CLASS TEST

S.No. : 01 JP_ALL_EM_270323

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Engineering Mathematics

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 If x is uniformly distributed over (0, 15), the probability that $5 < x < 9$ is

(a) $\frac{1}{3}$ (b) $\frac{4}{15}$

(c) $\frac{2}{5}$ (d) $\frac{7}{15}$

Q.2 Let $f(x) = 2x^2 - 5x - 6$, then which of the following is true?

(a) $f(x)$ has maxima at $\frac{5}{4}$

(b) $f(x)$ has minima at $\frac{-5}{2}$

(c) $f(x)$ has maxima at $\frac{4}{5}$

(d) $f(x)$ has minima at $\frac{5}{4}$

Q.3 If $y = \frac{2\ln x}{3x}$, then y has a maximum at x equal to

(a) 0 (b) 1

(c) $\frac{1}{e}$ (d) e

Q.4 The area between the parabola $y^2 = 2ax$ and $x^2 = 2ay$ is

(a) $\frac{2a^2}{3}$

(b) $\frac{4a^2}{3}$

(c) $3a^2$

(d) $\frac{8a^2}{3}$

Q.5 The degree of differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{5/7} + \left(\frac{d^3y}{dx^3}\right)^{7/5} = 0 \text{ is}$$

(a) 12 (b) 16

(c) 18 (d) 24

Q.6 Let the eigen values of matrix $[A]_{2\times 2}$ are γ and δ, then eigen values of $(A + 7I)^{-1}$ are

(a) $(\gamma + 7), (\delta + 7)$

(b) $\frac{1}{\gamma} + 7, \frac{1}{\delta} + 7$

(c) $\frac{1}{\gamma + 7}, \frac{1}{\delta + 7}$

(d) Can't be determined

Q.7 The derivative of x^x with respect to x is

(a) $x^x (1 + \log x)$ (b) $1 + \log x^x$

(c) $x (1 + \log x)$ (d) $x^x (1 + x\log x)$

Q.8 A curve $y = \frac{1}{2\sqrt{x}}$ is allowed to revolve around x-axis. The volume of solid of revolution for $3 \le x \le 4$ is

(a) $\pi\left(\sqrt{3} + \frac{2}{\sqrt{3}}\right)$

(b) $\frac{\pi}{2}\ln\left(\frac{8}{3}\right)$

(c) $\frac{\pi}{4}\ln\left(\frac{4}{3}\right)$

(d) $\frac{\pi}{2}\left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$

Q.9 If $A \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I$, where I is a unit matrix of order 2 × 2, then A is

(a) $\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) None of these

Q.10 If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) none of these

Q.No. 11 to Q.No. 30 carry 2 marks each

Q.11 Consider the following equations,

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$

If the given equations are consistent, then the largest value of λ is

(a) 1 (b) 2
(c) 3 (d) 4

Q.12 The value of $\int_{-7}^{7} |2x+3|\,dx$ is

(a) 95.5 (b) 102.5
(c) 109.5 (d) 114.5

Q.13 The area of the segment made by the parabola $x^2 = 4y$ by the line $x - 2y + 4 = 0$ is

(a) 9 unit^2 (b) 12 unit^2
(c) 16 unit^2 (d) 24 unit^2

Q.14 A function $f(x)$ is designed as $f(x) =$

$$\begin{cases} 0 & ; \quad x < 3 \\ \frac{1}{A}(3x+5); & 3 < x < 6 \\ 0 & ; \quad x > 6 \end{cases}.$$

For $f(x)$ to be a valid probability density function the value of A must be

(a) 55.5 (b) 40.5
(c) 50 (d) 63.5

Q.15 Consider the matrix $[A] = \begin{bmatrix} 2 & 2 & 6 \\ 2 & 10 & 2 \\ 6 & 2 & 2 \end{bmatrix}$. It has eigen values as p, q, r. The value of $pq + qr + rp - pqr$ is

(a) 288 (b) 144
(c) 432 (d) 576

Q.16 A government official finds that on an average, 16% of tender applications received are rejected because they are either incomplete or incorrect. The probability that a file containing 8 tender applications will face at least one rejection is

(a) 0.2479 (b) 0.7521
(c) 0.3621 (d) 0.8421

Q.17 X is a uniformly distributed random variable that takes values between 0 and 1. The value of $E(X^4)$ will be

