CLASS TEST

Sl. : 01_SK_A_EC+EE_140223

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NETWORK THEORY

EC+EE

Date of Test : 14/02/2023

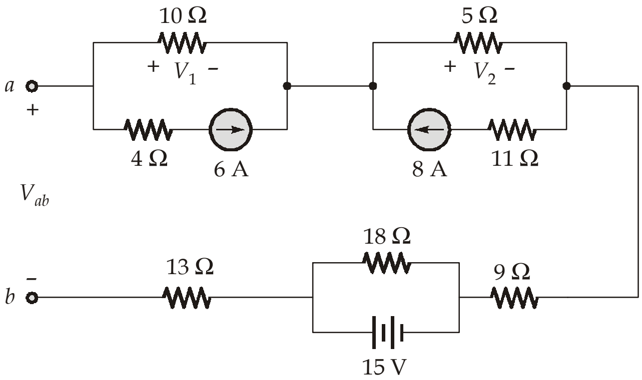
ANSWER KEY

1.	(c)	7.	(c)	13.	(a)	19.	(b)	25.	(c)
2.	(b)	8.	(a)	14.	(b)	20.	(c)	26.	(a)
3.	(a)	9.	(b)	15.	(b)	21.	(c)	27.	(b)
4.	(a)	10.	(c)	16.	(b)	22.	(a)	28.	(c)
5.	(c)	11.	(b)	17.	(a)	23.	(d)	29.	(d)
6.	(a)	12.	(d)	18.	(b)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. **(c)**

Since terminal *a* and *b* forms is open circuit thus, no current flows through circuit is zero, thus the current only flows into the loops.



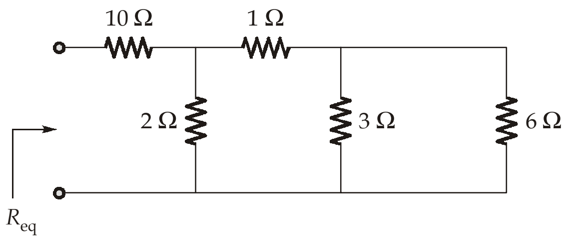
$$V_{ab} = V_1 + V_2 - 15\text{ V}$$

∴ $$V_{ab} = (-6\text{ A})(10\ \Omega) + (8\text{ A} \times 5\ \Omega) - 15$$

$$V_{ab} = -35\text{ V}$$

2. **(b)**

The resistance $6\,\Omega \| 3\,\Omega$ and $12\,\Omega \| 4\,\Omega$ also 1 Ω is in series with 5 Ω, thus, the circuit can be redrawn as



∴ $$R_{eq} = 10\,\Omega + 2\,\Omega \| (1 + 3\,\Omega \| 6\,\Omega)$$

$$R_{eq} = 11.2\ \Omega$$

3. **(a)**

$$Z_L = j\omega L = j\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(1)(0.05)} = -j20\,\Omega$$

∴ $$Z_{eq} = j + 2 \| (-j20) = 1.98 + j0.802\,\Omega$$

and $$Z_L(5j) = 5j\,\Omega$$

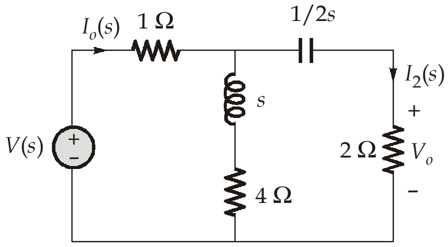
$$Z_C(5j) = -j4\ \Omega$$

∴ $$Z_{eq}(j5) = j5 + 2 \| (-j4) = 1.6 + j4.2\,\Omega$$

Now, $$I(j\omega) \propto \frac{1}{Z(j\omega)}$$

∴ $$\left|\frac{I_o(j)}{I_o(j5)}\right| = \frac{Z(j5)}{Z(j)} = \left|\frac{1.6 + j4.2}{1.98 + j0.802}\right| = 2.104$$

4. (a)



