CLASS TEST

Sl.: 01 SP_ME_ABC_160323

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STRENGTH OF MATERIALS

MECHANICAL ENGINEERING

Date of Test : 16/03/2023

ANSWER KEY

1.	(b)	7.	(c)	13.	(c)	19.	(a)	25.	(d)
2.	(b)	8.	(a)	14.	(b)	20.	(b)	26.	(b)
3.	(c)	9.	(d)	15.	(a)	21.	(a)	27.	(c)
4.	(d)	10.	(b)	16.	(b)	22.	(a)	28.	(b)
5.	(c)	11.	(c)	17.	(c)	23.	(c)	29.	(d)
6.	(d)	12.	(c)	18.	(d)	24.	(b)	30.	(d)

DETAILED EXPLANATIONS

1. (b)

Thickness of cylindrical portion, t_1 = 3 mm

Thickness of hemispherical ends = t_2

For no distortion of the junction under pressure,

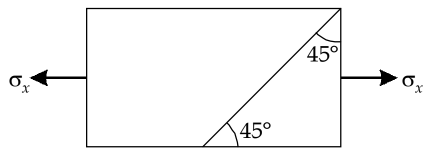
$$\frac{t_2}{t_1} = \frac{1-\mu}{2-\mu} = \frac{1-0.3}{2-0.3}$$

$$t_2 = 1.235 \text{ mm}$$

2. (b)

Given: P = 11 kN, A = 150 × 75 mm²

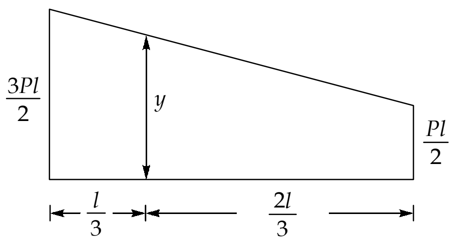
$$\sigma_x = \frac{P}{A} = \frac{11000}{150\times75} = 0.977 \text{ MPa}$$



$$(\sigma_n)_{\theta = 45°} = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2} = \frac{0.977}{2} = 0.488 \text{ MPa}$$

3. (c)



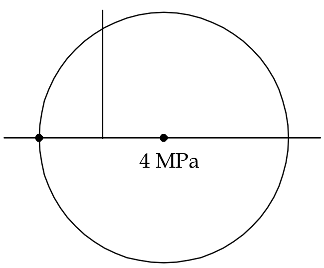
At $= \frac{2l}{3}$, $\quad y = \frac{Pl}{2} + \frac{2Pl}{3} = \frac{7Pl}{6}$

$$\theta_x - \theta_0 = \text{Area of } \frac{M}{EI} \text{ diagram}$$

$$= \frac{1}{2}\left(\frac{3Pl}{2} + \frac{7Pl}{6}\right)\frac{l}{3EI}$$

$$= \frac{4}{9}\frac{Pl^2}{EI}$$

4. (d)



$$\text{Radius of Mohr circle} = 4 - (-4) = 8 \text{ MPa}$$
$$\text{Maximum principal stress} = 4 + 8 = 12 \text{ MPa}$$

5. (c)

$$\frac{K}{G} = \frac{2(1+\mu)}{3(1-2\mu)}$$
$$= \frac{2(1.25)}{3(0.5)} = 1.67$$

6. (d)

$$R_A + R_B = P + wl$$
$$R_A + 6 = P + 6$$
$$R_A = P$$

$\Sigma M_A = 0$

$$6 \times 3 = P \times 2 + 2 \times \frac{3^2}{2}$$
$$\Rightarrow \quad 18 = 2P + 9$$
$$P = 4.5 \text{ kN}$$
$$\therefore \quad R_A = 4.5 \text{ kN}$$

7. (c)

$$w = 5 \text{ kN/m}, \ q = (10 - 5) = 5 \text{ kN/m}$$
$$\text{B.M.} = \frac{wl^2}{2} + \frac{ql^2}{6}$$
$$= \frac{5 \times 4^2}{2} + \frac{5 \times 4^2}{6} = 53.33 \text{ kNm}$$

