CLASS TEST

S.No. : 01_ IG_CE_A+C_230323

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STRENGTH OF MATERIALS

CIVIL ENGINEERING

Date of Test : 23/03/2023

ANSWER KEY

1.	(a)	7.	(b)	13.	(a)	19.	(c)	25.	(b)
2.	(b)	8.	(c)	14.	(c)	20.	(d)	26.	(c)
3.	(c)	9.	(b)	15.	(b)	21.	(c)	27.	(c)
4.	(a)	10.	(d)	16.	(c)	22.	(c)	28.	(b)
5.	(a)	11.	(a)	17.	(c)	23.	(c)	29.	(c)
6.	(a)	12.	(a)	18.	(b)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

2. (b)

We know deflection of spring,

$$\delta = \frac{64 W R^3 n}{G d^4}$$

where, $W = 100$ N, $R = 25$ mm, $n = 12$, $G = 80$ GPa, $d = 5$ mm

So,
$$\delta = \frac{64 \times 100 \times (25)^3 \times 12}{80 \times 10^3 \times 5^4} = 24 \text{ mm}$$

3. (c)

$$d = 2 \text{ mm}$$
$$\sigma_{b(\max)} = 80 \text{ N/mm}^2$$
$$E = 100 \times 10^3 \text{ N/mm}^2$$

Distance between the neutral axis of wire and its extreme fibre

$$y = \frac{2}{2} = 1 \text{ mm}$$

So, minimum radius of the drum

$$R = \frac{y}{\sigma_{b(\max)}} E \qquad \left(\because \frac{f}{y} = \frac{E}{R}\right)$$

$$= \frac{1}{80} \times 100 \times 10^3$$

$$= 1.25 \times 10^3 \text{ mm} = 1.25 \text{ m}$$

4. (a)

Internal hinge in given beam will become hinged support in conjugate beam.

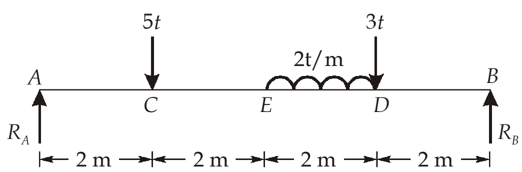
5. (a)

$$\tau_{\max} = \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

$$= \left[\frac{16}{\pi (100)^3}\sqrt{(8)^2 + (6)^2}\right] \times 10^6$$

$$= \frac{16}{\pi} \times \frac{10 \times 10^6}{10^6} = 50.93 \text{ MPa}$$

6. (a)



Taking moments about B,

$$R_A = \left(\frac{6}{8}\right)\times 5+\frac{3}{8}\times 4+3\times\frac{2}{8}$$

$$= 3.75 + 1.5 + 0.75 = 6t$$

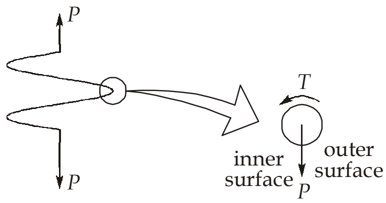
Taking moment about A,

$$R_B = (5+3+2\times 2)-R_A$$

$$= 12t - 6t = 6t$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{6t}{6t}=1$$

7. (b)



Due to combined effect of torque and shear force, inner surface will have more shear stress compared to outer surface.

On dividing a spring into m parts, the no. of turns on each spring will be m times less. Since stiffness is inversely proportional to no. of turns, the stiffness will become mk.

8. (c)

$$Z = \frac{I}{y}=\frac{225\times 10^6}{\frac{300}{2}}=1.5\times 10^6 \text{ mm}^3$$

$$M = \sigma_{max} \times Z = 120 \times 1.5 \times 10^6$$

$$= 180 \times 10^6 \text{ N-mm}$$

$$M_{max} = M = \frac{wl^2}{8}$$

So $$\frac{wl^2}{8} = 180 \times 10^6 = \frac{w(4\times 10^3)^2}{8}$$

$$\Rightarrow \quad w = \frac{180}{2}=90 \text{ N/mm} = 90 \text{ kN/m}$$

9. (b)

Flitched beam has a composite section made of two or more materials joined together in such a manner that they behave as a unit piece and each material bends to the same radius of curvature. The total moment of resistance of a flitched beam is equal to the sum of the moments of resistance of individual sections.