(a) 0.25 (b) 0.3
(c) 0.4 (d) 0.20

Q.18 If –8 and 5 are the eigen value of a non-singular matrix A of order 2, then the eigen values of adj A are

(a) 16, –10 (b) –5, 8
(c) 5, –8 (d) –16, 10

Q.19 If $2x + 3y = \lambda$ is normal to $y^2 = 9x$, then value of λ is

(a) 9 (b) 10
(c) 11 (d) 12

Q.20 Consider the differential equation

$$\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y = 0$$

with initial conditions $y(0) = 0$ and $y(1) = e^2$. The value of $y(2)$ is

(a) 109.2 (b) 89.2
(c) 119.2 (d) 79.2

Q.21 In a game, 30 cards are drawn at a time out of 80 playing cards numbered from 1 to 80. The expected value of the sum of numbers on the cards drawn is

(a) 2215 (b) 1215
(c) 615 (d) 815

Q.22 If the probability of a smart-phone getting a hanging problem after downloading a certain application is 0.002, then the probability that out of 1500 smart-phone more than two will get a hanging problem is

(a) $-1 + \frac{17}{2e^2}$ (b) $1 - \frac{17}{2e^3}$

(c) $1 - \frac{17}{2e^2}$ (d) $1 + \frac{17}{2e^3}$

Q.23 For the given linear differential equation $(2x + 1)dy - ydx - e^{4x}(2x + 1)^2 dx = 0$, the integrating factor (IF) is

(a) e^{2x+1}
(b) $\frac{1}{\sqrt{2x+1}}$
(c) e^{1-2x}
(d) $-(2x + 1)$

Q.24 If

$$X = \begin{bmatrix} \frac{9}{2} & -6 \\ \frac{3}{2} & \frac{-3}{2} \end{bmatrix}, B = \begin{bmatrix} \frac{15}{2} & 3 \\ -3 & \frac{3}{2} \end{bmatrix} \text{ and } A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

satisfy the equation $AX = B$, then the determinant of matrix A is

(a) 6
(b) 8
(c) 9
(d) 12

Q.25 Let C be the curve $x = 2 - 3y^2$ from (0, –1) to (0, 1). Then the value of integral $\int_c \left(2y^3 dx + 3x^2 dy\right)$ is

(a) 3
(b) 4
(c) 5
(d) 6

Q.26 In a factory, machine A produces 40% of the output and machine B produces 60%. On an average, 9 items in 1000 produced by A are defective and 1 item in 250 produced by B is defective. An item drawn at random from a day's output is defective. What is the probability that it was produced by A?

(a) 0.4
(b) 0.5
(c) 0.6
(d) 0.7

Q.27 A curve $y = \frac{1}{2\sqrt{x}}$ is allowed to revolve around x-axis. The volume of solid of revolution for $3 \le x \le 4$ is

(a) $\pi\left(\sqrt{3} + \frac{2}{\sqrt{3}}\right)$
(b) $\frac{\pi}{2}\ln\left(\frac{8}{3}\right)$
(c) $\frac{\pi}{4}\ln\left(\frac{4}{3}\right)$
(d) $\frac{\pi}{2}\left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$

Q.28 The probability of India winning a test match is 0.45 and of losing is 0.25. The probability of India neither winning nor losing is

(a) 0.70
(b) 0.60
(c) 0.40
(d) 0.30

Q.29 Particular integral for differential equation

$$\frac{d^2y}{dx^2} = \cos x - y \text{ is}$$

(a) $x \cos x$
(b) $\frac{1}{2} x \cos x$
(c) $x \sin x$
(d) $\frac{1}{2} x \sin x$

Q.30 A ball thrown vertically upward satisfies the equation $S = 160t - 20t^2$, where S is in meters and t is in seconds. The maximum height achieved by the stone is

(a) 80 m
(b) 160 m
(c) 320 m
(d) 480 m

■■■■

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ENGINEERING MATHEMATICS

Date of Test : 27/03/2023

ANSWER KEY

1.	(b)	7.	(a)	13.	(a)	19.	(c)	25.	(d)
2.	(d)	8.	(c)	14.	(a)	20.	(a)	26.	(c)
3.	(d)	9.	(c)	15.	(a)	21.	(b)	27.	(c)
4.	(b)	10.	(c)	16.	(b)	22.	(b)	28.	(d)
5.	(d)	11.	(c)	17.	(d)	23.	(b)	29.	(d)
6.	(c)	12.	(b)	18.	(c)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

$$f(x) = \frac{1}{15-0} = \frac{1}{15}$$

$$P\{5 < x < 9\} = \int_5^9 \frac{1}{15}dx = \frac{4}{15}$$

2. (d)

