

**MADE EASY**

India's Best Institute for IES, GATE &amp; PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Bhubaneswar | Kolkata

Web: [www.madeeasy.in](http://www.madeeasy.in) | E-mail: [info@madeeasy.in](mailto:info@madeeasy.in) | Ph: 011-45124612

# COMMUNICATIONS

## ELECTRONICS ENGINEERING

**Date of Test : 15/04/2023****ANSWER KEY ➤**

1. (d)	7. (d)	13. (c)	19. (d)	25. (b)
2. (b)	8. (b)	14. (b)	20. (d)	26. (c)
3. (a)	9. (c)	15. (d)	21. (c)	27. (c)
4. (c)	10. (d)	16. (c)	22. (b)	28. (b)
5. (d)	11. (c)	17. (c)	23. (b)	29. (c)
6. (d)	12. (c)	18. (b)	24. (c)	30. (d)

## Detailed Explanations

1. (d)

The maximum and minimum values of the envelope of an AM modulated signal can be given as,

$$E_{\max} = A_c(1 + \mu)$$

$$E_{\min} = A_c(1 - \mu)$$

Given that,

$$A_c = 10 \text{ V and } \mu = 0.4$$

So,

$$E_{\max} = 10(1 + 0.4) \text{ V} = 14 \text{ V}$$

$$E_{\min} = 10(1 - 0.4) \text{ V} = 6 \text{ V}$$

2. (b)

The deviation ratio,  $D = \frac{\Delta f}{f_{m(\max)}}$

Maximum frequency deviation,

$$\Delta f = k_f |m(t)|_{\max}$$

Given that,

$$m(t) = \text{sinc}(1000t) \text{ V}$$

$$k_f = 1 \text{ kHz/V}$$

$$f_{m(\max)} = \frac{1000}{2} = 500 \text{ Hz}$$

$$|m(t)|_{\max} = 1$$

So,

$$D = \frac{1(1000)}{500} = 2$$

3. (a)

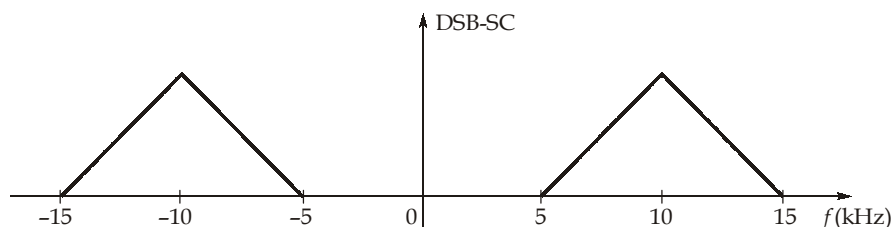
$$(BW)_{\text{FM}} = \left(1 + \frac{\Delta f}{f_m}\right)(2f_m) = \left(1 + \frac{A_m k_f}{f_m}\right)(2f_m)$$

$$(BW)_{\text{PM}} = (1 + A_m k_p)(2f_m)$$

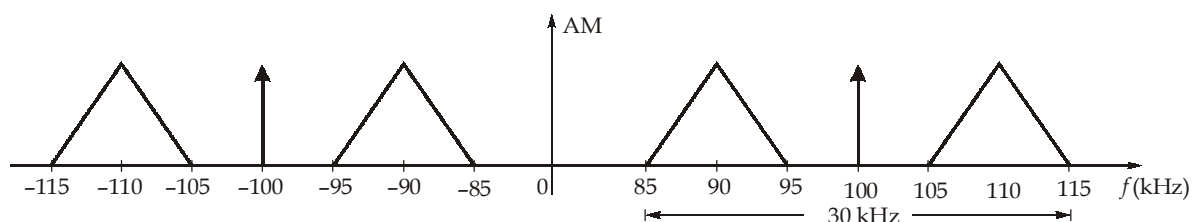
When only  $f_m$  is increased,  $(BW)_{\text{PM}}$  will be increased by higher factor than that of  $(BW)_{\text{FM}}$ .