Now, applying current division rule, we get,

$$I_2(s) = \frac{(s+4)I_o(s)}{s+4+2+\dfrac{1}{2s}}$$

$$I_2(s) = \frac{2s(s+4)}{2s^2+12s+1}\cdot I_o(s)$$

$$V_o(s) = 2I_2(s) = \frac{4s(s+4)}{2s^2+12s+1}I_o(s)$$

$$\frac{V_o(s)}{I_o(s)} = \frac{4s(s+4)}{2s^2+12s+1}$$

5. (c)

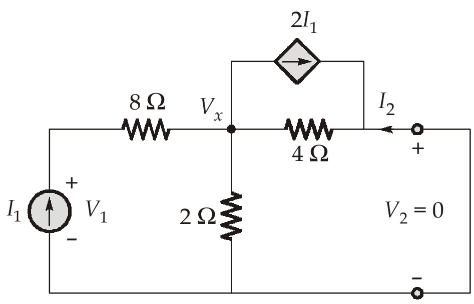
$$v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)]\, e^{-t/\tau}$$

where, $\tau = RC = 4 \times 2 = 8, v(0)$ =4 V and $v_c(\infty) = 20$ V

∴ $$v_c(t) = 20 + (4 - 20)e^{-t/8}$$

$$v_c(t) = 20 - 16\, e^{-t/8} \text{ V}$$

6. (a)



$$I_1 = \frac{V_x}{2}+\frac{V_x}{4}+2I_1$$

$$-I_1 = 0.75V_x \quad \text{...(i)}$$

$$I_2 = -\frac{V_x}{4}-2I_1 = -\frac{V_x}{4}+1.5V_x = 1.25\,V_x$$

Now, $$V_1 = 8I_1 + V_1$$

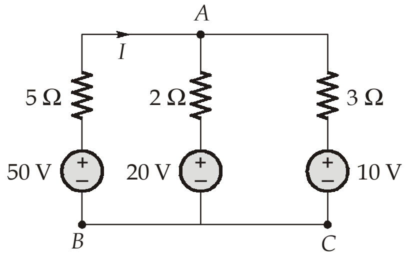
$$V_1 = -6V_x + V_x = -5\,V_x$$

∴ $$\frac{I_2}{V_1} = -\frac{1.25}{5} = -0.25\,\text{S}$$

7. (c)

$$R_L = \left|Z_s^*\right| = |5 - j10| = \sqrt{5^2 + 10^2} = 11.18\ \Omega$$

8. (a)



Using nodal analysis,
KCL at node 'A'

$$\frac{50 - V_A}{5} = \frac{V_A - 20}{2} + \frac{V_A - 10}{3}$$

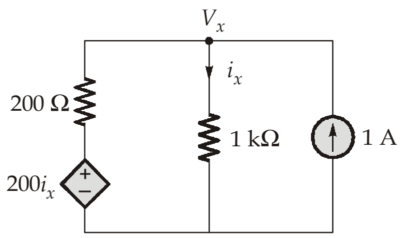
$$V_A = 22.580\text{ V}$$

So, $$I = \frac{50 - 22.580}{5} = 5.483\text{ A}$$

So, power delivered by 50 V is,

$$50 \times I = 50 \times 5.483 = 274.2\text{ Watts}$$

9. (b)



Since, $$i_x = \frac{V_x}{1\text{ k}\Omega} = \frac{V_x}{1000}$$

$$200 i_x = 0.2 V_x$$

and $$\frac{V_x}{1000} + \frac{V_x - 0.2V_x}{200} = 1\text{ A}$$

$$V_x = 200\text{ V}$$

$$R_{eq} = \frac{V_x}{1\text{ A}} = 200\ \Omega$$

10. (c)

Voltage across capacitor

$$v_c(t) = V_{final} + (V_{initial} - V_{final})e^{-t/RC}$$
$$= 0 + (5 - 0)e^{-t/RC}$$