10. (d)

By stress invariant law, $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

$\Rightarrow$ $32 + (-10) = 40 + \sigma_2$

$\Rightarrow$ $\sigma_2 = -18$ MPa

11. (a)

Direct longitudinal stress,

$$\sigma_x = \frac{90\times10^3}{30\times30} = 100 \text{ MPa} \qquad \text{(Compressive)}$$

$$\in_x = \frac{1}{E}\left[-\sigma_x + \nu(\sigma_y + \sigma_z)\right] \qquad \text{...(i)}$$

$$\in_y = \in_z = \frac{1}{E}\left[-\sigma_y + \nu(\sigma_x + \sigma_z)\right] = 0 \qquad \text{...(ii)}$$

As we know $\sigma_y = \sigma_z$

Also on solving equation (ii)

$$\sigma_y = \frac{\nu}{1-\nu}\sigma_x = \frac{0.25}{1-0.25}\sigma_x = \frac{\sigma_x}{3}$$

So,

$$\in_x = \frac{1}{E}\left[-\sigma_x + \nu(\sigma_y + \sigma_z)\right] = \frac{1}{E}\left[-\sigma_x + \nu\times2\sigma_y\right]$$

$$= \frac{1}{E}\left[-\sigma_x + 0.25\times2\times\frac{\sigma_x}{3}\right] = \frac{1}{E}\left[-\sigma_x + \frac{0.5\sigma_x}{3}\right]$$

$$= \frac{1}{100\times10^3}\left[-100 + \frac{0.5\times100}{3}\right] = \frac{1}{100\times10^3}\left[-\frac{250}{3}\right]$$

$$\delta l = l\in_x = \frac{1}{100\times10^3}\left[-\frac{250}{3}\right]\times100$$

$$= 0.083 \text{ mm} \qquad \text{(Reduction in length)}$$

12. (a)

Strain energy stored in hollow shaft, $U = \dfrac{\tau_{max}^2}{4G}\left[\dfrac{D^2+d^2}{D^2}\right]V$

$$= \frac{80^2}{4\times100\times10^3}\left[\frac{80^2+60^2}{80^2}\right]\times10^6 = \frac{10000\times10^6}{4\times10^5} = 2.5\times10^4 \text{ N-mm}$$

$$= 25 \text{ N-m}$$

13. (a)

Bending moment at A and B is zero. It increases in the form of cubic curve.

Maximum value of bending moment in beam AB occurs where shear force changes sign.

Now we know

$$R_A = \frac{wl}{3}$$

$$R_B = \frac{wl}{6}$$

Let at point x from B, shear force will be zero

$$V = R_B - \frac{1}{2}\times\frac{w}{l}x\cdot x$$

$$\Rightarrow \quad \frac{wl}{6} - \frac{wx^2}{2l} = 0$$

$$\Rightarrow \quad x = \frac{l}{\sqrt{3}}$$

So, bending moment at $x = \frac{l}{\sqrt{3}}$ from B is

$$M_{max} = R_B \cdot x - \frac{1}{2}\times\frac{w}{l}x\cdot x\times\frac{x}{3} = \frac{wl}{6}\times\frac{l}{\sqrt{3}} - \frac{1}{2}\times\frac{w}{l}\times\frac{1}{3}\times\frac{l^3}{(\sqrt{3})^3}$$

$$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{6\times3\sqrt{3}} = \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{18\sqrt{3}} = \frac{2wl^2}{18\sqrt{3}}$$

$$= \frac{wl^2}{9\sqrt{3}}$$

14. (c)

We know that area of triangular beam section

$$A = \frac{\sqrt{3}}{4}a^2 \text{ for equilateral triangle}$$

$$= \frac{\sqrt{3}}{4}(100)^2 = 2500\sqrt{3}\ \text{mm}^2$$

Average shear stress across the section

$$\tau_{avg} = \frac{F}{A} = \frac{13\times10^3}{2500\sqrt{3}} = 3\ \text{MPa}$$

So maximum shear stress for triangular section

$$\tau_{max} = 1.5\ \tau_{avg}$$

$$= 1.5 \times 3 = 4.5\ \text{MPa}$$

15. (b)

Let us split up the trapezoidal load into a uniformly distributed load (w_1) of 50 N/mm and a triangular load (w_2) of 100 N/mm at A to zero at B.