8. (a)

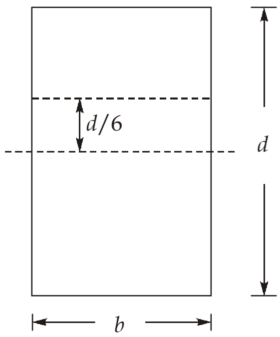
$$\frac{M}{I} = \frac{E}{R}$$
$$M = \frac{200 \times 10^3}{2000} \times 42 \times \frac{\pi}{180} \times \frac{18 \times 6^3}{12}$$
$$= 23760 \text{ Nmm} \simeq 23.76 \text{ N.m}$$

9. (d)

10. (b)

$$\text{Minimum moment of inertia} = \frac{12 \times 6^3}{12} = 216 \text{ mm}^4$$
$$\text{Radius of gyration} = \sqrt{\frac{I}{A}} = \sqrt{\frac{216}{12 \times 6}} = \sqrt{3}$$
$$\text{Slenderness ratio} = \frac{L_e}{k}$$
$$L_e = 100 \times \sqrt{3} = 173.21 \text{ mm}$$

11. (c)



Maximum shear force in beam $= \frac{P}{2}$

$$\text{Shear stress, } \tau = \frac{FA\bar{y}}{Ib} = \frac{\frac{P}{2}\times\frac{bd}{3}\times\frac{d}{3}}{\frac{bd^3}{12}\times b} = \frac{2P}{3bd}$$

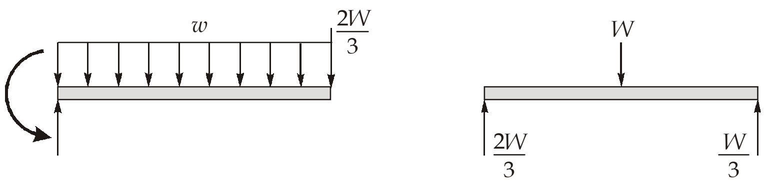
12. (c)

$$\delta_{\text{impact}} = \delta_{st}\left(1+\sqrt{1+\frac{2h}{\delta_{st}}}\right)$$

$$\delta_{\text{st}} = \frac{Pl}{AE} = \frac{500\times10^3\times1000}{50\times10^2\times200\times10^3} = 0.5 \text{ mm}$$

$$\frac{\delta_{\text{impact}}}{\delta_{\text{gradually}}} = \left(1+\sqrt{1+\frac{2\times12}{0.5}}\right) = 8$$

13. (c)



$$\delta_{\text{centre}} = \frac{wa^4}{8EI}+\frac{2Wa^3}{9EI}$$

14. (b)

Strain energy stored in AB, CD

$$= \frac{P^2L}{2\frac{\pi}{4}D^2E}$$

$$\text{Strain energy stored in BC} = \frac{P^2(2L)}{2\frac{\pi}{4}\frac{D^2}{4}E} = \frac{8P^2L}{2\frac{\pi}{4}\times D^2E}$$

$$\% \text{ Strain energy stored in BC} = \frac{\dfrac{8P^2L}{2\dfrac{\pi}{4}D^2E}}{\dfrac{P^2L}{2\dfrac{\pi}{4}D^2E}+\dfrac{8P^2L}{2\dfrac{\pi}{4}D^2E}+\dfrac{P^2L}{2\dfrac{\pi}{4}D^2E}} = \frac{8}{1+8+1} = \frac{8}{10}$$

$$= 80\%$$

15. (a)

As rod and tube are in parallel, so angle of twist will be same

$$\frac{T_R l}{G\dfrac{\pi}{32}d^4} = \frac{T_T l}{\dfrac{G}{3}\dfrac{\pi}{32}\left((2d)^4 - d^4\right)}$$

$$5T_R = T_T$$

$$T_R + T_T = T$$

$$T_R + 5T_R = T$$

$$T_R = \frac{T}{6}$$

$$\theta_R = \theta_T = \frac{T}{6}\times\frac{32l}{G\pi d^4} = \frac{16Tl}{3\pi G d^4}$$