But given, $$v_c(t) = \frac{5}{e} = 5e^{-t/RC}$$

$$\frac{5}{e} = 5e^{-(0.1/40\,k\Omega\times C)}$$

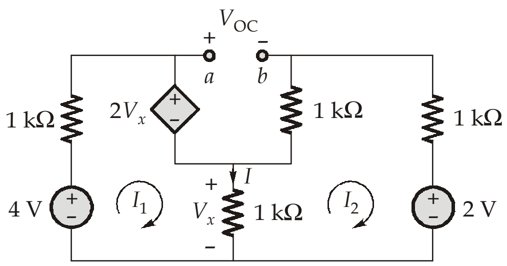
$$\frac{0.1}{40\,k\Omega\times C} = 1$$

$\therefore$ $$C = 2.5\ \mu F \quad (or) \quad 2.5\times 10^{-6}\ F$$

11. (b)

$$R_{ab} = \frac{V_{OC}}{I_{SC}}$$

To find open circuit voltage



$$I = I_1 - I_2$$

Applying KVL in loop 1, we get,

$$(1\ k\Omega)I_1 + 2V_x + I(1\ k\Omega) = 4$$

$$V_x = I(1\ k\Omega)$$

$\therefore$ $$(4\ k\Omega)I_1 - 3\ k\Omega\ I_2 = 4 \quad ...(i)$$

Applying KVL in loop 2, we get,

$$(1\ k\Omega)I_2 + (1\ k\Omega)I_2 - (1\ k\Omega)I = -2$$

$$-(1\ k\Omega)I_1 + (3\ k\Omega)I_2 = -2 \quad ...(ii)$$

Solving equations (i) and (ii), we get,

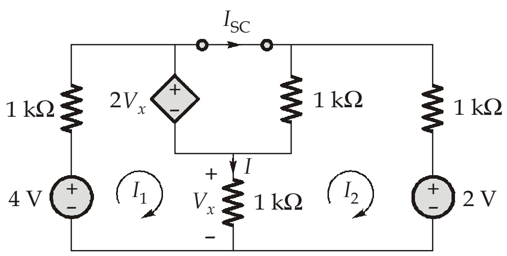
$$I_1 = 0.667\ mA$$

$$I_2 = -0.444\ mA$$

$\therefore$ $$V_{OC} = 2V_x + (1\ k\Omega)I_2$$

$$V_{OC} = 1.78\ V$$

Finding short circuit current



$$I_{SC} = I' + I_2$$

$$I' = I_{SC} - I_2$$

$$I + I_2 = I_1$$

$$I = I_1 - I_2$$

Applying KVL in loop (1)

$$(4\ k\Omega)I_1 + 2V_x - (3\ k\Omega)I_2 = 4 \quad ...(i)$$

Applying KVL in loop (ii)

$$(-1\text{ k}\Omega)I_{SC} - (1\text{ k}\Omega)I_1 + (3\text{ k}\Omega)I_2 = -2 \quad \text{...(ii)}$$

$$V_x = (1\text{ k}\Omega)I = (1\text{ k}\Omega)(I_1 - I_2)$$

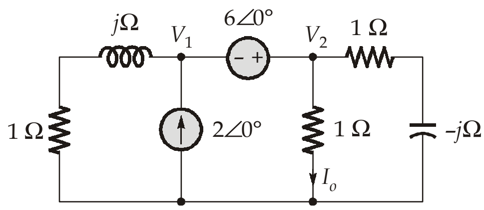
Thus,
$$I_1 = 1.43\text{ mA}$$
$$I_2 = 0.57\text{ mA}$$

∴ $$I_{SC} = \frac{(-1\text{ k}\Omega)I_1 + (3\text{ k}\Omega)I_2 + 2}{1\text{ k}\Omega} = 2.29\text{ mA}$$

∴ $$R_{Th} = \frac{1.78}{2.29}\times 10^3\ \Omega = 777\ \Omega$$

12. (d)

Applying KCL on supernode



$$\frac{V_1}{1+j} - 2 + \frac{V_2}{1} + \frac{V_2}{1-j} = 0$$

and
$$V_1 + 6 = V_2$$

∴ $$\begin{bmatrix} 0.5-0.5j & 1.5+0.5j \\ 1 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

