

CLASS TEST

S.No. : 01 LS1_EC_S+D_050819

Signal & System



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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

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ANSWER KEY > Signal & System

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|--------|---------|---------|---------|---------|
| 1. (d) | 7. (a) | 13. (d) | 19. (b) | 25. (a) |
| 2. (d) | 8. (b) | 14. (c) | 20. (a) | 26. (a) |
| 3. (b) | 9. (a) | 15. (a) | 21. (b) | 27. (a) |
| 4. (a) | 10. (a) | 16. (a) | 22. (a) | 28. (a) |
| 5. (a) | 11. (b) | 17. (a) | 23. (c) | 29. (d) |
| 6. (a) | 12. (a) | 18. (b) | 24. (c) | 30. (d) |

Detailed Explanations

1. (d)

$$2^n u[n] \xleftrightarrow{z} \frac{1}{1-2z^{-1}}$$

$$\therefore \text{ROC} \Rightarrow |z| > 2$$

and $(4)^n u[-n-1] \xleftrightarrow{z} \frac{1}{1-4z^{-1}}$

$$\therefore \text{ROC} \Rightarrow |z| < 4$$

\therefore for the signal to converge

$$\text{ROC} \Rightarrow 2 < |z| < 4$$

2. (d)

Since the transfer function has one pole on the R.H.S thus the system is unstable, so the final value theorem is not applicable in this case.

3. (b)

$$y[n] = x[n] * \delta[n-2] = x[n-2]$$

$$\therefore y[2] = x[0] = 1$$

4. (a)

$$\therefore j \frac{d}{dt} [F(j\omega)] \longleftrightarrow t[f(t)]$$

thus at $t = 0$ the answer will be zero.

6. (a)

$$\therefore X[k] = W_N^{n_0 k}$$

for $N = 4$, $X[k] = W_N^{n_0 k}$ for $k = 0, 1, 2, 3$

taking IDFT, we get, $x[n] = \delta[n-n_0]$

$$\therefore \text{energy} = \sum |x[n]|^2 = 1$$

7. (a)

$$\begin{aligned} X(e^{j\omega}) &= e^{j\omega} + e^{-j\omega} + 2(e^{2j\omega} - e^{-2j\omega}) + 3(e^{3j\omega} + e^{-3j\omega}) \\ &= 2\cos\omega + 4j\sin(2\omega) + 2\cos3\omega = 2\cos(\pi) + 4j\sin(2\pi) + 6\cos(3\pi) \\ &= -2 + 0 - 6 = -8 \\ |Xe^{j\pi}| &= 8 \end{aligned}$$

8. (b)

Hilbert transform will shift the signal by $\frac{-\pi}{2}$ thus $x(t) = e^{j(2\pi ft - \pi/2)} = -je^{j2\pi ft}$

9. (a)

\therefore For causal system degree of numerator < degree of denominator. (degree is highest power of z)

10. (a)

$$\operatorname{Re}\{x(t)\} = \frac{x(t) + x^*(t)}{2}$$

∴ The Fourier coefficient of $x^*(t)$ are

$$b_K = \frac{1}{T} \int_T x^*(t) e^{-jk\frac{2\pi}{T}t} dt$$

Taking conjugate on both sides

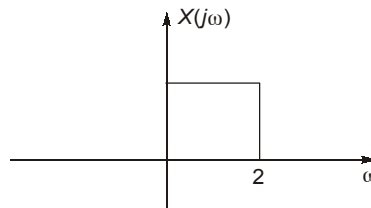
$$b_K^* = \frac{1}{T} \int_T x(t) e^{-j(-K)\frac{2\pi}{T}t} dt$$

$$\therefore a_{-K} = b_K^*$$

$$\therefore \text{Fourier series Coefficient of } \operatorname{Re}\{x(t)\} = \frac{a_K + a_{-K}^*}{2}$$

11. (b)

The signal can be represented as



Since the signal is neither even nor odd symmetric in frequency domain thus it will be a complex signal in time domain.