4. (c)

Spectrum of the DSB-SC signal,



Spectrum of the final AM signal,



So, the bandwidth of the resultant AM signal is 30 kHz.

5. (d)

The variance of a random variable "X" can be given as,

$$\sigma_X^2 = E[X^2] - (E[X])^2$$

$$E[X] = \sum_{i=0}^1 x_i P(x_i) = 1(p) + 0(q) = p$$

$$E[X^2] = \sum_{i=0}^1 x_i^2 P(x_i) = (1)^2(p) + (0)^2(q) = p$$

$$\text{So the variance, } \sigma_X^2 = p - (p)^2 = p(1 - p) = pq$$

$$\because p + q = 1$$

Hence, option (d) is correct.

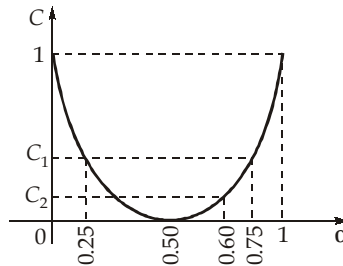
6. (d)

For a real WSS process  $X(t)$ , the power spectral density  $S_X(f)$  should be an even function of "f".  
i.e.,  $S_X(f) = S_X(-f)$

Only the PSD given in option (d) satisfied the above condition.

7. (d)

For a BSC, the variation of C with  $\alpha$  can be plotted as,



The plot of C versus  $\alpha$  is symmetric about  $\alpha = 0.50$ . Hence,  $C = C_1$  for both  $\alpha = 0.25$  and  $\alpha = 0.75$ .  
So, the correct relation between  $C_1$  and  $C_2$  is  $C_2 < C_1$ .

8. (b)

The average probability of error can be given as,

$$\begin{aligned} P_e &= P(r_0 | m_1) P(m_1) + P(r_1 | m_0) P(m_0) \\ &= (0.2) (0.5) + (0.2) (0.5) \\ &= 0.2 \end{aligned}$$

9. (c)

The condition to detect upto ' $e_d$ ' bit errors and simultaneously correct upto ' $e_c$ ' bit errors is,

$$d_{\min} \geq (e_d + e_c + 1)$$

For the given LBC,  $d_{\min} = 4$

$$\text{So, } e_d + e_c + 1 \leq 4$$

$$e_d + e_c \leq 3$$

Only option (c) satisfies the above condition.

10. (d)

$$\text{Total bandwidth, } B_t = 25 \text{ MHz}$$

$$\text{Bandwidth of each radio channel, } B_c = 200 \text{ kHz}$$

$$\text{Number of time slots for each radio channel, } m = 8$$

$$\text{Total number of radio channels possible} = \frac{25 \times 1000}{200} = 125$$

$$\text{Maximum number of simultaneous users} = 125 \times 8 = 1000$$

11. (c)

The angle of the modulated signal  $s(t)$  can be given as,

$$\theta(t) = 2\pi f_c t + 4 \sin(3000\pi t) + 3 \cos(3000\pi t)$$

The instantaneous frequency of the modulated signal can be given as,

$$\begin{aligned} f_i &= \frac{1}{2\pi} \frac{d\theta(t)}{dt} \\ &= f_c + \frac{1}{2\pi} [12000\pi \cos(3000\pi t) - 9000\pi \sin(3000\pi t)] \\ &= f_c + [6000 \cos(3000\pi t) - 4500 \sin(3000\pi t)] \text{ Hz} \\ &= f_c + 1500 [4 \cos(3000\pi t) - 3 \sin(3000\pi t)] \text{ Hz} \end{aligned}$$

$$= f_c + 1500 [5 \cos(3000\pi t + \alpha)] \text{ Hz ;} \quad \text{Where, } \alpha = \tan^{-1} \left( \frac{3}{4} \right)$$

$$f_i = f_c + 7500 \cos(3000\pi t + \alpha) \text{ Hz}$$

Maximum frequency deviation of the signal  $s(t)$  is,

$$(\Delta f)_{\max} = 7500 \text{ Hz} = 7.5 \text{ kHz}$$

12. (c)

