CLASS TEST

Sl. : 01 SP_ME_ABCD_270323

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ENGINEERING MECHANICS

MECHANICAL ENGINEERING

Date of Test : 27/03/2023

ANSWER KEY

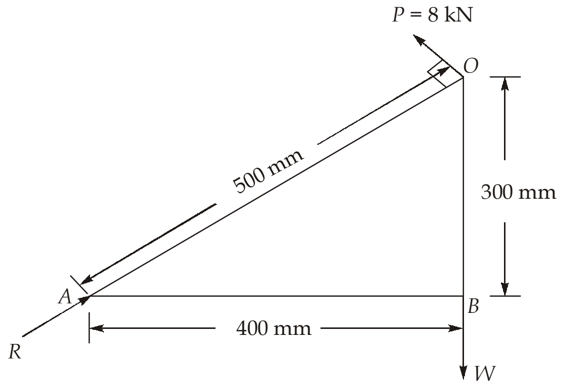
1.	(d)	7.	(d)	13.	(d)	19.	(d)	25.	(c)
2.	(b)	8.	(b)	14.	(a)	20.	(c)	26.	(a)
3.	(a)	9.	(b)	15.	(c)	21.	(d)	27.	(a)
4.	(a)	10.	(b)	16.	(a)	22.	(b)	28.	(c)
5.	(b)	11.	(c)	17.	(d)	23.	(d)	29.	(b)
6.	(d)	12.	(c)	18.	(d)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. (d)

For minimum pull, force P must be perpendicular to OA.

$$AB = \sqrt{500^2 - 300^2} = 400 \text{ mm}$$



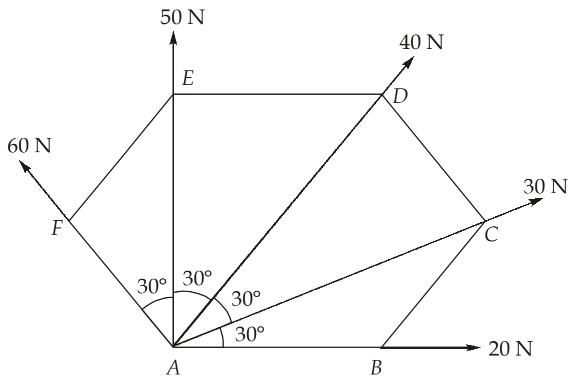
Taking moments about point A,

$$W \times 400 = P \times 500$$

$$W = \frac{8 \times 500}{400} = 10 \text{ kN}$$

2. (b)

The system of forces is shown in figure.



Resolving forces in horizontal direction,

$$\Sigma F_H = 20\cos 0° + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120°$$

$$= (20 \times 1) + \left(30 \times \frac{\sqrt{3}}{2}\right) + \left(40 \times \frac{1}{2}\right) + (50 \times 0) + \left(60 \times \frac{-1}{2}\right)$$

$$= 20 + 15\sqrt{3} + 20 - 30$$

$$\Sigma F_H = 35.98 \text{ N}$$

$$\Sigma F_V = 20\sin 0° + 30\sin 30° + 40\sin 60° + 50\sin 90° + 60\sin 120°$$

$$= (20 \times 0) + \left(30 \times \frac{1}{2}\right) + \left(40 \times \frac{\sqrt{3}}{2}\right) + (50 \times 1) + \left(60 \times \frac{\sqrt{3}}{2}\right)$$

$$= 0 + 15 + 20\sqrt{3} + 50 + 30\sqrt{3}$$

$$\Sigma F_V = 151.60 \text{ N}$$

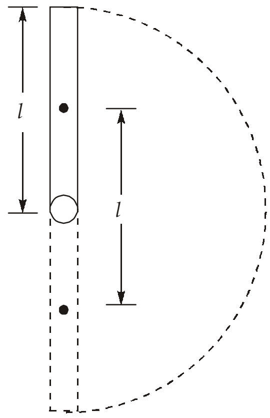
$$R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2} = \sqrt{(35.98)^2 + (151.60)^2}$$

$$\text{Resultant force, } R = 155.81 \text{ N}$$

3. (a)

As the rod reaches it lowest position, the center of mass is lowered by a distance l. Its gravitational potential energy is decreased by mgl.