16. (b)

Change of length of bar = Compression of spring

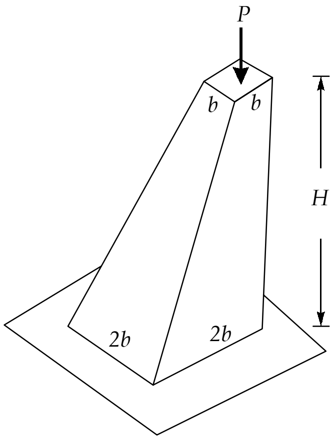
$$\int_0^L \alpha\Delta T(x)\,dx - \frac{R_A L}{AE} = \frac{R_A}{K}$$

$$\alpha\Delta T_0\left(x - \frac{x^3}{3L^2}\right)_0^L = \frac{R_A L}{AE} + \frac{R_A}{K}$$

$$\frac{2}{3}\alpha\Delta T_0 L = R_A\left(\frac{L}{AE}+\frac{1}{K}\right)$$

$$R_A = \frac{2}{3}\frac{\alpha\Delta T_0 LKAE}{(KL+AE)}$$

17. (c)



$$\delta = \int_0^H \frac{Pdx}{AE} = \int_0^H \frac{Pdx}{\left(b+\frac{bx}{H}\right)^2 E}$$

Let $$b+\frac{bx}{H} = t$$

$$\frac{dt}{dx} = \frac{b}{H}$$

$$dx = dt\left(\frac{H}{b}\right)$$

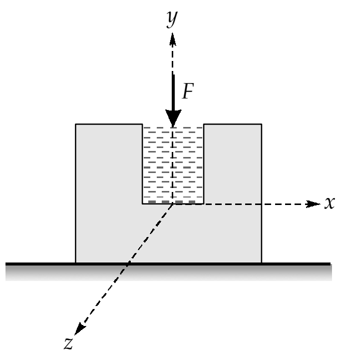
when $x = 0$, $t = b$; $x = H$, $t = 2b$

$$\delta = \frac{H}{b}\int_b^{2b} \frac{P}{t^2}\frac{dt}{E} = \frac{HP}{bE}\times\left(-\frac{1}{t}\right)_b^{2b}$$

$$\delta = \frac{HP}{Eb}\left(\frac{1}{b}-\frac{1}{2b}\right) = \frac{HP\times 1}{Eb\times 2b}$$

$$\delta = \frac{HP}{2b^2E}$$

18. (d)



$$\sigma_x = -\sigma$$

$$\sigma_z = -\sigma$$

$$\varepsilon_x = 0 = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y+\sigma_z)$$

$$= -\sigma - \mu\left(\frac{-F}{A}-\sigma\right)$$

$$\sigma = +\mu\frac{F}{A}+\mu\sigma = \frac{\mu}{1-\mu}\frac{F}{A}$$

$$\varepsilon_y = -\frac{F}{AE}-\frac{\mu}{E}(-2\sigma)$$

$$\varepsilon_y = -\frac{F}{AE} + \frac{\mu}{E}\left(\frac{2\mu}{1-\mu}\frac{F}{A}\right)$$

$$\varepsilon_y = \frac{F}{AE}\left(\frac{2\mu^2}{1-\mu} - 1\right)$$

$$\varepsilon_y = \frac{F}{AE}\left(\frac{2\mu^2 - 1 + \mu}{1-\mu}\right)$$

$$\delta = \frac{Fl}{AE}\left(\frac{2\mu^2 + \mu - 1}{1-\mu}\right)$$

19. (a)

$$\text{Strain energy} = \frac{T^2 l}{2GJ}$$

$$= \frac{T_0^2}{2G} \times \left(\frac{2l}{3}\right) \times \frac{32}{\pi(d^4)} + \frac{T_0^2}{2G} \times \left(\frac{4l}{3}\right) \times \frac{32 \times 16}{\pi \times d^4}$$

$$= \frac{T_0^2 l}{2Gd^4}\left[\frac{2}{3} \times \frac{32}{\pi} + \frac{4}{3} \times \frac{32}{\pi} \times 16\right]$$

$$= 112\frac{T_0^2 l}{Gd^4}$$

20. (b)