The required condition is,

$$RC \leq \frac{1}{\omega_m} \left( \frac{\sqrt{1-\mu^2}}{\mu} \right) = \frac{1}{\omega_m} \sqrt{\frac{1}{\mu^2} - 1}$$

From the given AM wave equation,

$$\begin{aligned} \omega_m &= 2000\pi \\ \mu &= 0.50 \end{aligned}$$

So,

$$RC \leq \frac{1}{2000\pi} \sqrt{\left( \frac{1}{0.5} \right)^2 - 1} = \frac{\sqrt{3}}{2000\pi} \text{ sec}$$

$$RC \leq 275.66 \text{ } \mu\text{sec}$$

13. (c)

$$x(t) = s(t) A \cos(\omega_c t + 30^\circ)$$

$$= A^2 [\cos(\omega_c t + 30^\circ) \cos(\omega_c t)] m(t) = \frac{A^2}{2} [\cos(2\omega_c t + 30^\circ) + \cos 30^\circ] m(t)$$

After passing through LPF, we get,

$$y(t) = \frac{A^2}{2} \cos(30^\circ) m(t) = \frac{\sqrt{3} A^2}{4} m(t)$$

Average power of  $y(t)$ ,

$$P_y = \left( \frac{\sqrt{3}}{4} A^2 \right)^2 P_m = \frac{3A^4}{16} P_m$$

14. (b)

$$M(f) = 10^{-3} \text{rect}\left(\frac{f}{2000}\right)$$

So,  $f_{m(\max)} = 1000 \text{ Hz} = 1 \text{ kHz}$

$$m(t) = 2 \text{sinc}(2000t) \text{ V}$$

So,  $|m(t)|_{(\max)} = 2 \text{ V}$

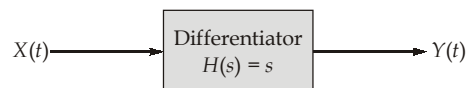
Maximum frequency deviation,

$$\Delta f = k_f |m(t)|_{\max} = (10 \text{ kHz/V}) \times 2 \text{ V} = 20 \text{ kHz}$$

Deviation ratio,  $D = \frac{\Delta f}{f_{m(\max)}} = \frac{20}{1} = 20$

Bandwidth,  $\text{BW} = (1 + D)2f_{m(\max)}$   
 $= (1 + 20)(2)(1) \text{ kHz}$   
 $= 42 \text{ kHz}$

15. (d)



For a differentiator,  $H(s) = s$

$$H(j\omega) = j\omega$$

$$|H(j\omega)|^2 = \omega^2$$

PSD of  $Y(t)$ ,  $S_Y(\omega) = S_X(\omega)|H(\omega)|^2 = \omega^2 S_X(\omega)$

$$R_X(\tau) \xleftrightarrow{CTFT} S_X(\omega)$$

$$\frac{dR_X(\tau)}{d\tau} \xleftrightarrow{CTFT} (j\omega)S_X(\omega)$$

$$\frac{d^2 R_X(\tau)}{d\tau^2} \xleftrightarrow{CTFT} (j\omega)^2 S_X(\omega) = -\omega^2 S_X(\omega)$$

$$-\frac{d^2 R_X(\tau)}{d\tau^2} \xleftrightarrow{CTFT} \omega^2 S_X(\omega) = S_Y(\omega)$$

So,  $R_Y(\tau) = -\frac{d^2 R_X(\tau)}{d\tau^2}$

16. (c)

$$Y = aX + b$$

$$x = \frac{y - b}{a}$$

$$\frac{dy}{dx} = a$$

So, 
$$f_Y(y) = \frac{1}{a} f_X\left(\frac{y-b}{a}\right)$$

Differential entropy of  $Y$ ,

$$\begin{aligned} H(Y) &= - \int_{-\infty}^{\infty} f_Y(y) \log_2 f_Y(y) dy \\ &= - \int_{-\infty}^{\infty} \frac{1}{a} f_X\left(\frac{y-b}{a}\right) \log_2 \left[ \frac{1}{a} f_X\left(\frac{y-b}{a}\right) \right] dy \end{aligned}$$