Given

$$f(x) = 2x^2 - 5x - 6$$
$$f'(x) = 4x - 5$$
$$f''(x) = 4$$

For minima/maxima,

$$f'(x) = 0$$
$$4x - 5 = 0$$
$$x = \frac{5}{4}$$
$$f''(x) = 4 > 0 \Rightarrow \text{Minima}$$

3. (d)

$$y = \frac{2}{3}\frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{2}{3x}.\frac{1}{x} + \frac{2}{3}\ln x\left(\frac{-1}{x^2}\right) = \frac{2}{3x^2}(1 - \ln x)$$

For maxima, $$\frac{dy}{dx} = 0$$

$$\ln x = 1 \quad \Rightarrow \quad x = e \text{ is a stationary point}$$

$$\frac{d^2y}{dx^2} = \frac{-2}{3x^3}(3 - 2\ln x)$$

At $x = e$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = \frac{-2}{3e^3} < 0$$

Hence maxima at $x = e$

4. (b)

$$y^2 = 2ax$$

$$x = \frac{y^2}{2a} \quad \text{...(i)}$$

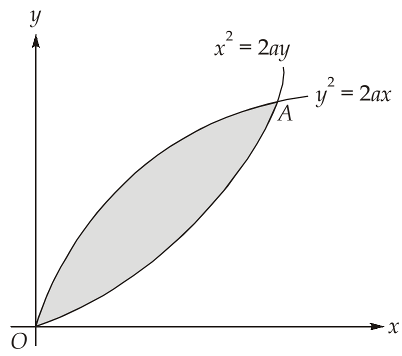
$$x^2 = 2ay$$

Using equation (i)

$$\frac{y^4}{4a^2} = 2ay$$

$$y^4 - 8a^3y = 0$$
$$y = 0, y = 2a$$

The parabolas intersect at 0 (0, 0) and A ($2a$, $2a$)



$$\text{Required area} = \int_0^{2a}\int_{x^2/2a}^{\sqrt{2ax}} dy\,dx$$

$$= \int_0^{2a}\left(\sqrt{2ax} - \frac{x^2}{2a}\right)dx$$

$$= \left|\sqrt{2a}\,\frac{2}{3}x^{3/2} - \frac{1}{2a}.\frac{x^3}{3}\right|_0^{2a}$$

$$= \sqrt{2a}\,\frac{2}{3}(2a)^{3/2} - \frac{1}{2a}\frac{(2a)^3}{3}$$

$$= \frac{8a^2}{3} - \frac{4a^2}{3} = \frac{4a^2}{3}$$

5. **(d)**

$$1 + \left(\frac{d^3y}{dx^3}\right)^{7/5-5/7} = 0$$

$$\left(\frac{d^3y}{dx^3}\right)^{24/35} = -1$$

Raising power 35 on both sides.

$$\left(\frac{d^3y}{dx^3}\right)^{24} = -1$$

From here degree of equation is 24.

6. **(c)**

Eigen values of $(A + 7I)$ are $\gamma + 7$ and $\delta + 7$

Eigen values of $(A + 7I)^{-1} = \frac{1}{\gamma + 7}$ and $\frac{1}{\delta + 7}$

7. **(a)**

Let $y = x^x$

Taking logarithm on both sides, we get

$$\log y = x \log x$$

Differentiating w.r.t. x, we get

$$\frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \quad \frac{dy}{dx} = y(1 + \log x)$$

$$= x^x (1 + \log x)$$

8. (c)

$$\text{Volume of solid} = \int_a^b \pi y^2 dx$$

Given $$y = \frac{1}{2\sqrt{x}}$$

$$\text{Volume of the solid} = \int_3^4 \frac{\pi}{4x} \cdot dx = \frac{\pi}{4}(\ln x)_3^4 = \frac{\pi}{4}\ln\left(\frac{4}{3}\right)$$

9. (c)

We know, $$AA^{-1} = I,$$

$$A \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I$$

$$\frac{A}{6} \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = I$$

$$\frac{A}{6} = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

10. (c)

$$AA^{-1} = I$$

$$\therefore \quad \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2x = 1$$

$$\therefore \quad x = \frac{1}{2}$$

11. (c)