∴ $$V_2 = \frac{\begin{bmatrix} 0.5-0.5j & 2 \\ 1 & -6 \end{bmatrix}}{\begin{bmatrix} 0.5-0.5j & j0.5+0.5j \\ 1 & -1 \end{bmatrix}}$$

$$V_2 = \frac{5.83095\angle 149.036}{2\angle 180°}$$

$$V_2 \approx 2.915\angle -30.96°$$

Thus,
$$I_o = \frac{V_2}{1\ \Omega} = 2.915\angle -30.96°$$

13. (a)

$$V_c(t) = \frac{-1}{10\times 10^{-6}}\left[\int_0^t 0.2e^{-800\tau}d\tau - \int_0^t 0.04e^{-200\tau}d\tau\right] + 5$$
$$= 25e^{-800t} - 20e^{-200t}$$

$$V_L(\text{t}) = 150\times 10^{-3}\frac{dI_o}{dt} = 150(-160e^{-800t} + 8e^{-200t})\times 10^{-3}$$
$$= -24\,e^{-800t} + 1.2e^{-200t}$$

$$V_o(t) = V_c(t) - V_L(t)$$
$$= (25e^{-800t} - 20e^{-200t}) - (-24e^{-800t} + 1.2e^{-200t})$$

$$V_o(t) = (49e^{-800t} - 21.2e^{-200t})\text{ V}$$

14. (b)

The 20 Ω impedance can be reflected to the primary side as

$$Z_R = \frac{20}{n^2} = \frac{20}{4} = 5\,\Omega$$

$$\therefore \quad Z_i = 4 - 6j + 5$$

$$= 9 - 6j = 10.82\angle{-33.69^\circ}\ \Omega$$

$$I_1 = \frac{120\angle 0^\circ}{Z_{in}} = \frac{120\angle 0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ$$

$$\therefore \quad I_2 = -\frac{1}{n} I_1 = -5.545\angle 33.69^\circ\ \text{A}$$

$$V_o = 20I_2 = 110.9\angle 213.69^\circ\ \text{V}$$

15. (b)

The transfer function, $H(s) = \frac{v_o}{v_i} = \frac{R \| 1/sC}{sL + R \| 1/sC}$

$$H(s) = \frac{\frac{R}{1+sRC}}{sL + \frac{R}{1+sRC}} = \frac{R}{s^2RLC + sL + R}$$

$$H(j\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

At corner frequency, $|H(j\omega)| = \frac{1}{\sqrt{2}}|H(j\omega)|_{max}$

Now, $|H(j\omega)| = \frac{R}{\sqrt{(R-\omega^2RLC)^2 + \omega^2L^2}}$

$$|H(j\omega)|^2 = \frac{1}{2} = \frac{R^2}{(R-\omega^2RLC)^2 + \omega^2L^2}$$

$$2 = (1-\omega_o^2LC)^2 + \left(\frac{\omega_o L}{R}\right)^2$$

$$2 = (1-\omega_o^2 4\times10^{-6})^2 + (\omega_o \times 10^{-3})^2$$

$$16\omega_o^4 - 7\omega_o^2 - 1 = 0 \text{(where } \omega_o \text{ is in Krad/s)}$$

$$\therefore \quad \omega_o = 0.742\ \text{K rad/sec} = 742\ \text{rad/sec}$$

16. (b)

$$Z_{AB} = \left(\frac{23}{6}\right) + \left[(3+j4)\|(3-j4)\right]$$

$$= \frac{23}{6} + \frac{(3+j4)(3-j4)}{6} = \frac{23+25}{6} = \frac{48}{6}\,\Omega = 8\,\Omega$$

$$\therefore \quad Z_{AB} = 8\ \Omega$$

17. (a)

$$L_{eq} = L_1 + L_2 - 2M = 4 + 4 - 2 \times 2 = 4 \text{ mH}$$

Resonant frequency, $$f_o = \frac{1}{2\pi\sqrt{L_{eq}C}}$$

$$f_o = \frac{1}{2\pi\sqrt{4\times0.1\times10^{-9}}} = 7.96 \text{ kHz}$$

18. (b)

Impedance matrix for 'N' $= \frac{1}{[Y]} = \frac{1}{5}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix}$