Rotation occurs about the horizontal axis through the clamped end.



$$\text{Moment of inertia, } I = \frac{ml^2}{3}$$

Now, by energy conservation;

$$\frac{1}{2}I\omega^2 = (mgl)$$

$$\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = (mgl)$$

$$\omega^2 = \frac{6g}{l}$$

$$\omega = \sqrt{\frac{6g}{l}}$$

Linear speed of the free end at given instant, $v = l\omega$

$$V = l \times \sqrt{\frac{6g}{l}}$$

$$V = \sqrt{6gl}$$

4. (a)

The acceleration of the centre of mass,

$$a_{cm} = \left(\frac{F}{m+m}\right) = \frac{F}{2m}$$

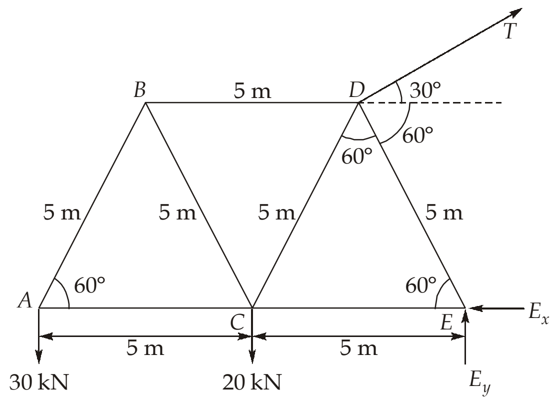
The change in position of the centre of mass at time t,

$$x = \frac{1}{2}(a_{cm}) \times t^2 = \frac{1}{2} \times \left(\frac{F}{2m}\right) \times t^2 = \frac{Ft^2}{4m} \qquad \text{(initial velocity is zero)}$$

5. (b)

Taking moment about point E.

Let tension in member DF is T.



$$\Sigma M_E = 0$$

$$5T - 5 \times 20 - 10 \times 30 = 0$$

$$T = 80 \text{ kN}$$

$\because$ ΔABC, ΔBCD and ΔCDE are equilateral triangles.

$$\Sigma F_x = 0$$

$$80 \cos 30° = E_x$$

$$E_x = 69.282 \text{ kN}$$

$$\Sigma F_y = 0$$

$$80 \sin 30° + E_y - 20 - 30 = 0$$

$$E_y = 50 - 40 = 10 \text{ kN}$$

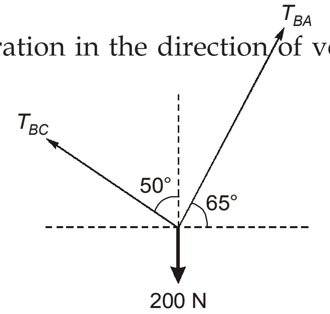
$$\text{Total reaction at } E = \sqrt{E_x^2 + E_y^2} = \sqrt{(69.282)^2 + 10^2} = 69.99 \text{ kN} = 70 \text{ kN}$$

6. **(d)**

$$\text{Rate of change of speed} = \frac{\vec{v}\cdot\vec{a}}{|\vec{v}|} = \frac{(\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{i}+3\hat{j})}{\sqrt{1+4+9}} \simeq 2.14 \text{ m/s}^2$$

Note: Rate of change of speed means the component of acceleration in the direction of velocity.

7. **(d)**

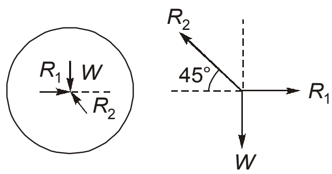


Applying Lami's theorem,

$$\frac{T_{BC}}{\sin(90°+65°)} = \frac{200}{\sin(50°+25°)}$$

$$\Rightarrow \quad T_{BC} = 87.5 \text{ N}$$

8. **(b)**



$$R_2 \cos 45° = R_1$$

$$R_2 \sin 45° = W$$

$$\Rightarrow \quad R_2 = W\sqrt{2}$$

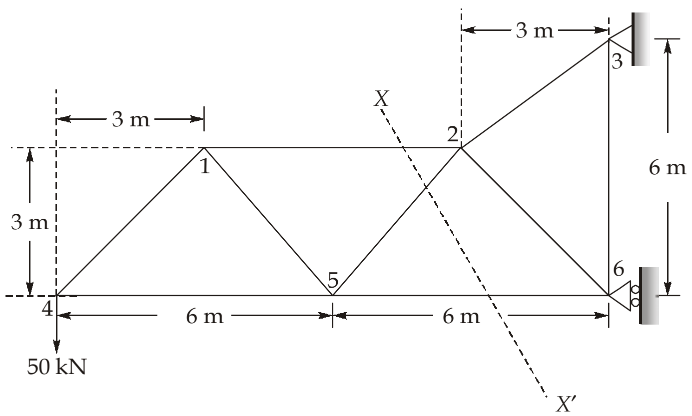
$$\therefore \quad R_1 = W\sqrt{2}\times\frac{1}{\sqrt{2}} = W$$

$$W = 50 \text{ N}$$

$$\therefore \quad R_1 = 50 \text{ N}$$

9. **(b)**

Cutting section about, 1-2, 2-5 and 5-6:



