CLASS TEST

S.No.: 01 SK_ABC_100323

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CONTROL SYSTEMS

EC | EE

Date of Test : 10/03/2023

ANSWER KEY

1.	(a)	7.	(b)	13.	(d)	19.	(b)	25.	(b)
2.	(a)	8.	(b)	14.	(a)	20.	(a)	26.	(d)
3.	(b)	9.	(b)	15.	(c)	21.	(d)	27.	(c)
4.	(b)	10.	(c)	16.	(b)	22.	(b)	28.	(a)
5.	(a)	11.	(b)	17.	(a)	23.	(d)	29.	(c)
6.	(a)	12.	(d)	18.	(a)	24.	(b)	30.	(b)

DETAILED EXPLANATIONS

1. (a)

After taking Laplace transform, the series combination of R_1 and C_1 gives: $R_1 + \dfrac{1}{sC_1}$.

Similarly, the parallel combination of R_2 and C_2 gives: $\dfrac{\dfrac{R_2}{sC_2}}{R_2 + \dfrac{1}{sC_2}}$

Substituting, $R_1 = R_2 = C_1 = C_2 = 1$ makes the above combinations $1+\dfrac{1}{s}$ and $\dfrac{\dfrac{1}{s}}{1+\dfrac{1}{s}}$

On applying the voltage divider rule, we get

$$V_0(s) = \frac{\dfrac{\frac{1}{s}}{1+\frac{1}{s}}}{1+\dfrac{1}{s}+\dfrac{\frac{1}{s}}{1+\frac{1}{s}}} V_i(s)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{\dfrac{1}{s+1}}{\dfrac{s(s+1)+(s+1)+s}{s(s+1)}}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{s}{s^2+3s+1}$$

2. (a)

The general form of transfer function of a 2nd order system is given by,

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

On comparing:

$$\omega_n^2 = 64$$

Natural frequency, $\omega_n = 8 \text{ rad/sec}$

$$2\,\xi\omega_n = 8$$

Damping ratio, $\xi = \dfrac{8}{16} = 0.5$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 8\sqrt{1-0.5^2}$$

$$\omega_d = 6.93 \text{ rad/sec}$$

3. **(b)**

For impulse response, $R(s) = 1$

The impulse response is given by,

$$y(t) = L^{-1}\left\{\frac{1}{(s+1)(s+10)}\right\}$$

$$y(t) = L^{-1}\left\{\frac{1}{9(s+1)} - \frac{1}{9(s+10)}\right\}$$

$$y(t) = \frac{1}{9}e^{-t} - \frac{1}{9}e^{-10t}$$

4. **(b)**

Close loop transfer function of the system is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K(s^2+1)}{(K+1)s^2+3s+K+2}$$

$\therefore$ the characteristic equation is given by

$$(K+1)s^2 + 3s + K + 2 = 0$$

For the system to be stable all the coefficient should be of same sign

$\therefore$ $K+1>0 = K>-1$

and $K+2>0 = K>-2$

$\therefore$ The system is stable for $K > -1$.

5. **(a)**

$$G(s)H(s) = \frac{(1-s)}{(1+s)(3+s)}$$

So, $$|G(j\omega)H(j\omega)| = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}\sqrt{9+\omega^2}} = \frac{1}{\sqrt{9+\omega^2}}$$

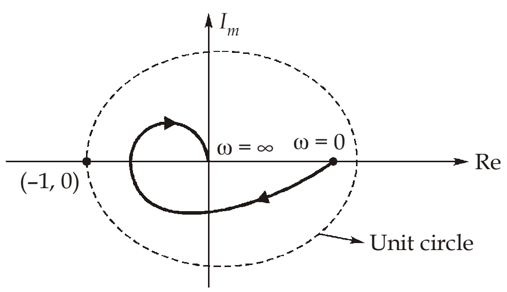
Phase angle is,

$$\phi = -2\tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

At $\omega = 0$; $|G(j\omega)H(j\omega)| = \frac{1}{3}$; $\phi = 0°$

At $\omega = 4$; $|G(j\omega)H(j\omega)| = \frac{1}{5}$; $\phi = -205°$

At $\omega = \infty$; $|G(j\omega)H(j\omega)| = 0$; $\phi = -270°$



Polar plot does not cross unit circle, for all values of ω (0 to ∞), magnitude will be less than unity. In this case, gain crossover frequency does not exist and phase margin becomes ∞.