The given equation will be consistent, if

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ 0 & \lambda-3 & 3-\lambda \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$$

$C_3 \rightarrow C_3 + C_2$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 5\lambda+1 \\ 0 & \lambda-3 & 0 \\ 2 & 3\lambda+1 & 6\lambda-2 \end{vmatrix} = 0$$

$$(\lambda-3)\begin{vmatrix} \lambda-1 & 5\lambda+1 \\ 2 & 2(3\lambda-1) \end{vmatrix} = 0$$

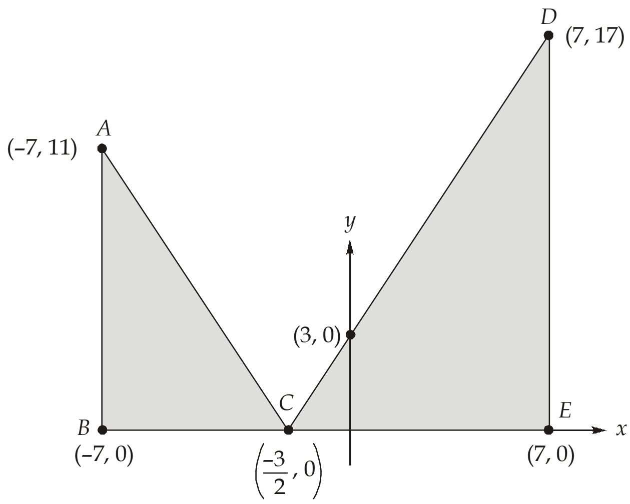
$$2(\lambda-3)[(\lambda-1)(3\lambda-1)-(5\lambda+1] = 0$$

$$6\lambda(\lambda-3)^2 = 0$$

$$\lambda = 0 \text{ or } 3$$

The largest value $\lambda = 3$

12. (b)



The value of integral is equal to area of shaded region.

$$\text{Area} = \frac{1}{2}\times AB\times BC+\frac{1}{2}\times DE\times EC = \frac{1}{2}\times 11\times\frac{11}{2}+\frac{1}{2}\times 17\times\frac{17}{2}$$

$$= \frac{410}{4} = 102.5$$

13. (a)

Parabola given : $x^2 = 4y$ ...(i)

Straight line is $x - 2y + 4 = 0$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$

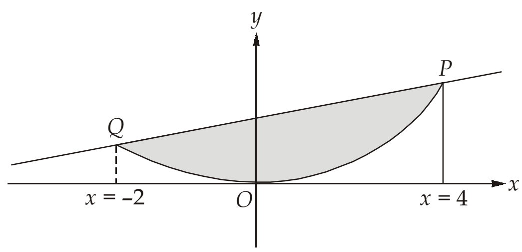
$\Rightarrow$ $x^2 = 2(x+4)$

$\Rightarrow$ $x^2 - 2x - 8 = 0$

$\Rightarrow$ $x^2 - 4x + 2x - 8 = 0$

$\Rightarrow$ $x(x-4) + 2(x-4) = 0$

$\Rightarrow$ $x = 4, -2$



Required area $= POQ$

$$= \int_{-2}^{4} y\,dx \text{ from straight line} - \int_{-2}^{4} y\,dx \text{ from parabola}$$

$$= \int_{-2}^{4}\left(\frac{x+4}{2}\right)dx - \int_{-2}^{4}\frac{x^2}{4}dx = \frac{1}{2}\left|\frac{x^2}{2}+4x\right|_{-2}^{4} - \frac{1}{4}\left|\frac{x^3}{3}\right|_{-2}^{4}$$

$$= \frac{1}{2}\{8+16-(-6)\} - \frac{1}{12}(64+8) = \frac{1}{2}\times 30 - \frac{1}{12}\times 72 = 15-6 = 9 \text{ unit}^2$$

14. (a)

For $f(x)$ to be probability density function $= \int_{-\infty}^{\infty} f(x)dx = 1$

$$\frac{1}{A}\int_{3}^{6}(3x+5)\,dx = 1 \Rightarrow \frac{1}{A}\left|\frac{3x^2}{2}+5x\right|_{3}^{6} = 1$$

$$A = \left(\frac{3}{2}\cdot 6^2 - \frac{3}{2}\cdot 3^2\right) + 5(6-3) = \frac{3}{2}\cdot 27 + 15 = 55.5$$

15. (a)