Taking moment of left part about 5.

$$F_{1\text{-}2} = \frac{50\times 6}{3} = 100 \text{ kN(T)}$$

Cutting section through 1-2, 1-5 and 4-5 and balancing vertical forces for left part only,

$\Sigma F_V = 0$

$$\Rightarrow \quad F_{1\text{-}5}\times\cos 45° + 50 = 0$$

$$F_{1\text{-}5}\cos 45° = -50$$

$$\frac{F_{1-5}}{\sqrt{2}} = -50 \Rightarrow F_{1\text{-}5} = 50\sqrt{2} \text{ kN(C)}$$

10. (b)

Given: $\vec{P} = \left(4\hat{i} + 5\hat{j} + 3\hat{k}\right)$ kg-m/s, $\vec{r} = \left(1\hat{i} + 2\hat{j} + 5\hat{k}\right)$ m

$$\text{Angular momentum, } \vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 4 & 5 & 3 \end{vmatrix} = -19\hat{i} - \hat{j}(-17) + \hat{k}(-3)$$

$$= -19\hat{i} + 17\hat{j} - 3\hat{k}$$

$$\Rightarrow \quad (\vec{L}) = \sqrt{19^2 + 17^2 + 3^2} = 25.67 \text{ kg m}^2/\text{s}$$

11. (c)

Given: Span = 10 m

Let, R_A and R_B are reaction at supports A and B respectively. The perpendicular distance between the support A and the line of action of the loads at D is

$$AD = \frac{10}{2 \times \cos 30°} = \frac{5}{\cos 30°} = 5.77 \text{ m}$$

The perpendicular distance between the support A and the line of action of the load at C.

$$AC = \frac{AD}{2} = \frac{5.77}{2} = 2.885 \text{ m}$$

Taking moment about A,

$$R_B \times 10 = (4 \times 2.885) + (2 \times 5.77) \Rightarrow R_B = 2.308 \text{ kN}$$

$$\text{Total wind load} = 2 + 4 + 2 = 8 \text{ kN}$$

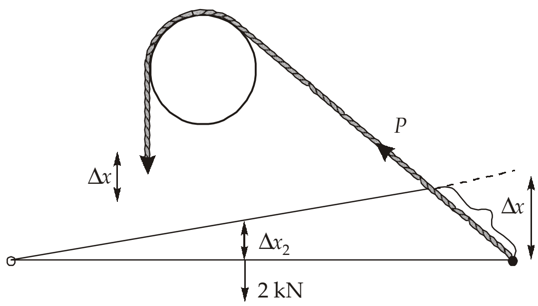
Horizontal component of total load, $(F_H)_{net} = 8\cos 60° = 4$ kN

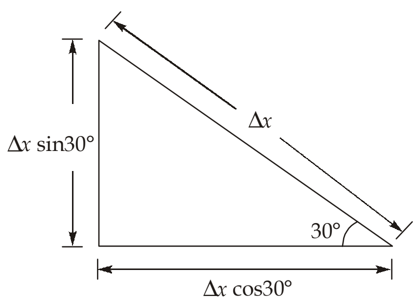
Vertical component of total load = 8 sin60° = 6.928 kN

Balance vertical reaction at A, $(R_A)_y = 4.620$ kN

$$\text{Total reaction at } A,\ R_A = \sqrt{(R_A)_x^2 + (R_A)_y^2} = \sqrt{4^2 + 4.62^2} = 6.111 \text{ kN}$$

12. (c)





$$\delta x_2 = \frac{\Delta x \sin 30°}{2}$$

$$F_2 = 2 \text{ kN}$$

$$\delta x_1 = \Delta x \sin 30°$$

$$F_1 = P \sin 30°$$

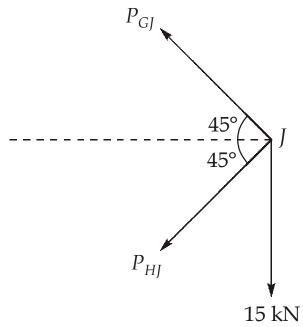
$$F_1 \times \Delta x_1 = F_2 \times \Delta x_2$$

$$P\sin 30° \times \Delta x \sin 30° = 2 \times \frac{\Delta x \sin 30°}{2}$$

$$\Rightarrow \quad P = 2 \text{ kN}$$

13. (d)