In series connection: individual impedance parameters are added

$\therefore$ For individual network $= \frac{1}{2}\begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix}$

19. (b)

We know that, for a transformer

$$R_s = \left(\frac{n_1}{n_2}\right)^2 R_L$$

$$100 \text{ k}\Omega = \left(\frac{n_1}{n_2}\right)^2 10$$

$$\left(\frac{n_1}{n_2}\right)^2 = 10^4$$

$$\therefore \quad \frac{n_1}{n_2} = 100$$

20. (c)

$$v(t) = 2[u(t) - u(t-2)] \text{ V}$$

$$i(t) = [r(t) - r(t-2)] \text{ A}$$

$$v(t) = 2\frac{di(t)}{dt}$$

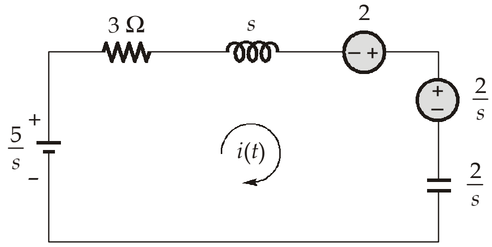
For inductor, $$v(t) = L\frac{di(t)}{dt}$$

$\therefore$ The element is inductor of 2 H

21. (c)

At $t = 0$, switch is closed

For $t > 0$, the circuit in s-domain becomes,



Applying KVL, we get,

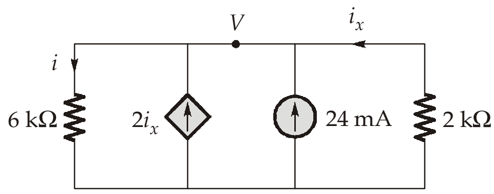
$$\frac{5}{s} - \frac{2}{s} + 2 = \left(3 + s + \frac{2}{s}\right) I(s)$$

$$I(s) = \frac{2s+3}{(s+1)(s+2)}$$

Using partial fractions, $I(s) = \frac{1}{(s+1)} + \frac{1}{(s+2)}$

or $\quad i(t) = L^{-1}[I(s)] = (e^{-t} + e^{-2t})\text{ A} \;;\; \text{for } t > 0$

22. (a)



Applying KCL, $\quad i = 2i_x + 24\text{ mA} + i_x \quad$ ...(i)

where, $\quad i = \frac{V}{6000}$ and $i_x = \frac{-V}{2000}$ ...(ii)

Therefore, from equations (i) and (ii)

$$\frac{V}{6000} + \frac{V}{2000} - 2\left(-\frac{V}{2000}\right) = 24\text{ mA}$$

$\Rightarrow \quad V = (600)(24 \times 10^{-3}) = 14.4\text{ V}$

Hence, power supplied by independent current source

$$P = V \times 24\text{ mA} = 14.4 \times 24 \times 10^{-3} = 345.6\text{ mW}$$

23. (d)

Given,
$f = 1.5$ MHz
$C = 150$ pF
BW = 10 kHz

For series *RLC* circuit, $Q = \frac{f_o}{\text{BW}} = \frac{1.5 \times 10^6}{10 \times 10^3} = 150$

$$Q = \frac{1}{\omega RC}$$

$$\frac{1}{150} = 2\pi \times 1.5 \times 10^6 \times 150 \times 10^{-12} \times R$$

$$R = \frac{10^6}{2\pi \times 1.5 \times 150 \times 150} = 4.71\ \Omega$$

24. (b)

For t < 0, source $2\,u(t) = 0$

Therefore, $\quad i_L(0^-) = i_L(0^+) = 0$ A

$v_c(0^-) = v_c(0^+) = 0$ V

For $t > 0$, $\quad i_c(0^+) = 2$ mA

$$i_c(0^+) = c\frac{dv_c(0^+)}{dt}$$

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{2\times10^{-3}}{4\times10^{-3}} = 0.5\text{ V/sec}$$