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 2 & 6 \\ 2 & 10-\lambda & 2 \\ 6 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 14\lambda^2 + 288 = 0$$

From here

$$p+q+r = 14$$
$$pq+qr+rp = 0$$
$$pqr = -288$$
$$pq+qr+rp-pqr = 0-(-288) = 288$$

16. (b)

Let X be the number of rejections

$$n = 8$$
$$p = 0.16$$
$$q = 0.84$$

Probability of at least one rejection

$$= 1 - P(X \le 0)$$
$$= 1 - P(X_0)$$

$$P(X_0) = {}^nC_r p^r q^{n-r} = {}^8C_0(0.16)^0(0.84)^8 = 0.2479$$

Probability of at least one rejection = 1 – 0.2479 = 0.7521

17. (d)

$$f_x(X) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$E(X^4) = \int_{-\infty}^{\infty} x^4 \cdot f_x(X)\,dx$$

$$= \int_{0}^{1} x^4 = \left.\frac{x^5}{5}\right|_{0}^{1} = \frac{1}{5} = 0.2$$

18. (c)

$$A^{-1} = \frac{(\text{adj } A)}{|A|}$$

$$|A| = -8 \times 5 = -40$$

$$|A| \cdot \left(A^{-1}\right) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-40}{-8}, \frac{-40}{5}$$

$$= 5, -8$$

19. (c)

$$y^2 = 9x$$

$$2y\frac{dy}{dx} = 9$$

$$\frac{dy}{dx} = \frac{9}{2y} = \text{slope of tangent } (m_1)$$

$$\text{Slope of normal } (m_2) = \frac{-2y}{9} \qquad [as\ m_1 m_2 = -1]$$

Slope of the given line is $\frac{-2}{3}$

$$\frac{-2y}{9} = -\frac{2}{3}$$

$$y = 3$$

For $y = 3$

$$3^2 = 9x$$

$$x = 1$$

For (1, 3) to lie on the given line

$$\lambda = 2x + 3y = 2 + 9 = 11$$

20. (a)

$$D^2 - 4D + 4 = 0$$

$$(D - 2)(D - 2) = 0$$

$$D = 2, 2$$

$$y = \left(C_1 + C_2 x\right)e^{2x}$$

$$y(0) = C_1 = 0$$

$$y(1) = e^2 = C_2 \cdot e^2 \Rightarrow C_2 = 1$$

$$y = xe^{2x}$$

$$y(2) = 2e^4 = 109.196$$

21. (b)

Probability of drawing a card = $\frac{1}{80}$

$$E(x_i) = 1 \times \frac{1}{80} + 2 \times \frac{1}{80} + ... + 80 \times \frac{1}{80}$$

$$= \frac{1}{80} \times \frac{(80)(80+1)}{2} = \frac{81}{2}$$

Expected value of the sum of numbers on the ticket drawn:

$$E(x_1 + x_2 + x_3 + ... + x_{30}) = E(x_1) + E(x_2) + ... + E(x_{30})$$

$$30\,E(x_i) = 30 \times \frac{81}{2} = 1215$$

22. **(b)**

Since the probability of occurrence is very small, this follows Poisson distribution.

$$\begin{aligned} \text{mean} &= m = np \\ &= 1500 \times 0.002 \\ &= 3 \end{aligned}$$

Probability that more than 2 will get a hanging problem

$$\begin{aligned} &= 1 - P(0) - P(1) - P(2) \\ &= 1 - \left[e^{-m} + \frac{e^{-m} \cdot m^1}{1!} + \frac{e^{-m} \cdot m^2}{2!}\right] \\ &= 1 - \left[e^{-3} + \frac{e^{-3} \cdot 3}{1} + \frac{e^{-3} \cdot 3^2}{2}\right] \\ &= 1 - \left[\frac{1}{e^3} + \frac{3}{e^3} + \frac{9/2}{e^3}\right] = 1 - \frac{17}{2e^3} \end{aligned}$$

23. **(b)**

Rearranging the equation,

$$\frac{dy}{dx} - \frac{y}{(2x+1)} = e^{4x}(2x+1)$$

The equation is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\begin{aligned} IF &= e^{\int P(x)dx} = e^{\int \frac{-1}{2x+1}dx} \\ &= e^{-\frac{\ln(2x+1)}{2}} = \frac{1}{\sqrt{2x+1}} \end{aligned}$$