At joint J,



$$\Sigma F_H = 0$$

$$P_{GJ}\cos 45° + P_{HJ}\sin 45° = 0$$

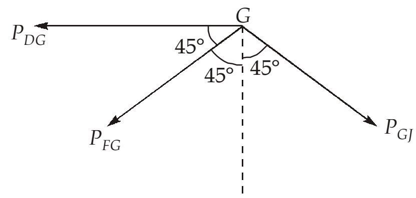
$$P_{GJ} = -P_{HJ} \quad \text{...(1)}$$

$$\Sigma F_V = 0$$

$$P_{GJ}\sin 45° - P_{HJ}\sin 45° - 15 = 0 \quad \{\because P_{GJ} = -P_{HJ}\}$$

$$2P_{GJ}\sin 45° = 15$$

$$P_{GJ} = \frac{15\times\sqrt{2}}{2} = 10.606 \text{ kN}$$



At joint G, $\Sigma F_V = 0$

$$P_{FG}\cos 45° + P_{GJ}\cos 45° = 0$$

$$P_{FG} = -P_{GJ} = -10.606 \text{ kN}$$

$$\Sigma F_H = 0$$

$$P_{DG} + P_{FG}\sin 45° = P_{GJ}\sin 45°$$

$$P_{DG} = 10.606\sin 45° - (-10.606\sin 45°)$$

$$= 2 \times 10.606 \sin 45°$$

$$P_{DG} = 15 \text{ kN (Tension)}$$

14. (a)

Let the angular speed be ω.

As the ball moves in a horizontal circle of radius $L + L\sin\theta$. Its acceleration is $\omega^2(L + L\sin\theta)$ towards the centre.

Resolving the forces along the radius and applying Newton's second law,

$$T\sin\theta = m\omega^2 L(1 + \sin\theta) \quad \text{...(i)}$$

In vertical direction,

$$T\cos\theta = mg \quad \text{...(ii)}$$

Now, by equation (i) ÷ (ii)

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m\omega^2 L(1+\sin\theta)}{mg}$$

$$\tan 30° = \frac{\omega^2 \times 0.2(1+\sin 30°)}{10} \quad (\because \theta = 30°)$$

$$\omega^2 = \frac{10}{0.2 \times 1.5 \times \sqrt{3}}$$

$$\omega^2 = 19.245$$

$$\omega = 4.387 \text{ rad/s}$$

15. (c)

Given: h_o = 1 m = 100 cm; h_1 = 81 cm

Let, coefficient of restitution is e.

Velocity with which ball impinges, $u = \sqrt{2gh_o} = \sqrt{2g \times 100} = 10\sqrt{2g}$ cm/s

Velocity with which the ball rebounds, $v = \sqrt{2gh_1} = \sqrt{2g \times 81} = 9\sqrt{2g}$ cm/s

We know that,

$$v = eu$$

$$9\sqrt{2g} = e\left(10\sqrt{2g}\right)$$

$$e = 0.9$$

Velocity with which which the ball impinges second time, $u_2 = \sqrt{2gh_1} = \sqrt{2g \times 81} = 9\sqrt{2g}$ cm/s

Now, velocity with which ball rebounds, $v = eu$

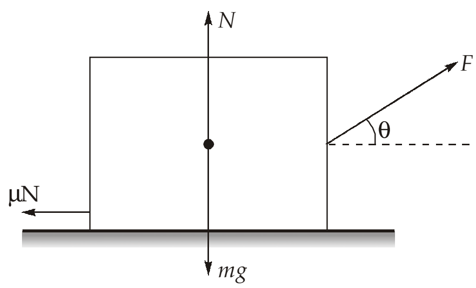
$$\sqrt{2gh_2} = 0.9 \times 9\sqrt{2g}$$

$$h_2 = (8.1)^2$$

Expected height of second bounce, h_2 = 65.61 cm

16. (a)

Taking horizontal and vertical component.



