_2L_2} \times 32$$

$$F_{U_2L_2} = 0.75 V_{L_1}$$

$$= 0.75\left(1 - \frac{x}{48}\right)$$

At L_3 i.e. x = 16,

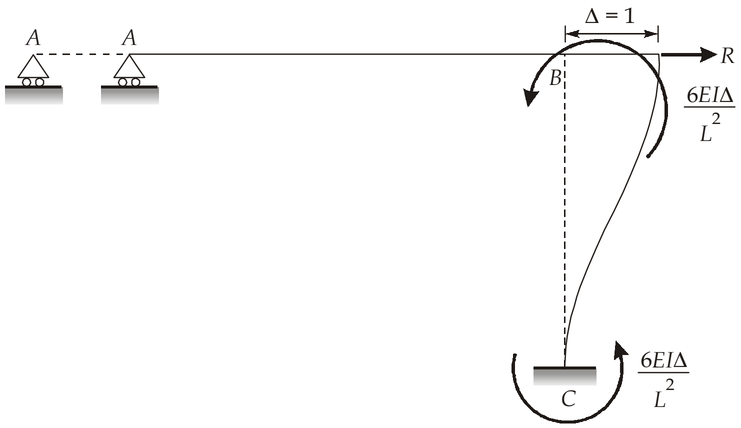
$$F_{U_2L_2} = 0.5 \text{ kN (Compressive)}$$

27. **(a)**

For the given frame, let stiffness matrix be:

$$k = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

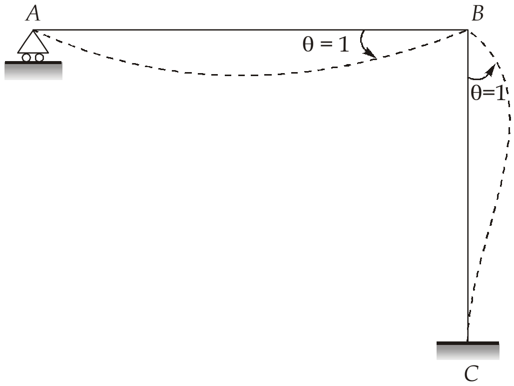
For k_{11} and k_{12}



$$R \times L = \frac{12EI}{L^2}$$

$$\therefore \quad k_{11} = R = \frac{12EI}{L^3}$$

$$k_{12} = k_{21} = \frac{6EI}{L^2}$$



For k_{22}

$$k_{22} = \frac{4EI}{L} + \frac{3EI}{L} = \frac{7EI}{L}$$

$$\therefore \quad k = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{7EI}{L} \end{bmatrix}$$

28. (b)

Applying Betti's theorem

$$25 \times 0.002 + 15 \times \frac{9}{1000} = 15 \times \theta_A + 22 \times 0.004$$

$$\Rightarrow \quad \theta_A = 0.00647 \text{ radian}$$

29. (a)

30. (c)

ILD for shear force at the fixed end of a cantilever and SFD due to unit load at the free end are same.

■■■■