24. **(c)**

$$AX = B$$

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} \frac{9}{2} & -6 \\ \frac{3}{2} & \frac{-3}{2} \end{bmatrix} = \begin{bmatrix} \frac{15}{2} & 3 \\ -3 & \frac{3}{2} \end{bmatrix}$$

$$\frac{9}{2}p + \frac{3}{2}q = \frac{15}{2} \qquad -6p - \frac{3}{2}q = 3$$

$$p = -7 \qquad q = 26$$

$$\frac{9}{2}r + \frac{3}{2}s = -3 \qquad -6r - \frac{3}{2}s = \frac{3}{2}$$

$$r = 1 \qquad s = -5$$

$$\Rightarrow \qquad A = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -7 & 26 \\ 1 & -5 \end{vmatrix} = 35 - 26 = 9$$

25. (d)

We parameterize the curve using $t = y$

$$x = 2 - 3t^2 \qquad -1 \le t \le 1$$

$$y = t$$

Then $\quad dx = -6t\, dt$

$$dy = dt$$

$$\int_c 2y^3 dx + 3x^2 dy = \int_{-1}^{1} \left[2t^3(-6t) + 3\left(2 - 3t^2\right)^2\right] dt$$

$$= \int_{-1}^{1} \left(15t^4 - 36t^2 + 12\right) dt$$

$$= \left[\frac{15t^5}{5} - \frac{36t^3}{3} + 12t\right]_{-1}^{1} = \left[3t^5 - 12t^3 + 12t\right]_{-1}^{1}$$

$$= 3 + 3 = 6$$

26. (c)

Output produced by A = 40%

$\therefore \quad P(A) = 0.4$

Output produced by B = 60%

$\therefore \quad P(B) = 0.6$

Let, $\quad P\left(\frac{D}{A}\right)$ = probability that item produced by A is defective

$$\therefore \quad P\left(\frac{D}{A}\right) = \frac{9}{1000} = 0.009$$

similarly, $\quad P\left(\frac{D}{B}\right) = \frac{1}{250} = 0.004$

$P\left(\frac{A}{D}\right)$ = Probability that product is produced by A given that it is defective.

$$P\left(\frac{A}{D}\right) = \frac{P(A) \times P\left(\frac{D}{A}\right)}{P(A) \times P\left(\frac{D}{A}\right) + P(B) \times P\left(\frac{D}{B}\right)}$$

$$= \frac{0.4 \times 0.009}{0.4 \times 0.009 + 0.6 \times 0.004}$$

$$P\left(\frac{A}{D}\right) = \frac{0.0036}{0.0036 + 0.0024} = \frac{0.0036}{0.006} = 0.6$$

$$\therefore \quad P\left(\frac{A}{D}\right) = 0.6$$

27. (c)

$$\text{Volume of solid} = \int_a^b \pi y^2 dx$$

Given $$y = \frac{1}{2\sqrt{x}}$$

$$\text{Volume of the solid} = \int_3^4 \frac{\pi}{4x} \cdot dx = \frac{\pi}{4}(\ln x)_3^4 = \frac{\pi}{4}\ln\left(\frac{4}{3}\right)$$

28. (d)

$$P(W \cup L) = P(W) + P(L) - P(W \cap L)$$

$$P(W \cup L) = 0.45 + 0.25 = 0.70$$

$$P(W' \cup L') = 1 - P(W \cup L)$$

$$= 1 - 0.70 = 0.3$$

29. (d)

$$\frac{d^2y}{dx^2} + y = \cos x$$

$$(D^2 + 1)y = \cos x$$

$$PI = \frac{\cos x}{D^2 + 1}$$

Putting $$D^2 = -1$$

$$PI = \frac{\cos x}{-1+1} \quad [\text{Makes denominator zero}]$$

$\therefore$ Differentiating numerator and denominator

$$PI = x \cdot \frac{\cos x}{2D}$$

$$= \frac{1}{2} x \int \cos x \, dx = \frac{1}{2} x \sin x$$

30. (c)

$$S = 160t - 20t^2$$

For maximum height

$$\frac{dS}{dt} = 160 - 40t = 0$$

and $$t = 4 \text{ sec}$$

$$\frac{d^2S}{dt^2} = -40 < 0 \quad \Rightarrow \text{Maxima}$$

$$S_{max} = 160 \times 4 - 20 \times 4^2$$

$$\text{Maximum height} = 320 \text{ m}$$

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