$$\Sigma F_V = 0$$

$$F\sin\theta + N = mg$$

$$N = mg - F\sin\theta \quad ...(i)$$

$$\Sigma F_H = 0$$

$$F\cos\theta = \mu N$$

By eq. (i), $$F\cos\theta = \mu(mg - F\sin\theta)$$

$$F(\cos\theta + \mu\sin\theta) = \mu mg$$

$$F = \left(\frac{\mu mg}{\cos\theta + \mu\sin\theta}\right)$$

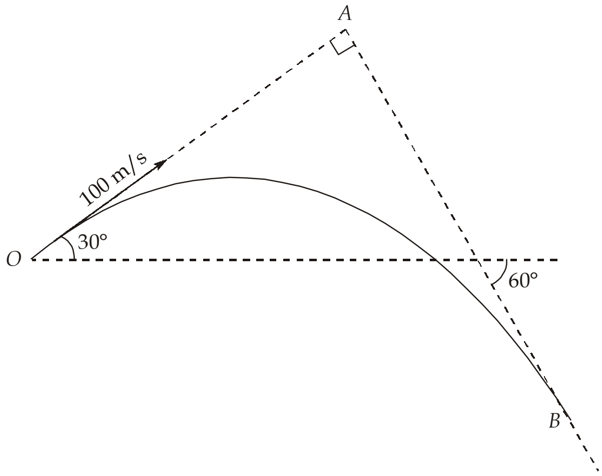
Work done by this force for a displacement d will be,

$$W = F \times d\cos\theta = \frac{\mu mgd\cos\theta}{(\cos\theta + \mu\sin\theta)}$$

$$W = \frac{\mu mgd}{(1 + \mu\tan\theta)}$$

17. (d)

Given: $u = 100$ m/s, $\alpha = 30°$



Let, t is the time from the instant of projection, when the particle will move perpendicular to its initial direction.

$\theta = -60°$

(When the particle will move perpendicular to its initial direction. Here minus sign is for downward direction or for clockwise direction)

In horizontal direction, $(V_B)\cos(-60°) = u\cos\alpha$

$$(V_B)\cos(-60°) = u\cos\alpha \quad ...(1)$$

In vertical direction,

$$(V_B)\sin(-60°) = u\sin\alpha - gt \quad ...(2)$$

By equation (2) ÷ (1)

$$\frac{\sin(-60°)}{\cos(-60°)} = \frac{u\sin 30° - gt}{u\cos 30°}$$

$$\tan(-60°) = \frac{100\times 0.5 - 9.81t}{\left(100\times\frac{\sqrt{3}}{2}\right)}$$

$$-\sqrt{3}\times 50\times\sqrt{3} = 50 - 9.81t$$

$$9.81t = 50 + 150$$

$$t = \frac{200}{9.81} = 20.387 \text{ seconds}$$

18. (d)

Given; $N_1 = 10$ rev/s; $\omega_1 = 2\pi N_1 = 20\pi$ rad/s, $N_2 = 18$ rev/s; $\omega_2 = 2\pi N_2 = 36\pi$ rad/s

t_1 = 4 second, t_2 = 8 second, N = 400 revolutions

α = Angular acceleration of the shaft

We know that, $\omega_2 = \omega_1 + \alpha t_1$

$$36\pi = 20\pi + \alpha\times 4$$

$$\alpha = 4\pi \text{ rad/s}^2$$

α remains same for (8 + 4) = 12 second. Total displacement in 12 seconds.

$$\theta_1 = \omega_1 t_2 + \frac{1}{2}\alpha t_2^2$$

$$\theta_1 = 20\pi\times 12 + \frac{1}{2}\times 4\pi\times 12^2 = 240\pi + 2\pi\times 144$$

$\theta_1 = 528\pi$ radian

Angular velocity after 12 seconds,

$$\omega_f = \omega_1 + \alpha t_1$$
$$\omega_f = 20\pi + 4\pi \times 12$$
$$\omega_f = 68\pi \text{ rad/s}$$

Now, total revolutions completed in first 12 seconds $= \frac{\theta_1}{2\pi} = \frac{528\pi}{2\pi} = 264$ revolutions

Number of revolutions to be completed = 400 – 264 = 136 revolutions

Now, $\theta = \omega_f t$ $\quad [\because \alpha = 0]$

$$[136 \times 2\pi] = 68\pi \times t_3$$
$$t_3 = 4 \text{ seconds}$$

Total time required = 12 + 4 = 16 seconds

19. **(d)**

We know that,

$$\text{Work done, } dW = \vec{F}\cdot\vec{dr} = \vec{F}\left(dx\hat{i} + dy\hat{j}\right) = k\int\left[\frac{xdx}{\left(x^2+y^2\right)^{3/2}} + \frac{ydy}{\left(x^2+y^2\right)^{3/2}}\right]$$