Now slope at free end

$$\theta_B = \frac{w_1 l^3}{6EI} + \frac{w_2 l^3}{24EI} = \left[\frac{50\times(2\times10^3)^3}{6\times10^{13}} + \frac{100\times(2\times10^3)^3}{24\times10^{13}}\right]\text{rad}$$

$$= 0.0067 + 0.0033 = 0.01\ \text{rad}$$

16. (c)

Free expansion of rod $= \delta l = \alpha l \Delta t = 12\times10^{-6}\times20\times10^3\ (65-20) = 10.8$ mm

When the rod is permitted to expand by 5.8 mm in this case, expansion prevented = 10.8 – 5.8

= 5 mm

$$\therefore \quad \text{Strain prevented} = \frac{\text{Expansion Prevented}}{\text{Original length}}$$

$$= \frac{5}{20\times10^3} = \frac{1}{4000}$$

$$\therefore \quad \text{Thermal stress} = \text{Strain prevented} \times E$$

$$= \frac{1}{4000}\times200\times10^3 = 50\ \text{MPa}$$

17. (c)

From bending equation, $\frac{f}{y} = \frac{f_{max}}{y_{max}}$

$$\therefore \quad f = \frac{f_{max}}{y_{max}} \times y$$

$$\therefore \quad \text{Force on shaded area} = \frac{f_{max}}{y_{max}} \times \Sigma Ay$$

$$= \frac{f_{max}}{y_{max}} (A\bar{y})$$

[where A is shaded area, $\bar{y}$ = distance of centroid of shaded area from N.A.]

$$= \frac{90}{12} \times \left[\frac{15}{2} \times 12\right] \times \frac{2}{3} \times 12 = 5400 \text{ kg}$$

18. (b)

$$\sigma_x = \frac{2.5 \times 10^3}{25} = 100 \text{ MPa (T)}$$

$$\sigma_y = \frac{1250}{25} = 50 \text{ MPa (T)}$$

$$\sigma_z = \frac{625}{25} = 25 \text{ MPa (T)}$$

$$\tau_{xy} = \frac{1000}{25} = 40 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{100 + 50}{2} \pm \frac{1}{2}\sqrt{(100 - 50)^2 + 4(40)^2}$$

$$\sigma_1 = 122.17 \text{ MPa}$$

$$\sigma_2 = 27.83 \text{ MPa}$$

$$\sigma_3 = 25 \text{ MPa}$$

Now according to maximum shear stress theory

$$(\tau_{abs})_{max} \leq \frac{\sigma_y}{2(\text{FOS})}$$

$$\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right\}_{max} = \frac{\sigma_y}{2(\text{FOS})}$$

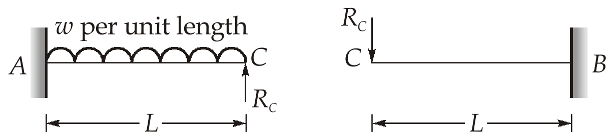
$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_y}{2(\text{FOS})}$$

$$\text{FOS} = \frac{75}{122.17 - 25} = 0.77$$

19. (c)

We know that at internal hinge, deflection will be same at just left and right of hinge.

So,



$$(\delta_c)_{left}\downarrow = \frac{wL^4}{8EI} - \frac{R_cL^3}{3EI} \quad \text{...(i)}$$

$$(\delta_c)_{right}\downarrow = \frac{R_cL^3}{3EI} \quad \text{...(ii)}$$

So from eq. (i) and (ii)

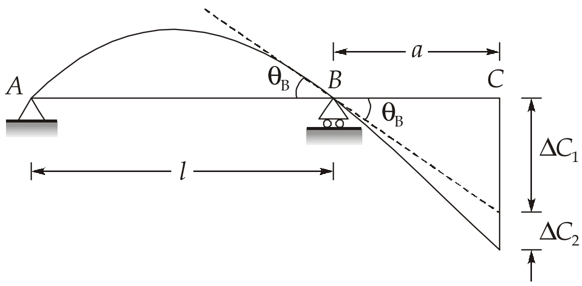
$$\frac{wL^4}{8EI} - \frac{R_cL^3}{3EI} = \frac{R_cL^3}{3EI}$$

$$\Rightarrow \quad \frac{2R_cL^3}{3EI} = \frac{wL^4}{8EI}$$

$$\Rightarrow \quad R_c = \frac{3}{16}wL$$

20. (d)

The deformation of the beam will be as shown below.