12. (a)

$$Y(z) = X(z) \cdot H(z) = 2[z^3 + z^2 - 3z^{-1} + 5z^{-3}] \cdot [2(z^{-2})]$$

$$= 2[z + 2z^{-1} - 3z^{-3} + 5z^{-5}]$$

$$\therefore z^{-2} = 0$$

$$\text{thus } Y[2] = 0$$

13. (d)

1. $x(t) = 0$ for $t < 0$ thus the signal is causal

∴ $x(t)$ is real thus $X(j\omega)$ is an even symmetric signal

$$\text{now } \operatorname{Re}\{X(j\omega)\} \xrightarrow{F.T} \text{even}\{x(t)\}$$

$$\therefore |t|e^{-|t|} = \text{even}\{x(t)\}$$

$$\therefore \frac{x(t) + x(-t)}{2} = |t|e^{-|t|}$$

$$\therefore x(t) = 2te^{-|t|}$$

{∵ right sided $x(-t) = 0$ }

$$x(t) = 2te^{-t}u(t)$$

14. (c)

$$x(t) \xrightarrow{L.T} X(s) = \frac{s+3}{(s+3)^2+4}$$

$$y(t) \Rightarrow \int x(\tau) d\tau \Rightarrow \frac{s+3}{s[(s+3)^2+4]}$$

15. (a)

$$\frac{\sin\left(\frac{3}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)} \xrightarrow{F.T} x_1[n] = \begin{cases} 1 & -1 < n < 1 \\ 0 & \text{otherwise} \end{cases}$$

now $\sum_{k=-\infty}^n x[k] \xrightarrow{F.T} \frac{1}{1-e^{-j\omega}} X_1(e^{j\omega}) + 3\pi\delta(\omega)$

$$2\pi\delta(\omega) \xrightarrow{F.T} 1$$

$$\therefore x[n] = 1 + \sum_{k=-\infty}^n x_1[k]$$

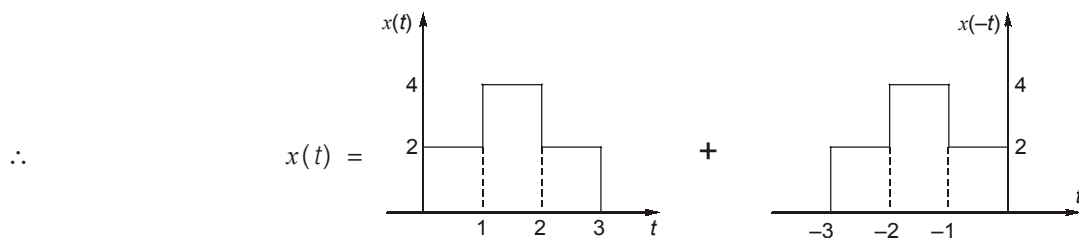
$$= \begin{cases} 1 & n < -4 \\ n+3 & -1 < n < 1 \\ 4 & n \geq 2 \end{cases}$$

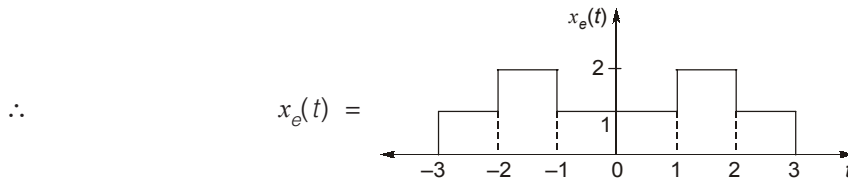
∴ at $n \rightarrow \infty$

$$x[n] = 4$$

16. (a)

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$





17. (a)