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6. (a)

The gain of function is 1, given system is all pass system, Hence option (a) is correct answer.

We can cross check:

$$GH(s) = \frac{1-s}{1+s}$$

$$GH(j\omega) = \frac{1-j\omega}{1+j\omega}$$

$$\angle GH(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(\omega) = -2\tan^{-1}(\omega)$$

By varying ω: $0 \to \infty$

The phase varies from 0° to –180° as shown in the plot.

7. (b)

$$\text{OLTF is } G(s)H(s) = \frac{-1}{2s(1-20s)}$$

(or) $$G(j\omega)H(j\omega) = \frac{-1}{2j\omega(1-20j\omega)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{1}{2\omega\sqrt{1+400\omega^2}}$$

$$\phi = 180^\circ - 90^\circ - \tan^{-1}\left(\frac{-20\omega}{1}\right)$$

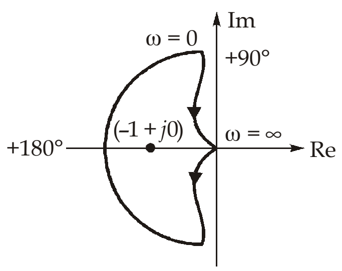
$$= 90^\circ + \tan^{-1}(20\omega)$$

At $\omega = 0$; $M = \infty$; $\phi = 90^\circ$

At $\omega = 0.1$; $M = 2.24$; $\phi = 153.43^\circ$

At $\omega = \infty$; $M = 0$; $\phi = 180^\circ$

We get the Nyquist plot as,



8. (b)

The characteristic equation is,

$$1 + G(s)H(s) = 0$$

$$1 + \left(K_P + \frac{K_I}{s}\right)\left(\frac{1}{(s+1)(s+2)}\right) = 0$$

(or) $$s^3 + 3s^2 + (2 + K_p)s + K_I = 0$$

Forming Routh Array:

$$\begin{array}{c|cc} s^3 & 1 & 2+K_P \\ s^2 & 3 & K_I \\ s^1 & \dfrac{6+3K_P-K_I}{3} & \\ s^0 & K_I & \end{array}$$

For stable system, $K_I > 0$

and $K_P > \dfrac{K_I}{3} - 2$

9. **(b)**

$$x(t) = \phi(t)\, x(0)$$

Where, $\phi(t)$ = state transition matrix

Given, $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s-1)(s-1)}\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{s-1} & 0 \\ \dfrac{1}{(s-1)^2} & \dfrac{1}{(s-1)} \end{bmatrix}$$

$$L^{-1}[sI - A]^{-1} = \phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$\therefore \quad x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

10. **(c)**

The transfer function of the system is,

$$H(s) = \frac{(s+a)}{[(s-(-a-jb))\,(s-(-a+jb))]^2}$$

$$= \frac{s+a}{[(s+a)^2+b^2]^2} = \frac{1}{2}\cdot\frac{2(s+a)}{[(s+a)^2+b^2]^2}$$

$$H(s) = \frac{1}{2}\left[-\frac{dF(s)}{ds}\right]$$

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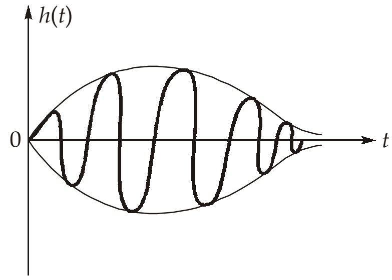
where, $F(s) = \dfrac{1}{(s+a)^2 + b^2} \xleftarrow[\text{I.L.T.}]{} f(t) = \dfrac{1}{b} e^{-at} \sin(bt)$