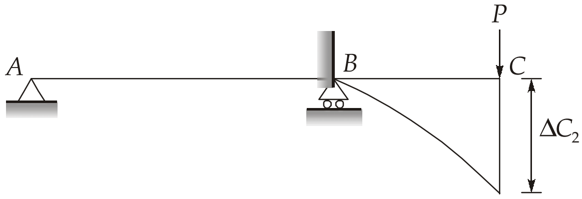
Now ΔC_1 is produced due to deflection of C as caused due to deformation of AB,

$$\Delta C_1 = \theta_B\,(BC) = \theta_B a$$

$$\theta_B = \frac{M_{BA}l}{3EI} = \frac{Pal}{3EI}$$

$$\therefore \quad \Delta C_1 = \frac{Pala}{3EI} = \frac{Pa^2l}{3EI}$$

ΔC_2 is produced due to deformation of BC



$$\Delta C_2 = \frac{Pa^3}{3EI}$$

So total deflection at $C, \Delta C = \Delta C_1 + \Delta C_2$

$$= \frac{Pa^2 l}{3EI} + \frac{Pa^3}{3EI}$$

21. (c)

Modulus of section for both timber sections

$$Z_T = 2\left[\frac{60 \times (200)^2}{6}\right] = 8 \times 10^5 \text{ mm}^3$$

Similarly modulus of section for the steel section

$$Z_S = \frac{15 \times (200)^2}{6} = 10^5 \text{ mm}^3$$

Now, moment of resistance of timber section,

$$M_T = Z_T \cdot \sigma_T$$
$$= 8 \times 10^5 \times 5$$
$$= 4 \times 10^6 \text{ N-mm} = 4 \text{ kN-m}$$

Similarly,

Moment of resistance of steel section,

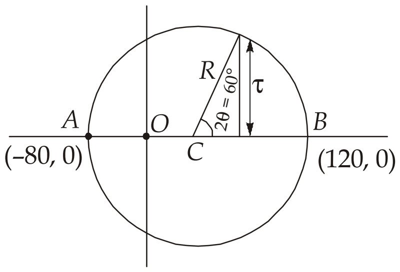
$$M_S = Z_S \cdot \sigma_S$$
$$= 10^5 \times 100$$
$$= 10 \times 10^6 \text{ N-mm}$$
$$= 10 \text{ kNm}$$

Total moment of resistance of the beam

$$M = M_S + M_T = 10 + 4 = 14 \text{ kNm}$$

22. (c)

Radius, $R = \dfrac{120-(-80)}{2} = 100$



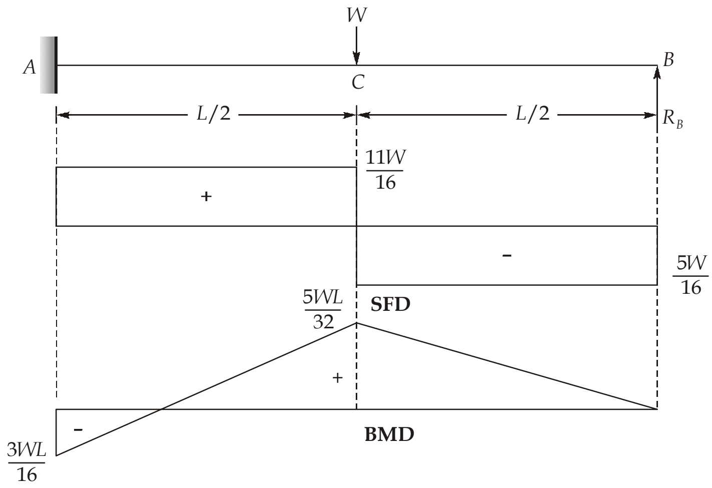
Tangential stress, $\tau = R \sin 2\theta$

$$= 100 \sin 60°$$

$$= 100 \times \frac{\sqrt{3}}{2}$$

$$= 50\sqrt{3}$$

23. (c)



Since end B is propped

Net deflection at B is zero.

$$\Rightarrow \qquad (\delta_B\downarrow)_W = (\delta_B\uparrow)_{R_B}$$

$$\frac{W\left(\frac{L}{2}\right)^3}{3EI} + \frac{W\left(\frac{L}{2}\right)^2}{2EI} \times \frac{L}{2} = \frac{R_B L^3}{3EI}$$

$$\frac{WL^3}{24EI} + \frac{WL^3}{16EI} = \frac{R_B L^3}{3EI}$$

$$\frac{2WL^3 + 3WL^3}{48EI} = \frac{R_B L^3}{3EI}$$

$$R_B = \frac{5W}{16}$$

$$\therefore \qquad R_A = W - R_B = W - \frac{5W}{16}$$

$$= \frac{11W}{16}$$

Bending moment diagram:

For BC: $\qquad M_x = R_B x = \dfrac{5Wx}{16}$

At $x = 0$; $\qquad M_B = 0 \Rightarrow$ Moment at propped end is zero.