Let, 
$$\frac{y-b}{a} = u \Rightarrow dy = a du$$

So, 
$$\begin{aligned} H(Y) &= - \int_{-\infty}^{\infty} f_X(u) \log_2 \left[ \frac{1}{a} f_X(u) \right] du \\ &= - \int_{-\infty}^{\infty} f_X(u) \log_2 f_X(u) du + \int_{-\infty}^{\infty} f_X(u) \log_2(a) du \\ &= H(X) + \log_2(a) \int_{-\infty}^{\infty} f_X(x) dx \\ H(Y) &= H(X) + \log_2(a) \end{aligned}$$

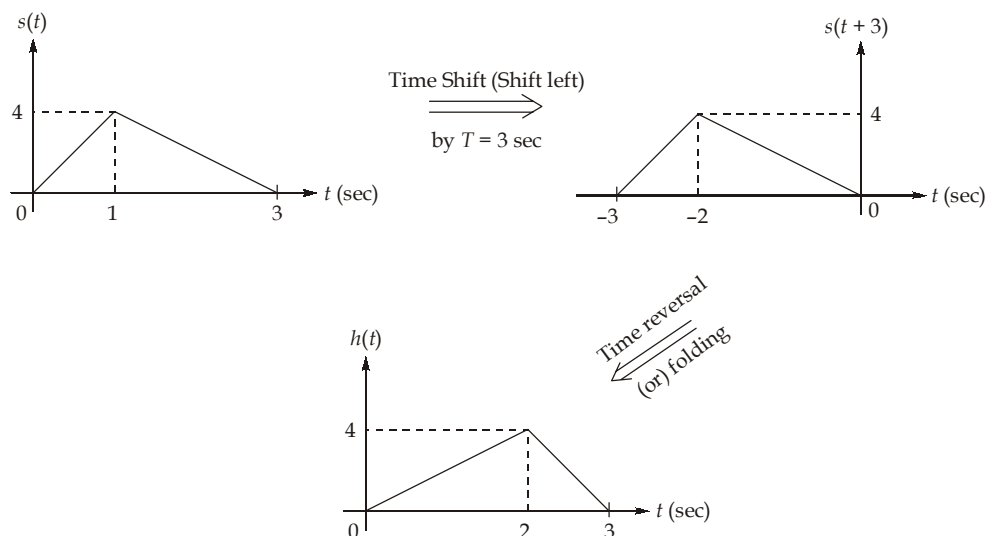
17. (c)

The impulse response of the filter matched to the signal  $s(t)$  can be given as,

$$h(t) = s(T - t); \quad \text{Where, } T = \text{duration of the signal}$$

For the given signal  $s(t)$ ,  $T = 3$  sec.

The signal  $h(t)$  can be obtained by doing following operations to the signal  $s(t)$ .



18. (b)

Number of voice signals,  $N = 4$

Maximum frequency of each signal,  $f_{m(\max)} = 4$  kHz

Bits per sample,  $n = 8$

Sampling rate of each voice signal,  $f_s = 2f_{m(\max)} = 8 \text{ kHz}$

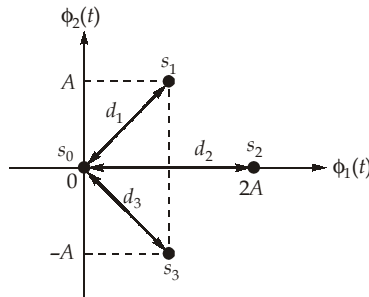
Sampling rate of the multiplexed signal,  $f_{s(\text{overall})} = Nf_s = 32 \text{ kHz}$

Overall transmission rate,  $R_b = nf_{s(\text{overall})} = 256 \text{ kbps}$

Theoretical minimum channel bandwidth required is,

$$(BW)_{\min} = \frac{R_b}{2} = 128 \text{ kHz}$$

19. (d)