$$-\frac{d}{ds} F(s) \longleftrightarrow t\, f(t)$$

$$h(t) = \frac{t}{2} f(t) = \frac{t}{2b} e^{-at} \sin(bt)$$

$$h(t) = K\, te^{-at} \sin(bt)$$

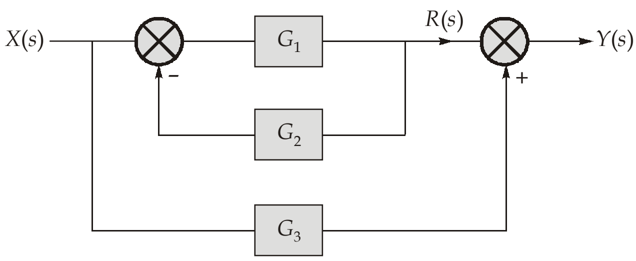


11. **(b)**

For given block diagram.

In inner loop feedback we can write,

$$\frac{R(s)}{X(s)} = \frac{G_1(s)}{1 + G_1(s) G_2(s)}$$



∴ Overall transfer function,

$$Y(s) = X(s)\, R(s) + G_3(s)\, X(s)$$

$$\frac{Y(s)}{X(s)} = R(s) + G_3(s)$$

$$= \frac{\dfrac{1}{s+1}}{1 + \dfrac{1}{(s+1)} \dfrac{1}{(s+4)}} = \frac{(s+4)}{(s+1)(s+4)+1} = \frac{s+4}{s^2 + 5s + 4 + 1}$$

$$\frac{s+4}{s^2 + 5s + 5} + \frac{s+2}{s+5} \Rightarrow \frac{(s+4)(s+5) + (s+2)(s^2 + 5s + 5)}{(s^2 + 5s + 5)(s+5)}$$

$$= \frac{s^2 + 9s + 20 + s^3 + 5s^2 + 5s + 2s^2 + 10s + 10}{s^3 + 5s^2 + 5s^2 + 25s + 5s + 25} =$$

$$\frac{s^3 + 8s^2 + 24s + 30}{s^3 + 10s^2 + 30s + 25}$$

12. (d)

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y = x(t)$$

$$s^2Y(s) + 5sY(s) + 4Y(s) = X(s)$$

$$x(t) = 8u(t) \quad \text{or} \quad X(s)=\frac{8}{s}$$

$$(s^2 + 5s + 4)\ Y(s) = \frac{8}{s}$$

$$Y(s) = \frac{8}{s(s^2+5s+4)}=\frac{A}{s}+\frac{B}{s+4}+\frac{C}{s+1}$$

$$Y(s) = \frac{2}{s}-\frac{8}{3(s+1)}+\frac{2}{3(s+4)}$$

$$y(t) = 2\left[1+\frac{1}{3}e^{-4t}-\frac{4}{3}e^{-t}\right]u(t)$$

13. (d)

$$\frac{Y(s)}{R(s)} = \frac{K/s}{1-\left(\frac{3}{s}+\frac{K}{s}\right)}=\frac{K}{s-(3+K)}$$