Let $x'(t) = e^{-2t} u(t)$

$$X'(j\omega) = \frac{1}{2 + j\omega}$$

now applying time reversal property

$$x(-t) \xrightarrow{FT} X(-j\omega)$$

$$\therefore e^{2t} u(-t) \xrightarrow{FT} \frac{1}{2 - j\omega}$$

18. (b)

$$Y(e^{j\omega}) = X(e^{j\omega}) * X(e^{j\omega - \pi/2})$$

∴ In time domain

$$Y(e^{j\omega}) = 2\pi x_1[n] \cdot x_2[n]$$

now, If $X(e^{j\omega}) \longleftrightarrow n \left(\frac{3}{4}\right)^{|n|}$

Then $X(e^{j\omega - \pi/2}) \longleftrightarrow n \cdot e^{j\pi/2} \left(\frac{3}{4}\right)^{|n|}$

$$\therefore y[n] \longleftrightarrow 2\pi n^2 e^{j\pi n/2} \left(\frac{3}{4}\right)^{2|n|}$$

19. (b)

$$X(z) = \frac{z}{z-1} \quad |z| > 1$$

$$Y(z) = \frac{2z}{z - \frac{1}{3}} \quad |z| > \frac{1}{3}$$

$$H(z) = \frac{2(z-1)}{z - \frac{1}{3}} \quad |z| > \frac{1}{3}$$

$$X'(z) = \frac{z}{z - \frac{1}{2}} \quad |z| > \frac{1}{2}$$

$$Y'(z) = H(z) \cdot X'(z)$$

$$= \frac{2z(z-1)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \quad |z| > \frac{1}{2}$$

Taking inverse z transform

$$y[n] = \left[-6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{3}\right)^n \right] u[n]$$

$$k_1 = -6, \quad k_2 = 8$$

so,

$$k_1 + k_2 = 2$$

20. (a)

$$\begin{aligned} x(t) &= \cos\left(2\left(t-\frac{1}{2}\right)\right) \\ &= \frac{1}{2}\left[e^{j2(t-1/2)} + e^{-j2(t-1/2)}\right] \\ &= \frac{1}{2}\left[e^{j2t} \cdot e^{-j} + e^{-j2t} \cdot e^j\right] \end{aligned}$$

According to the given condition the output is

$$= \frac{1}{2}\left[e^{-j}e^{j3t} + e^je^{-j3t}\right] = \frac{1}{2}\left[e^{j3(t-1/3)} + e^{-j3(t-1/3)}\right]$$

$$y(t) = \cos\left[3\left(t-\frac{1}{3}\right)\right]$$

$$\begin{aligned} \therefore y\left(\frac{1}{3}\right) &= \cos\left[3\left(\frac{1}{3}-\frac{1}{3}\right)\right] \\ &= \cos 0 = 1 \end{aligned}$$

21. (b)

If $x[n]$ is real

$$\text{odd}[x[n]] \xrightarrow{FT} j\text{Im}[X(e^{j\omega})]$$

$$\begin{aligned} \therefore \text{odd}[x[n]] &= F^{-1}\left[\frac{1}{2}\left(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}\right)\right] \\ &= \frac{1}{2}\left[\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right] \end{aligned}$$

$$\therefore \text{odd}[x[n]] = \frac{x[n] - x[-n]}{2}$$

Since,

$$\begin{aligned} x[n] &= 0 \text{ for } n > 0 \\ x[n] &= 2 \text{ odd}[x[n]] \\ &= \delta[n+1] - \delta[n+2] \text{ for } n < 0 \end{aligned}$$

using Parseval's theorem

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$(x[0])^2 - 2 = 3$$

$$x[0] = \pm 1$$

$$\therefore x[0] > 0$$

$$\therefore x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

22. (a)

$$|H(j\omega)| = \left| 3 \cos\left(\frac{\omega}{2}\right) \right|; \quad \angle H(j\omega) = -\omega/2$$