Given, It is moving along a circular path of radius 'a'. So, $x^2 + y^2 = a^2$

$$\int_0^W dW = k\left[\int_a^0 \frac{xdx}{a^3} + \int_0^a \frac{ydy}{a^3}\right]$$

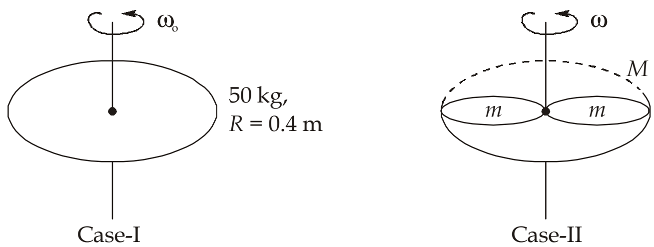
$$W = \frac{k}{a^3}\left[\left[\frac{x^2}{2}\right]_a^0 + \left[\frac{y^2}{2}\right]_0^a\right]$$

$$W = \frac{k}{a^3}\left[0 - \frac{a^2}{2} + \frac{a^2}{2} - 0\right]$$

$$W = 0$$

20. **(c)**

Given, $M = 50$ kg, $m = 6.25$ kg, $R = 0.4$ m, $r = 0.2$ m



Applying angular momentum conservation, about vertical axis of rotation,

$$\left(\frac{MR^2}{2}\right)\times\omega_0 = \left[\frac{MR^2}{2} + 2mr^2 + \left(2mr^2\right)\right]\omega'$$

$$\left(\frac{50\times 0.4^2}{2}\right)\times 10 = \left(\frac{50\times 0.4^2}{2} + 4\times 6.25\times(0.2)^2\right)\omega'$$

$$\frac{40}{5} = \omega'$$

$$\Rightarrow \quad \omega' = 8 \text{ rad/s}$$

21. (d)

Using work energy theorem:

(Work done by all forces) = (Change in kinetic energy)

$$W_{mg} + W_F = \Delta kE$$
$$(-mgh) + Fd = \Delta kE$$
$$-1 \times 10 \times 4 + 18 \times 5 = \Delta kE$$
$$\Delta kE = 50 \text{ J}$$

22. (b)

Let speed of the car at the topmost point of the loop is V and m is the mass of the car. By energy conservation between top most point of loop and at starting point,

$$0 + 0 = -mgh + \frac{1}{2}mv^2$$
$$mv^2 = 2mgh \quad ...(1)$$

At top most point of the circle normal reaction will be minimum.

For successful looping,

Normal reaction, $N \geq 0$

By Newton's IInd law at topmost point,

$$N + mg = \frac{mv^2}{R}$$
$$mg + N = \frac{2mgh}{R}$$

For minimum value of h, N should be minimum i.e. 0

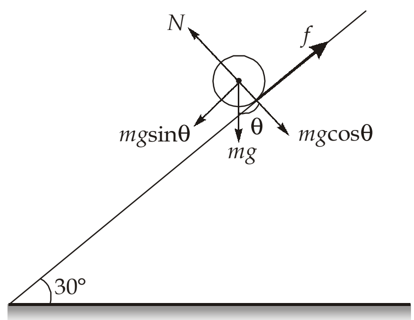
$$mg + 0 = \frac{2mgh_{min}}{R}$$
$$h_{min} = \frac{R}{2}$$

23. (d)

Given, m = 5 kg, θ = 30°

Let, linear acceleration of the sphere down the plane be a and radius of sphere is r.

Along the plane by Newton's IInd law,



$$mg\sin\theta - f = ma \quad ...(1)$$

If sphere rolls without slipping, angular acceleration about the center will be $\frac{a}{r}$.

Taking moment about center of mass,

$$f \times r = (I)\alpha$$
$$f \times r = \left(\frac{2}{5}mr^2\right) \times \left(\frac{a}{r}\right)$$

$$\text{Friction force, } f = \frac{2}{5}ma \qquad ...(2)$$

By equation (1) and (2),

$$mg\sin\theta = \left(\frac{7}{5}\right)ma$$

$$a = \frac{5}{7}g\sin\theta$$

$$\text{Frictional force, } f = \frac{2}{5}ma = \frac{2}{5}\times\frac{5}{7}mg\sin\theta = \frac{2}{7}mg\sin\theta$$

Maximum friction force will be $mg\cos\theta$. Where μ is coefficient of static friction.