The displacement of point 'C'

$$\delta_C = (\delta_{AB})_{self\ weight} + (\delta_{AB})_{weight\ of\ BC} + (\delta_{BC})_{Self\ weight}$$

$$= \left.\frac{WL}{2AE}\right|_{AB} + \left.\frac{P_{ext}L}{AE}\right|_{AB} + \left.\frac{WL}{2AE}\right|_{BC}$$

From given data,

$$W_{BC} = W_{AC} - W_{AB}$$
$$= 3W - W = 2W$$
$$W_{AB} = W$$
$$(P_{\text{ext}})_B = 2W$$

So,
$$\delta_C = \frac{WL}{2AE} + \frac{2WL}{AE} + \frac{(2W) \times L}{2(2A)E}$$

$$\delta_C = \frac{WL}{AE}\left[\frac{2+8+2}{4}\right] = \frac{12WL}{4AE}$$

$$\delta_C = \frac{3WL}{AE}$$

21. (a)

The applied stress in the direction of thickness of plate,

$$\sigma_z = 0$$

Strain along thickness direction,

$$\Rightarrow \qquad \varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y) \qquad \text{...(i)}$$

As we know that,

$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2)$$

So, we can write, $$\sigma_x = \frac{209\times10^3}{1-0.3^2}\left[50\times10^{-5} + 0.3\times15\times10^{-5}\right]$$

$$= 125.17 \text{ MPa}$$

Similarly, $$\sigma_y = \frac{209\times10^3}{1-0.3^2}\left[15\times10^{-5} + 0.3\times50\times10^{-5}\right]$$

$$\sigma_y = 68.9 \text{ MPa}$$

From equation (i)

$$\varepsilon_z = 0 - \frac{0.3}{209\times10^3}[125.17 + 68.9]$$

$$= 2.785 \times 10^{-4}$$

Reduction in thickness of plate,

$$\delta t = t \times \varepsilon_z$$

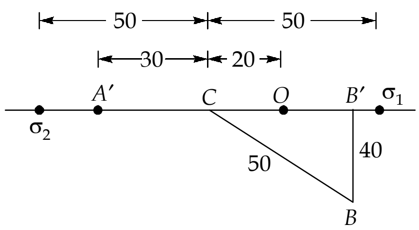
$$= 20 \times 2.785 \times 10^{-4}$$

$$= 5.571 \times 10^{-3} \text{ mm}$$

22. (a)

Point A and B on the Mohr's circle represents the complementary planes. So shear stress will be same, i.e. $\tau_A = \tau_B$

Radius of the Mohr's circle, CA



$$CA = \sqrt{30^2 + 40^2} = 50$$

$$CB' = \sqrt{50^2 - 40^2} = 30$$

$$OB' = CB' - CO = 30 - 20 = 10$$

$$\sigma_B = OB' = 10$$

$$\sigma_1 = 50 - 20 = 30$$

$$\sigma_2 = -(50 + 20) = -70$$

$$\sigma_A = OA' = 30 + 20 = 50$$

23. (c)

$$\text{Strain tensor} = \begin{bmatrix} \epsilon_{xx} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \epsilon_{yy} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

$$\gamma_{xy} = 0.004 \times 2 = 0.008$$

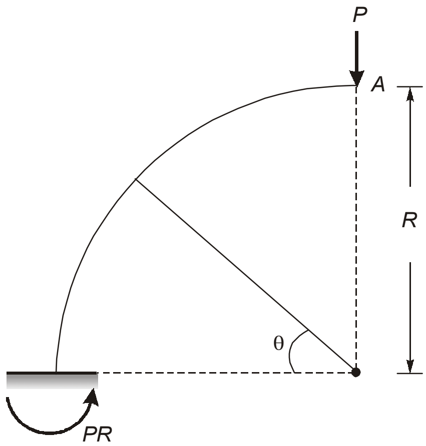
$$\gamma_{xz} = 0.006 \times 2 = 0.012$$