At $x = \dfrac{L}{2}$; $\qquad M_C = \dfrac{5WL}{32}$

For CA: $\qquad M_x = \dfrac{5Wx}{16} - W\left(x - \dfrac{L}{2}\right)$

At $x = \dfrac{L}{2}$; $\qquad M_C = \dfrac{5WL}{32}$

At $x = L$; $M_A = -\dfrac{3WL}{16}$

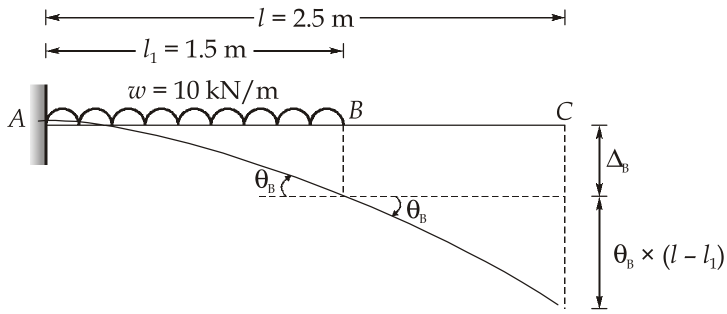
$BM_{xx} = 0$

$$\frac{5Wx}{16} = Wx - \frac{WL}{2}$$

$$x = \frac{8L}{11} \quad \text{(From prop end)}$$

$\therefore$ The point of contraflexure is at $\left(L - \dfrac{8L}{11}\right) = \dfrac{3}{11}$ from fixed end.

24. (a)



We know deflection at free end for the case given in question,

$$y = \frac{wl_1^4}{8EI} + \frac{wl_1^3}{6EI}(l - l_1)$$

where l = 2.5 m, l_1 = 1.5 m, w = 10 kN/m = 10 N/mm

$$y = \frac{10\left[1.5\times10^3\right]^4}{8\times1.9\times10^{12}} + \frac{10\left[1.5\times10^3\right]^3}{6\times1.9\times10^{12}}\times[2.5-1.5]\times10^3$$

$$= 6.29 \text{ mm} \simeq 6.3 \text{ mm}$$

25. (b)

Since section is symmetric about x-x and y-y, therefore centre of section will lie on the geometrical centroid of section.

The semi-circular grooves may be assumed together and consider one circle of diameter 60 mm.

So, $$I_{xx} = \frac{80\times(100)^3}{12} - \frac{\pi}{64}(60)^4$$

$$= 6.03 \times 10^6 \text{ mm}^4$$

Now for shear stress at neutral axis, consider the area above the neutral axis,

$$A\bar{y} = [80\times50\times25] - \frac{\pi}{2}(30)^2 \times \frac{4\times30}{3\pi}$$

$$= 100000 - 18000 = 82000 \text{ mm}^3$$

$$b = 20 \text{ mm}$$

So, $$\tau = \frac{VA\bar{y}}{Ib} = \frac{20\times10^3\times82000}{6.03\times10^6\times20}$$
$$= 13.60 \text{ MPa}$$

26. (c)

(i) Force on shaded area $= \dfrac{f_{\max}}{y_{\max}}Ay$

where, A is area of shaded portion, y is distance of centroid of shaded area from NA

$$Ay = 5\times5\times(5+2.5) = 187.5 \text{ cm}^3$$

So, $$\text{Force} = \frac{80}{10}\times187.5 = 1500 \text{ kg}$$

(ii) Moment of this force about the neutral axis

$$M = \frac{f_{\max}}{y_{\max}}I_o$$

(I_o = Moment of inertia of shaded area about neutral axis)

$$I_o = \frac{5\times5^3}{12}+5\times5\times(7.5)^2 = \frac{4375}{3}\text{ cm}^4$$