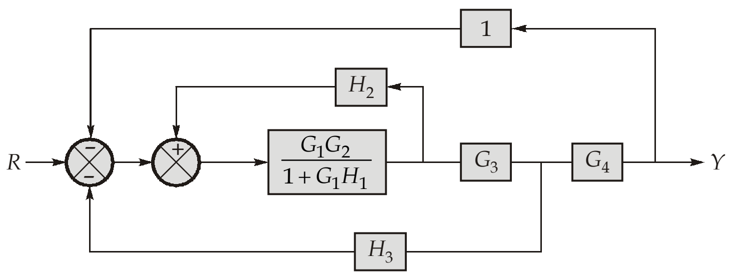
For system to be stable,

$$3 + K < 0$$

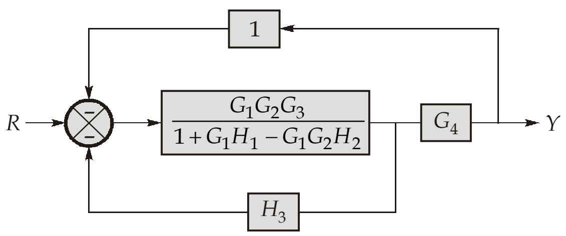
$$K < -3$$

14. (a)

G_1 and H_1 form a negative feedback loop and that loop is in cascade with G_2. The block diagram can be simplified as:



Now $\frac{G_1G_2}{1+G_1H_1}$ forms a positive feedback loop with H_2 and that loop is in cascade with G_3. The block diagram can be further simplified as:



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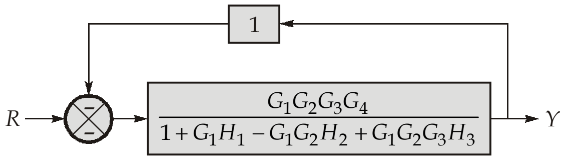
Now there is another negative feedback loop with H_3 and cascade with G_4. Block diagram becomes



$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4}$$

15. (c)

The Routh table for the characteristic equation corresponding to the given transfer function is:

s^5	1	0	4
s^4	3	5	3
s^3	–1.67	3	
s^2	10.4	3	
s^1	3.48		
s^0	3		

There are two sign changes in the fist column of Routh table. So there are 2 poles in RHP. Order of the characteristic equation is 5 = number of poles

∴ No. of poles in LHP = 5 – 2 = 3

16. (b)

Closed loop transfer function is,

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K(s+3)(s+5)}{(K+1)s^2+(8K-6)s+(15K+8)}$$

For stability all coefficients should be positive or all negative

Case-I: All coefficient are positive,

$$K+1>0 \Rightarrow K>-1$$
$$8K-6>0 \Rightarrow K>0.75$$
$$15K+8>0 \Rightarrow K>\frac{-8}{15}$$

∴ $K > 0.75$

Case-II : All coefficients are negative:

$$K+1<0 \Rightarrow K<-1$$
$$8K-6<0 \Rightarrow K<0.75$$
$$15K+8<0 \Rightarrow K<\frac{-8}{15}$$

∴ $K < -1$

So, for stability either $K > 0.75$ or $K < -1$

17. (a)

Transfer function is,

$$G(s)H(s) = \frac{Ks}{\left(\frac{s}{4}+1\right)\left(\frac{s}{10}+1\right)^2} = \frac{400Ks}{(s+4)(s+10)^2}$$

Magnitude of initial plot is given by,

$$M = 20 \log \omega + 20 \log K$$

and $$M = 0 \text{ dB at } \omega = 4$$

$$0 = 20 \log_{10} 4 + 20 \log K$$

(or) $$20\log\frac{1}{K} = 20 \log 4$$

(or) $$\frac{1}{K} = 4$$

(or) $$K = \frac{1}{4}$$

Hence, $$G(s)H(s) = \frac{100s}{(s+4)(s+10)^2}$$

18. (a)

Damped frequency, $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$\Rightarrow$ $$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{10}{\sqrt{1-(0.6)^2}}$$

The desired second order C.E. is,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 2(0.6)(12.5)s + (12.5)^2 = 0$$

$$s^2 + 15s + 156.25 = 0$$

The C.E. of given system is,

$$1+(K_p+sK_D)\left(\frac{1}{s(s+1)}\right) = 0$$

(or) $$s^2 + (1 + K_D)s + K_p = 0$$

Hence, $$K_p = 156.25$$

and $$1 + K_p = 15$$

$\Rightarrow$ $$K_D = 14$$

19. (b)