$$\text{For pure rolling, } \mu mg\cos\theta \geq \frac{2}{7}mg\sin\theta$$

$$\mu \geq \frac{2}{7}\times\tan 30°$$

$$\Rightarrow \quad \mu \geq 0.165$$

24. (c)

Let, N = Magnitude of the contact force between the particle and the pan

T = Tension in the string

By impulse momentum equation, for particle of mass 'm'.

$$\int Ndt = mv - m(kv) \qquad ...(1)$$

By impulse-momentum equation for pan,

$$\int (N-T)dt = m(kv) \qquad ...(2)$$

By impulse-momentum equation for block,

$$\int Tdt = m(kv) \qquad ...(3)$$

Adding equation (2) and (3),

$$\int Ndt = 2m(kv) \qquad ...(4)$$

From eq. (1) and (4)

$$2m(kv) = mv - m(kv)$$

$$v = 3kv$$

$$k = \frac{1}{3}$$

25. (c)

Given:

$$V_1 = 350 \text{ kmph} = 350\times\frac{5}{18} = 97.222 \text{ m/s}$$

$$V_2 = 200 \text{ kmph} = 200\times\frac{5}{18} = 55.555 \text{ m/s}$$

We know that,

$$F = ma$$

$$ma = -27.5V^2$$

$$m\times\left(\frac{VdV}{ds}\right) = -(27.5V^2) \qquad \left[\because a = \frac{dV}{dt} = \left(\frac{dV}{ds}\right)\times\left(\frac{ds}{dt}\right)\right]$$

$$8000 \times \left(\frac{dV}{V}\right) = -(27.5)ds$$

$$8000\int_{97.222}^{55.555}\left(\frac{dV}{V}\right) = -27.5\int_{0}^{s} ds$$

$$8000[\ln V]_{97.222}^{55.555} = -27.5 \times s$$

$$s = -\left(\frac{8000}{27.5}\right)\times \ln\left(\frac{55.555}{97.222}\right)$$

$$s = 162.8 \text{ m}$$

26. (a)

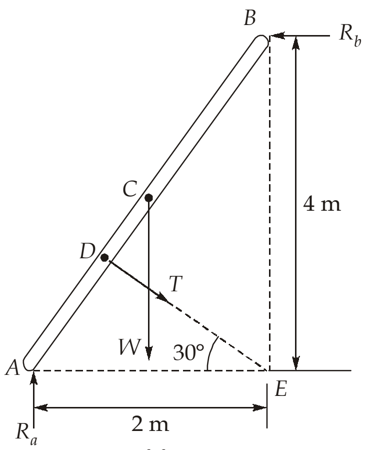
Given: W = 700 + 300 = 1000 N

Let, tension in string is T.

By equations of equilibrium, $\Sigma F_x = 0$

$$R_b = T\cos 30°$$

$$R_b = \left(\frac{\sqrt{3}}{2}\right)T \quad \text{... (i)}$$



FBD of forces

$\Sigma F_y = 0,$ $\quad W + T\sin 30° = R_a$

$$(0.5)T + 1000 = R_a \quad \text{... (ii)}$$

$\Sigma M_E = 0,$

$$R_b \times 4 + W \times 1 = R_a \times 2$$

$$2R_b + (0.5) \times 1000 = R_a \quad \text{... (iii)}$$

By equation (ii) and (iii)

$$2R_b + 500 = (0.5)T + 1000$$

By equation (i),

$$\left(2\times\frac{\sqrt{3}}{2}\right)T - 0.5T = 500$$

$$T = \frac{500}{\sqrt{3}-0.5} = 405.827 \text{ N}$$

27. (a)

There are three forces acting on the bar AB; pull Q at B, tension in string T and reaction at point A i.e. R_a.

For isosceles triangle ABC,

$$\beta = \gamma = \left(\frac{\pi-\alpha}{2}\right) = 90° - \left(\frac{\alpha}{2}\right)$$

If there is no friction on pulley, tension in string BC will be P.