So,
$$H(\omega) = 3 \cos\left(\frac{\omega}{2}\right) \cdot e^{-j\omega/2} = \frac{3}{2} [e^{j\omega/2} + e^{-j\omega/2}] \cdot e^{-j\omega/2}$$

$$= \frac{3}{2} [1 + e^{-j\omega}]$$

So,
$$H(z) = \frac{3}{2} [1 + z^{-1}]$$

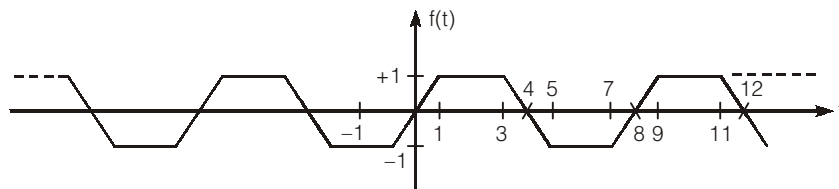
23. (c)

Taking Fourier transform of the signal, we obtain

$$H(e^{j\omega}) = \frac{\frac{3}{2} - \frac{1}{2} e^{-j\omega}}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-2j\omega}}$$

$$\therefore y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = \frac{3}{2} x[n] - \frac{1}{2} x[n-1]$$

24. (c)



The signal $f(t)$ has hidden, odd and half-wave symmetry.

So, $a_0 \neq 0$

$$a_n = 0; \quad \forall n$$

$$b_n \neq 0; \quad n = 1, 3, 5$$

Therefore, non zero Fourier series coefficients are

a_0 and $b_n, n = 1, 3, 5, \dots$

25. (a)

$$\begin{aligned} \text{As, } x(t) \cos^2 t &= x(t) \left[\frac{1}{2} + \frac{1}{2} \cos 2t \right] \\ &= \frac{1}{2} x(t) + \frac{1}{2} \cos 2t \end{aligned}$$

$$\begin{aligned} \text{Now, } g(t) &= x(t) \cdot \cos^2 t * \frac{\sin t}{\pi t} \\ &= x(t) \left[\frac{1 + \cos 2t}{2} \right] * \frac{\sin t}{\pi t} \end{aligned}$$

$$\therefore G(j\omega) = \left[\frac{1}{2} X(j\omega) + \frac{1}{4} X(\omega - 2) + \frac{1}{4} X(\omega + 2) \right] \times \text{rect} \left(\frac{\omega}{2} \right)$$

Thus the given solution will be

$$\therefore G(j\omega) = \frac{1}{2} X(j\omega)$$

$$\text{or } g(t) = \frac{1}{2} x(t)$$

Thus to get the desired result

$$h(t) = \frac{1}{2} \delta(t)$$

26. (a)

$$\begin{aligned} x[n] &= \delta[n] \\ X(e^{j\omega}) &= 1 \end{aligned}$$

$$\frac{dX(e^{j\omega})}{d\omega} = 0$$

$$\therefore Y(e^{j\omega}) = e^{-j\omega} X(e^{j\omega})$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega} \cdot e^{j\omega n} d\omega = \frac{\sin \pi(n-1)}{\pi(n-1)}$$

27. (a)

$$\therefore x^*(t) \xrightarrow{F} X^*(-j\omega)$$

$$\text{and } \int_{-\infty}^t x(\tau) d\tau \xrightarrow{F} \frac{X(j\omega)}{j\omega} + \pi X(0) \delta(\omega)$$

28. (a)

$$X(j0) = \int_{-\infty}^{\infty} x(t) dt = 7$$

29. (d)

$$H(j\omega) = \frac{1 + 2e^{-j\omega}}{1 + \frac{1}{2e^{-j\omega}}} = \frac{1 + 2e^{-j\omega}}{2e^{-j\omega} + 1} \cdot 2e^{-j\omega}$$

$$|H(j\omega)| = 2$$

30. (d)

The impulse response convolves with the input signal and the output can be broken up into shifted versions of input.