$\Rightarrow$ $$\tau_{xy} = G\,\gamma_{xy} = 100 \times 0.008 = 0.8 \text{ GPa}$$

and $$\tau_{xz} = G\,\gamma_{xz} = 100 \times 0.012 = 1.2 \text{ GPa}$$

So, $$\tau_{xy} + \tau_{xz} = 800 + 1200 = 2000 \text{ MPa}$$

24. (b)



$$M = PR\cos\theta$$

$$\frac{\partial M}{\partial P} = R\cos\theta$$

Now, $$\delta_v = \frac{\partial U}{\partial P}$$ (where U = strain energy)

$$\frac{\partial U}{\partial P} = \int_0^{\pi/2} \frac{M \times \left(\frac{\partial M}{\partial P}\right) R d\theta}{EI}$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2} \cos^2\theta d\theta$$

$$\delta_v = \frac{\pi PR^3}{4EI}$$

25. (d)

From equilibrium, $\Sigma V = 0$,

$\Rightarrow$ $R_A + R_B = 2 + 2 \times 4 = 10$ kN

$\Sigma M = 0$,

$\Rightarrow$ $2 \times 4 \times 2 + 2 \times 6 = R_B \times 4$

So, $$R_B = \frac{16+12}{4} = \frac{28}{4} = 7 \text{ kN}$$

and $R_A = 3$ kN

Bending moment at a distance 'x' from end A will be

$$(\text{BM})_x = 3x - 2 \times x \times \frac{x}{2} = 3x - x^2$$

Now, the BMD changes sign in section AB, so the point of contraflexure is where the BM is zero.

So, $3x - x^2 = 0$

$x = 3$ m

26. (b)

$$I = \frac{\pi R^4}{4} - 2\left[\frac{\pi}{4}\left(\frac{R}{3}\right)^4 + \pi\left(\frac{R}{3}\right)^2\left(\frac{R}{2}\right)^2\right]$$

$$I = 0.592\ R^4$$

Bending stress will be maximum at point A and B.

So, $$\sigma_{max} = \frac{P \times 1.5 \times 0.1}{0.592 \times (0.1)^4}$$

$\therefore$ $$P = 236.8 \text{ kN}$$

27. (c)

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{200+120}{2} + \sqrt{\left(\frac{200-120}{2}\right)^2 + 30^2}$$

$$= 210 \text{ MPa}$$

28. (b)

$$\text{Column I} = P_{cr} = \frac{2\pi^2 EI}{h_1^2}$$

$$\text{Column II} = P_{cr} = \frac{4\pi^2}{3}\frac{EI}{h_2^2}$$

$$\frac{2\pi^2 EI}{h_1^2} = \frac{4\pi^2 EI}{3h_2^2}$$

$$\frac{h_2}{h_1} = \sqrt{\frac{2}{3}} = 0.82$$

29. (d)

$$\gamma_{max} = r_{max}\frac{\theta}{l}$$

$$\gamma = 35 \times \frac{22 \times 1.8}{7 \times 180} \times \frac{1}{1500}$$

$$= 7.33 \times 10^{-4}$$

30. (d)

$$EI\frac{d^4y}{dx^4} = \sqrt{\frac{x}{L}}q$$

$$V_x = EI\frac{d^3y}{dx^3} = \frac{2x^{3/2}}{3\sqrt{L}}q + C_1$$

$$M_x = EI\frac{d^2y}{dx^2} = \frac{2}{3}\times\frac{2}{5}\times\frac{x^{5/2}}{\sqrt{L}}q + C_1x + C_2$$

$x = 0$ $\quad V_x = 0$

$\Rightarrow$ $\quad C_1 = 0$

$x = 0$ $\quad M_x = 0$

$\Rightarrow$ $\quad C_2 = 0$

$$M_x = \frac{4}{15}q\frac{x^{5/2}}{\sqrt{l}}$$

At $x = l$ $\quad M_x = M_{\max}$

$$M_{\max} = \frac{4}{15}ql^2$$

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