Taking moment about point A,

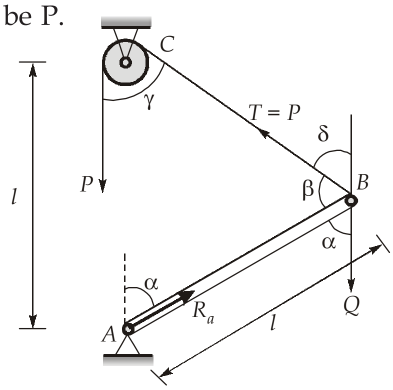
$$(P\cos\delta) \times (l\sin\alpha) + (P\sin\delta)(l\cos\alpha) = Ql\sin\alpha$$

$$Pl\sin(\alpha + \delta) = Ql\sin\alpha$$

$$P\sin(180° - \beta) = Q\sin\alpha$$

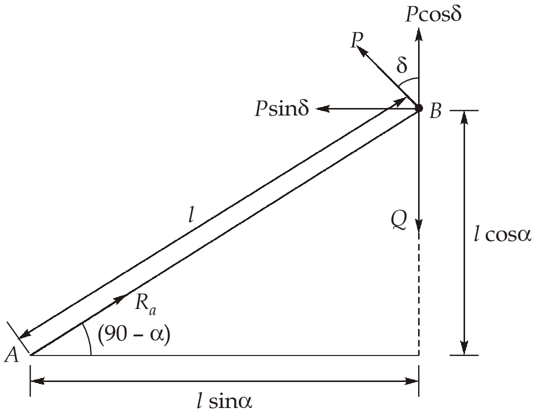
$$P\sin\left[180 - 90 + \frac{\alpha}{2}\right] = Q\sin\alpha$$

$$P\cos\frac{\alpha}{2} = 2Q\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$



$$\left(\cos\frac{\alpha}{2}\right)\left[P - 2Q\sin\frac{\alpha}{2}\right] = 0$$

or $$\sin\frac{\alpha}{2} = \frac{P}{2Q}$$



$$\alpha = 2\sin^{-1}\left(\frac{P}{2Q}\right) = 2\sin^{-1}\left(\frac{900}{2\times 2200}\right) = 23.6057°$$

$$\alpha = 23.6057\times\left(\frac{\pi}{180}\right) = 0.412 \text{ radian}$$

28. (c)

Given: M = 2000 kg, v_1 = 2 m/s, v_2 = 0, for drum m = 50 kg, k = 0.7 m, R = 0.75 m, h = 0.5 m

$$\Delta kE \text{ of mass} = \frac{1}{2}M\left(v_1^2 - v_2^2\right) = \frac{1}{2}\times 2000\times\left(2^2 - 0^2\right) = 4000 \text{ J}$$

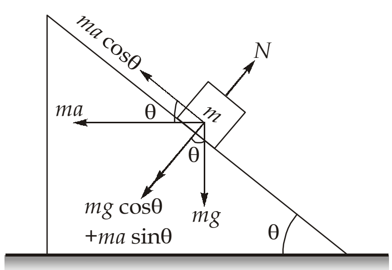
$$\Delta kE \text{ of drum} = \frac{1}{2}mk^2\left(\omega_1^2 - \omega_2^2\right) = \frac{1}{2}\times 50\times 0.7^2\times\left[\left(\frac{2}{0.75}\right)^2 - 0^2\right] = 87.11 \text{ J}$$

$$\Delta PE \text{ of the mass} = mgh = 2000 \times 9.81 \times 0.5 = 9810 \text{ J}$$

the total energy absorbed by the break is given by

$$E = \Delta kE \text{ of mass} + \Delta kE \text{ of drum} + \Delta PE \text{ of mass}$$
$$= 4000 + 87.11 + 9810$$
$$E = 13897.11 \text{ J} \simeq 13897 \text{ J}$$

29. (b)



For maximum acceleration, the applied force should be such that it balances the effect of frictional force and gravity. Hence applied force,

$$F_a = ma\cos\theta - mg\sin\theta$$

For no slipping $F_a \le f_e$ (Frictional force)

$$ma\cos\theta - mg\sin\theta \le \frac{1}{2\sqrt{3}}(mg\cos\theta + ma\sin\theta)$$

On solving, $$a = \frac{3\sqrt{3}g}{5}$$

30. (c)

$$\text{Lagrangian, } L = T - V$$
$$= v^2\dot{u}^2 + 2\dot{v}^2 - u^2 + v^2$$
$$= v^2(1 + \dot{u}^2) + (2\dot{v}^2 - u^2)$$

The equation of motion, using langrangian (L) for $q = u$,

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}}\right) - \frac{\partial L}{\partial u} = 0$$

$$\Rightarrow \quad \frac{d}{dt}(2v^2\dot{u}) - (-2u) = 0$$

$$\Rightarrow \quad 2\left[v^2\ddot{u} + 2v\dot{v}\dot{u}\right] + 2u = 0$$

$$\Rightarrow \quad 2v^2\ddot{u} + 4v\dot{v}\dot{u} + 2u = 0$$

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