Let the energy associated with the symbols  $s_0, s_1, s_2$  and  $s_3$  are  $E_0, E_1, E_2$  and  $E_3$  respectively.

$$E_i = (d_i)^2 ; \quad i = 0, 1, 2, 3$$

From the above diagram,

$$d_0 = 0$$

$$d_1 = d_3 = \sqrt{A^2 + A^2} = \sqrt{2}A$$

$$d_2 = 2A$$

So,

$$E_0 = 0$$

$$E_1 = E_3 = 2A^2$$

$$E_2 = 4A^2$$

The average symbol energy of the modulation scheme can be given as,

$$\begin{aligned} E_s &= \sum_{i=0}^3 E_i P(s_i) ; \quad P(s_i) = \text{probability of occurrence of the symbol } s_i \\ &= 0(0.3) + 2A^2(0.2) + 4A^2(0.4) + 2A^2(0.1) \\ &= (0.4 + 1.6 + 0.2)A^2 \\ &= 2.2 A^2 \end{aligned}$$

20. (d)

For a distortion free detection, the time constant ( $RC$ ) of an envelope detector should satisfy the following condition:

$$\frac{1}{f_c} \ll RC \ll \frac{1}{f_{m(\max)}}$$

Given that,  $f_c = 1 \text{ MHz}$  and  $f_{m(\max)} = 5 \text{ kHz}$

So,  $1 \mu\text{s} \ll RC \ll 200 \mu\text{s}$

For option "a"  $\Rightarrow RC = 100 \times 10^3 \times 10 \times 10^{-12} = 1 \mu\text{s}$

For option "b"  $\Rightarrow RC = 20 \times 10^6 \times 10 \times 10^{-12} = 200 \mu\text{s}$

For option "c"  $\Rightarrow RC = 20 \times 10^6 \times 20 \times 10^{-12} = 400 \mu\text{s}$

For option "d"  $\Rightarrow RC = 1 \times 10^6 \times 20 \times 10^{-12} = 20 \mu\text{s}$

So, the values given in option "d" are best suitable.

21. (c)

The modulation efficiency of an AM signal can be given as,

$$\% \eta = \frac{P_{SB}}{P_{total}} \times 100 = \frac{\mu^2}{2 + \mu^2} \times 100$$

Where,  $\mu$  = modulation index

For a multi-tone modulation,

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

For the given AM signal,  $\mu_1 = 0.5$  and  $\mu_2 = 0.4$

So, 
$$\mu^2 = \mu_1^2 + \mu_2^2 = (0.5)^2 + (0.4)^2 = 0.41$$

$$\% \eta = \frac{0.41}{2 + 0.41} \times 100 \approx 17\%$$

22. (b)

$$\beta_1 = \frac{\Delta f_1}{f_{m1}} = \frac{A_{m1} k_f}{f_1} = 10 \text{ rad} \quad \dots(i)$$

$$\beta_2 = \frac{\Delta f_2}{f_{m2}} = \frac{A_{m2} k_f}{f_2} = 20 \text{ rad} \quad \dots(ii)$$

From equations (i) and (ii), it is clear that,

$$\frac{A_{m2} k_f}{f_2} = 2 \times \frac{A_{m1} k_f}{f_1}$$

$$\frac{f_2}{f_1} = \frac{A_{m2}}{2 A_{m1}} = \frac{2}{2 \times 1} = 1$$

$\therefore$  Given that,  $A_{m1} = 1 \text{ V}$  and  $A_{m2} = 2 \text{ V}$

So,

$$f_2 = f_1$$

23. (b)

$$(\Delta f_{\max})_{FM} = A_m k_f$$

$$(\Delta f_{\max})_{PM} = \frac{k_p}{2\pi} \left| \frac{dm(t)}{dt} \right|_{\max}$$

So,

$$(\Delta f_{\max})_{PM} = \frac{k_p}{2\pi} \left( \frac{A_m}{2} \right) = \frac{k_p A_m}{4\pi}$$