So, $$M = \frac{80}{10}\times\frac{4375}{3} = 11666.67 \text{ kg cm}$$

27. (c)

$$U = \frac{P^2L}{2AE} \quad \text{(For axially loaded bar)}$$

$L_1 = 10$ cm, $L_2 = 20$ cm, $d_1 = 2$ cm and $d_2 = 4$ cm

$$U_A = \frac{P^2L_1}{2A_1E}+\frac{P^2L_2}{2A_2E}$$

$$U_B = \frac{P^2L_2}{2A_1E}+\frac{P^2L_1}{2A_2E}$$

$$\therefore \quad \frac{U_B}{U_A} = \frac{\dfrac{L_2}{A_1}+\dfrac{L_1}{A_2}}{\dfrac{L_1}{A_1}+\dfrac{L_2}{A_2}}$$

$$= \frac{L_1d_1^2+L_2d_2^2}{L_1d_2^2+L_2d_1^2}$$

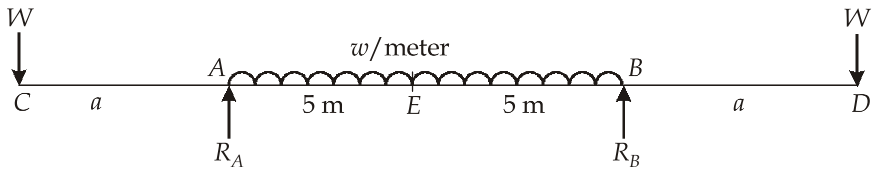
$$= \frac{10\times2^2+20\times4^2}{10\times4^2+20\times2^2} = \frac{3}{2} = 1.5$$

28. (b)

$$R_A = R_B = \frac{1}{2}(2W + 10w) = W + 5w$$

$$= 15w \qquad (W = 10\,w)$$

Now



$$M_E = R_A \times 5 - W(a+5) - w \times 5 \times 2.5 = 0$$

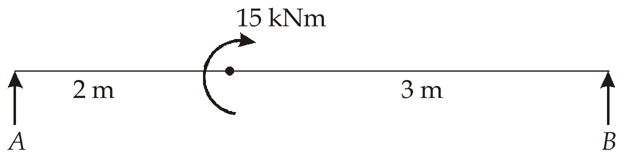
$\Rightarrow \quad 15w \times 5 - 10w\,(a+5) - 12.5w = 0$

$\Rightarrow \quad 75 - 10a - 50 - 12.5 = 0$

$\Rightarrow \quad 12.5 = 10a$

$\Rightarrow \quad a = 1.25$ m

29. (c)



$R_A + R_B = 0$

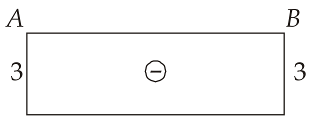
$\Sigma M_A = 0$

$\Rightarrow \quad R_B \times 5 = 15$

$\Rightarrow \quad R_B = 3$ kN

$R_A = -3$ kN

Now the for SFD will be as shon below.



30. (b)

$$\text{Total load} = 2\left[\frac{1}{2}\left(\frac{L}{2}\right)\times w_0\right] = \frac{w_0 L}{2}$$

By symmetry, $$R_1 = R_2 = \frac{1}{2}\times\text{Total load}$$

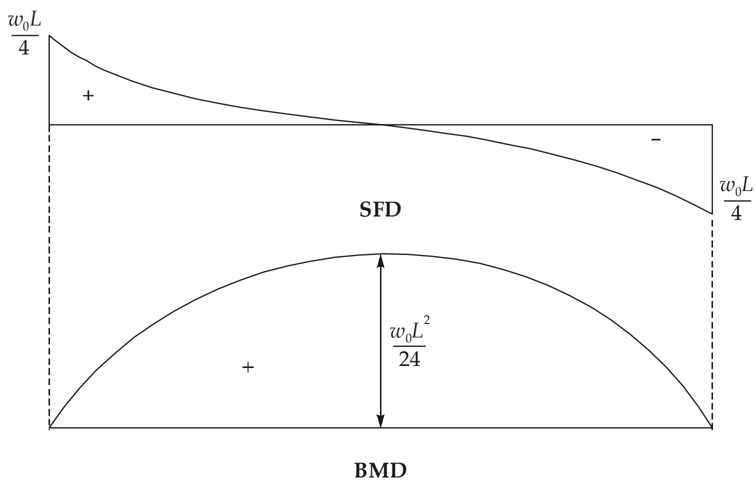
$$\Rightarrow \quad R_1 = R_2 = \frac{w_0 L}{4}$$

Bending moment at B,

$$(M_B) = R_1\times\frac{L}{2} - \frac{1}{2}w_0\times\frac{L}{2}\times\frac{2}{3}\left(\frac{L}{2}\right)$$

$$= \frac{w_0 L}{4}\times\frac{L}{2} - \frac{w_0}{2}\times\frac{L}{2}\times\frac{2}{3}\left(\frac{L}{2}\right)$$

$$= \frac{w_0 L^2}{8} - \frac{w_0 L^2}{12} = \frac{w_0 L^2}{24}$$



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