From the above Bode plot,

For section *de*, slope is –20 dB/dec

$\therefore$ $$-20 = \frac{y-0}{\log 8 - \log 16}$$

$$y = 6.02 \text{ dB}$$

Now, for section *bc*, slope is –20 dB/dec

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$$\therefore \quad -20 = \frac{16 - 6.02}{\log \omega_1 - \log 4}$$

$$\omega_1 = 1.268 \text{ rad/sec}$$

To find value of gain K

$$y = mx + c$$

$$16 = -40 \log 1.268 + 20 \log K$$

$$K = 10.14$$

From all the result, transfer function is,

$$T(s) = \frac{10.14\left(\frac{s}{1.268}+1\right)\left(\frac{s}{4}+1\right)}{s^2\left(\frac{s}{8}+1\right)}$$

$$T(s) = \frac{16(s+1.268)(s+4)}{s^2(s+8)}$$

20. (a)

Given, $$G(s)\,H(s) = \frac{1}{s^2(s-a)\,(s-b)\,(s-c)}$$

Put $s = j\omega$, $$G(j\omega)\;H(j\omega) = \frac{1}{(j\omega)^2(j\omega-a)\,(j\omega-b)\,(j\omega-c)}$$

$$|G(j\omega)\,H(j\omega)| = \frac{1}{\omega^2\sqrt{\omega^2+a^2}\cdot\sqrt{\omega^2+b^2}\cdot\sqrt{\omega^2+c^2}}$$

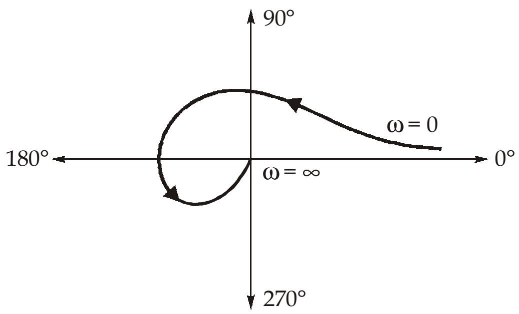
when, $\omega = 0$ Magnitude = ∞; Phase = $0°$

$\omega = \infty$ Magnitude = 0; Phase = $270°$

$$\angle G(j\omega)\;H(j\omega) = -\left[180° + \left(180° - \tan^{-1}\left(\frac{\omega}{a}\right)\right) + \left(180° - \tan^{-1}\left(\frac{\omega}{b}\right)\right) + \left(180° - \tan^{-1}\left(\frac{\omega}{c}\right)\right)\right]$$

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) + \tan^{-1}\left(\frac{\omega}{b}\right) + \tan^{-1}\left(\frac{\omega}{c}\right)$$

Note: Since a, b and c are positive values, for $\omega = 1$, phase of $G(j\omega)\;H(j\omega)$ will be a positive quantity. So, the polar plot will start into the first quadrant.



21. **(d)**

The transfer function from steady state model can be written as

$$T(s) = C[sI - A]^{-1} B$$

$$[sI - A]^{-1} = \frac{\text{adj}[sI - A]}{|sI - A|}$$

$$[sI - A] = \begin{bmatrix} s+5 & -1 \\ 0 & s+4 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+5)(s+4)}\begin{bmatrix} s+4 & 1 \\ 0 & s+5 \end{bmatrix}$$

$$T(s) = \frac{1}{(s+5)(s+4)}\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} s+4 & 1 \\ 0 & s+5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(s) = \frac{1}{(s+5)(s+4)}\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ s+5 \end{bmatrix}$$

$$T(s) = \frac{1+4(s+5)}{(s+4)(s+5)} = \frac{4s+21}{(s+4)(s+5)}$$

If two systems are connected in parallel

Overall transfer function,

$$T'(s) = T(s) + T(s) = \frac{2(4s+21)}{(s+4)(s+5)}$$

22. **(b)**

For the given system we can write,

$$Y(s)\,[s^3 + 4s^2 + 5s + 3] = U(s)[1] \quad \text{...(i)}$$

Let, $x_1 = y$

So, $x_2 = \dot{x}_1 = \dot{y}$ ...(ii)