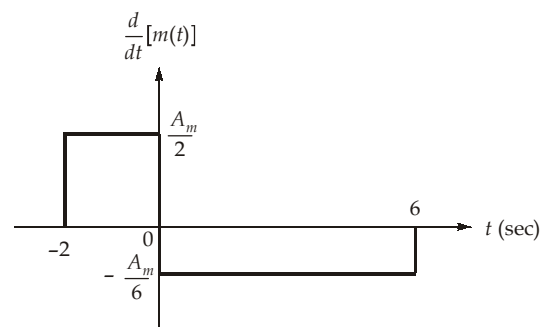
If,

$$(\Delta f_{\max})_{PM} = (\Delta f_{\max})_{FM}$$

Then,

$$\frac{k_p A_m}{4\pi} = k_f A_m$$

$$\frac{k_p}{k_f} = 4\pi \text{ rad/Hz}$$



24. (c)

$$\theta(t) = 2\pi f_c t + 5\sin(3000\pi t) + 10\sin(2000\pi t)$$

$$f_i = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + 7500 \cos(3000\pi t) + 10000 \cos(2000\pi t)$$

So,  $\Delta f = 7500 + 10000 = 17.5 \text{ kHz}$

$$f_{m(\max)} = 1500 \text{ Hz} = 1.5 \text{ kHz}$$

Deviation ratio,  $D = \frac{\Delta f}{f_{m(\max)}} = \frac{17.5}{1.5} = \frac{35}{3}$

Bandwidth,  $BW = (1 + D)2f_{m(\max)} = \left(1 + \frac{35}{3}\right)(3) \text{ kHz} = 38 \text{ kHz}$

25. (b)

Given that,

$$f_{LO} > f_c$$

So,

$$f_{LO} = f_c + f_{IF}$$

$$f_{LO(\max)} = f_{c(\max)} + f_{IF} = 1650 + 450 = 2100 \text{ kHz}$$

$$f_{LO(\min)} = f_{c(\min)} + f_{IF} = 550 + 450 = 1000 \text{ kHz}$$

For an LC oscillator,  $f_0 = \frac{1}{\sqrt{LC}}$

$$C \propto \frac{1}{f_0^2}$$

$$\frac{C_{\max}}{C_{\min}} = \left( \frac{f_{LO(\max)}}{f_{LO(\min)}} \right)^2 = \left( \frac{2100}{1000} \right)^2 = 4.41$$

26. (c)

$$E[X(t)] = 5$$

$$E[Y(t)] = H(0)E[X(t)]$$

$$H(0) = H(s) \big|_{s=0}$$

$$H(s) = \text{Transfer function of the given circuit}$$

From the given circuit,

$$H(0) = \frac{4 \text{ M}\Omega}{4 \text{ M}\Omega + 1 \text{ M}\Omega} = \frac{4}{5} = 0.80$$

So,  $[Y(t)] = 0.80 \times 5 = 4$

27. (c)

Minimum signal-to-noise ratio that has to be maintained in the channel is,

$$\begin{aligned} \text{SNR} &= \frac{S}{N_0 B} = \frac{S}{\left(\frac{N_0}{2}\right)(2B)} \\ &= \frac{100 \times 10^{-6}}{(10^{-10})(2 \times 4 \times 10^3)} = 125 \end{aligned}$$

The capacity of the channel can be given by,

$$\begin{aligned} C &= B \log_2(1 + \text{SNR}) \\ &= 4 \log_2(1 + 125) \simeq 27.91 \text{ kbps} \end{aligned}$$

28. (b)

Signal power, 
$$S = \int_{-\infty}^{\infty} x^2 f_X(x) dx = 2 \int_0^2 x^2 \left( \frac{2-x}{4} \right) dx$$

$$= \frac{1}{2} \int_0^2 (2x^2 - x^3) dx = \frac{1}{2} \left[ \frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2 = \frac{1}{2} \left[ \frac{16}{3} - \frac{16}{4} \right] = \frac{2}{3}$$

Quantization noise power,  $N_Q = \frac{\Delta^2}{12}$

$$\Delta = \frac{2 - (-2)}{64} = \frac{1}{16}$$

So, 
$$N_Q = \frac{(1/16)^2}{12} = \frac{1}{3072}$$

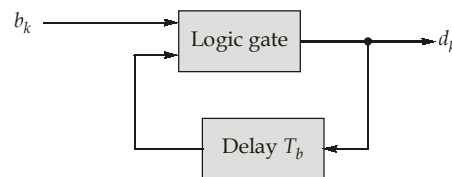
$$(SQNR) = \frac{S}{N_Q} = \frac{(2/3)}{(1/3072)} = \frac{2 \times 3072}{3} = 2048$$

In decibels,  $[SQNR] = 10 \log_{10} (SQNR) = 33.11 \text{ dB}$

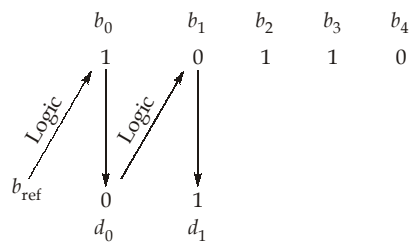
29. (c)

It is given that the carrier phase for first two encoded bits are  $\pi$ , 0. So first two encoded bits are 0, 1.

The logic diagram of DPSK encoder can be given as,



Finding the logic gate:



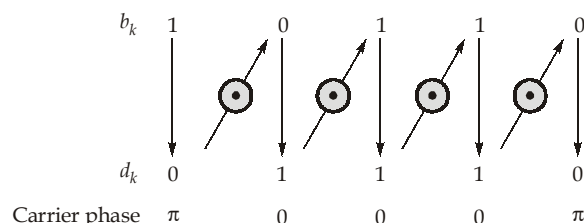
If we observe clearly,

$$d_0 (\text{logic}) b_1 = d_1$$

$$0 (\text{logic}) 0 = 1$$

So, the possible logic is, EX-NOR.

So, the complete encoding can be given as,



So, the carrier phases corresponding to the remaining three bits are 0, 0,  $\pi$ .

30. (d)

The given periodic signal  $s(t)$  has odd-symmetry. So, the Fourier series representation of  $s(t)$  has,

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{T} \left[ \int_{-T/2}^0 (-1) \sin(n\omega_0 t) dt + \int_0^{T/2} \sin(n\omega_0 t) dt \right]$$

Where,  $\omega_0 = \frac{2\pi}{T}$

So,

$$\begin{aligned} b_n &= \frac{2}{T} \left[ \left( \frac{\cos(n\omega_0 t)}{n\omega_0} \right)_{-T/2}^0 - \left( \frac{\cos(n\omega_0 t)}{n\omega_0} \right)_0^{T/2} \right] \\ &= \frac{2}{n\omega_0 T} \left[ 1 - \cos\left(\frac{n\omega_0 T}{2}\right) - \cos\left(\frac{n\omega_0 T}{2}\right) + 1 \right] \\ &= \frac{4}{n(2\pi)} [1 - \cos(n\pi)] = \frac{2}{n\pi} [1 - (-1)^n] \\ b_n &= \begin{cases} \frac{4}{n\pi} & ; \quad n \text{ is odd} \\ 0 & ; \quad n \text{ is even} \end{cases} \end{aligned}$$

So,  $x(t)$  can be represented as,

$$\begin{aligned} x(t) &= s(t) m(t) \\ &= \left[ \frac{4}{\pi} \sin\left(\frac{2\pi}{T} t\right) + \frac{4}{3\pi} \sin\left(\frac{6\pi}{T} t\right) + \frac{4}{5\pi} \sin\left(\frac{10\pi}{T} t\right) + \dots \right] m(t) \end{aligned}$$

After passing through BPF, we get,

$$y(t) = \frac{4}{\pi} \sin\left(\frac{2\pi}{T} t\right) m(t) = A_c m(t) \sin\left(\frac{2\pi}{T} t\right)$$

So,

$$A_c = \frac{4}{\pi}$$

■■■■