$x_3 = \dot{x}_2 = \ddot{y}$ ...(iii)

The equation (i) can be written as

$$\dot{x}_3 + 4x_3 + 5x_2 + 3x_1 = u$$

i.e. $\dot{x}_3 = -3x_1 - 5x_2 - 4x_3 + u$ ...(iv)

Using equation (ii), (iii) and (iv) we can write in state variable form as below,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -5 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u$$

So, $$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -5 & -4 \end{bmatrix}$$

and $$[B] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence option (b) is correct.

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23. (d)

The closed-loop transfer function for given system,

$$\frac{G(s)}{1+G(s)} = \frac{\frac{3(1+sK)}{s(s+4)}}{1+\frac{3(1+sK)}{s(s+4)}} = \frac{3(1+sK)}{s(s+4)+3(1+sK)}$$

$$= \frac{3(1+sK)}{s^2+4s+3(1+sK)} = \frac{3+3sK}{s^2+(4+3K)s+3}$$

Comparing the transfer function with standard second order transfer function.

$$\omega_n^2 = 3$$

$$\Rightarrow \qquad \omega_n = \sqrt{3}\,\text{rad/sec}$$

$$2\xi\omega_n = 4+3K$$

$$2\sqrt{3}\xi = 3K+4$$

$$\frac{2\sqrt{3}\xi-4}{3} = K$$

Given, $\xi = \sqrt{3}$, $\qquad K = \frac{6-4}{3} = \frac{2}{3}$

24. (b)

From plot –1

Number of poles on the right side of s-plane = 0

$$P = 0$$

Open-loop system is stable.

From plot –2

Number of encirclement about (–1, 0) is = 2 in clockwise

$$N = -2$$

$$N = P - Z$$

$$-2 = 0 - Z$$

$$Z = 2$$

Two closed loop poles on the right side of s-plane. Close-loop system is unstable.

25. (b)

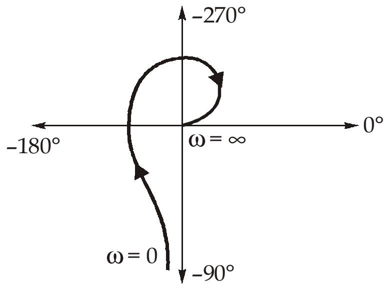
From the figure, the transfer function of the system is,

$$G(s)\,H(s) = \frac{K}{s(s+2)^2(s+4)}$$

$$|G(j\omega)H(j\omega)| = \frac{K}{\omega(4+\omega^2)\sqrt{16+\omega^2}}$$

$$\angle G(j\omega)\,H(j\omega) = -90° - 2\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

ω	M	φ
0	∞	–90°
⋮	⋮	⋮
∞	0	–360°



26. (d)

$$\text{T.F.} = \frac{K\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$$

To obtain the value of K,

$$30\text{ dB} = 20\log K - 20\,r\log(1) \qquad [\because\ r = 1]$$

$$1.5 = \log K$$

$$K = 10^{1.5} = 31.62$$

$$\text{T.F.} = \frac{31.62(0.5s+1)}{s(0.2s+1)(0.1s+1)}$$

27. (c)

$$\angle G(j\omega)\,H(j\omega) = \angle\frac{10}{(j\omega+1)(j\omega+20)} = -180°$$

$$= -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{20}\right) = -180°$$

$$-\tan^{-1}\left(\frac{\omega+\frac{\omega}{20}}{1-\frac{\omega^2}{20}}\right) = -180°$$

$$\frac{\omega+\frac{\omega}{20}}{1-\frac{\omega^2}{20}} = 0$$

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The above expression to become zero $1-\frac{\omega^2}{20}=\infty$.

$\therefore$
$$\omega_{pc} = \infty$$

$$|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{1+\omega^2}\sqrt{(20)^2+\omega^2}}$$

$$|G(j\omega)H(j\omega)|_{\omega_{pc}=\infty} = 0$$

$$\text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega_{pc}}} = \infty$$

$\therefore$ Gain crossover frequency does not exist.

$\therefore$ Phase margin is = ∞

28. (a)

For given state model,

$$A = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}$$

$$[sI-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s+4 & 0 \\ 0 & s+3 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{|sI-A|}\text{Adj}[sI-A] = \frac{1}{(s+3)(s+4)}\begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$$

So zero input response of the given system will be

$$x(t) = e^{At}\cdot x(0)$$

$$= \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 4e^{-4t} \\ -4e^{-3t} \end{bmatrix}$$

29. **(c)**

Method-I

For root locus point,

$$\angle G(s)\ H(s) = 180^o$$

$\therefore$ Substituting, $s = (\sigma + j\omega)$ we get,

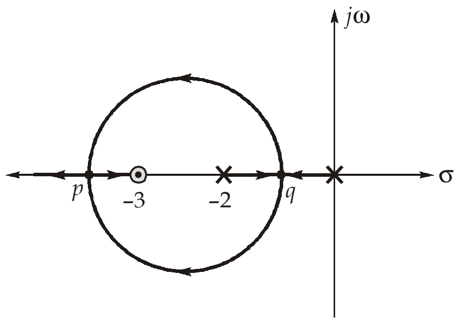
$$G(\sigma + j\omega)\ H(\sigma + j\omega) = \frac{K(\sigma + 3 + j\omega)}{(\sigma + j\omega)(\sigma + 2 + j\omega)}$$

$$\tan^{-1}\left(\frac{\omega}{\sigma+3}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right) = 180° + \tan^{-1}\left(\frac{\omega}{\sigma+2}\right)$$

$$\frac{\frac{\omega}{\sigma+3} - \frac{\omega}{\sigma}}{1 + \left(\frac{\omega}{\sigma+3}\right)\left(\frac{\omega}{\sigma}\right)} = \frac{\omega}{\sigma+2}$$

$\therefore$ $$(\sigma + 3)^2 + \omega^2 = 3$$

Method-II:



p and q are breakaway and breakin points, to obtain them we have to perform, $\frac{dK}{ds} = 0.$

$$K = -\frac{s(s+2)}{s+3}$$

$$\frac{dK}{ds} = \frac{d}{ds}\left[-\frac{s(s+2)}{s+3}\right] = 0$$

$$s^2 + 6s + 6 = 0$$

$$q = -1.268 \quad \text{to} \quad p = -4.732$$

As we know two points on the diameter, center of the circle is (–3, 0) and radius is 1.732.

Equation of the circle is $(\sigma + 3)^2 + \omega^2 = (\sqrt{3})^2$.

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30. **(b)**

The characteristic equation is,

$$q(s) = s^3 + 0.5s^2 + (K+3)\,s + (K+1) = 0$$

$$\begin{array}{c|cc} s^3 & 1 & K+3 \\ s^2 & 0.5 & K+1 \\ s^1 & (3+K)-2(K+1) & 0 \\ s^0 & (K+1) & \end{array}$$

For a system to oscillate a row should become zero.

$$\therefore \quad K + 3 - 2K - 2 = 0$$

$$K = 1$$

Given system is third order system $(s + a)\,(s^2 + bs + c) = 0$

For a marginally stable system, $\xi = 0$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + \omega_n^2 = 0$$

Take the coefficients of s^2 row.

$$0.5s^2 + (K+1) = 0$$

$$0.5s^2 + 2 = 0$$

$$s = \pm j2$$

$$\omega = 2 \text{ rad/s}$$

■■■■