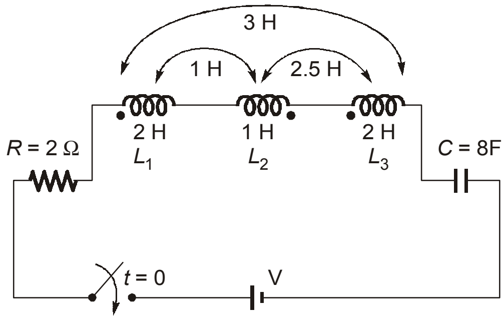
25. (c)

For a capacitor $i(t) = \frac{cdv(t)}{dt} = 10\times10^{-6}\frac{dv(t)}{dt}\times10^3\text{ A}$

$$= 10^{-2}\frac{dv(t)}{dt}\text{ A} = 10\frac{dv(t)}{dt}\text{ mA}$$

26. (a)

For the circuit



$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$
$$L_1 = 2\text{ H}$$
$$L_2 = 1\text{ H}$$
$$L_3 = 2\text{ H}$$
$$M_{12} = 1\text{ H}$$
$$M_{23} = 2.5\text{ H}$$
$$M_{13} = 3\text{ H}$$
$$L_{eq} = 2 + 1 + 2 - 2 - 5 + 6$$
$$= 11 - 7$$
$$= 4\text{ H}$$
$$C = 8\text{ F}$$
$$R = 2\ \Omega$$

$$\therefore \quad \omega_n = \frac{1}{\sqrt{L_{eq}C}} = \frac{1}{\sqrt{8\times4}} = 0.176\text{ rad/sec} \approx 0.18\text{ rad/sec}$$

Note : M_{12}, M_{23} is negative, because both L_1, L_2 and L_2, L_3 opposes the flux of respective loops.

27. (b)

$\because$ $p(t)$ varies with time, thus it can be concluded that the network is not purely resistive circuit.

$\therefore$ let, $v(t) = \sqrt{2}V_{rms}\cos(\omega t + \theta_v)$

$$i(t) = \sqrt{2}I_{rms}\cos(\omega t + \theta_I)$$

then, the instantaneous power into the network N is given as,

$$p(t) = v(t)\ i(t) = 2V_{rms}I_{rms}\cos(\omega t + \theta_v)\cos(\omega t + \theta_I)$$

$$= V_{rms}I_{rms}[\underbrace{\cos(\theta_v - \theta_I)}_{\text{constant}} + \underbrace{\cos(2\omega t + \theta_v + \theta_I)}_{\text{time varying}}]$$

thus, for minimum power delivered,

$$\cos(2\omega t + \theta_v + \theta_I) = -1$$

and for maximum power delivered

$$\cos(2\omega t + \theta_v + \theta_I) = 1$$

$$p(t)_{max} = V_{rms}\,I_{rms}\cos(\theta_v - \theta_I) + V_{rms}\,I_{rms} \quad \text{...(i)}$$

$$p(t)_{min} = V_{rms}\,I_{rms}\cos(\theta_v - \theta_I) - V_{rms}\,I_{rms} \quad \text{...(ii)}$$

Thus, from equation (i) and (ii), we get,

thus, $2V_{rms}\,I_{rms} = 2500$

$$V_{rms}\,I_{rms} = 1250$$

$$I_{rms} = \frac{1250}{V_{rms}} = \frac{1250}{100} = 12.5\text{ A}$$

28. (c)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}\times 0.4\times 10^{-6}}} = 50\text{ krad/sec}$$

$$Q = \frac{\omega_0 L}{R} = \frac{50\times 10^3\times 10^{-3}}{2} = 25$$

$$B = \frac{\omega_0}{Q} = \frac{50\times 10^3}{25} = 2\text{ krad/sec}$$

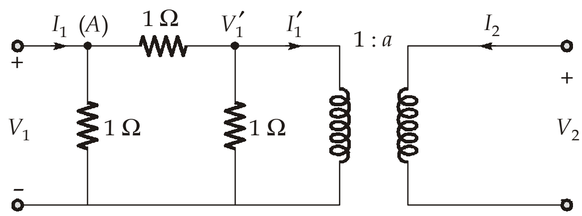
now, $\omega_1 = \omega_0 - \frac{B}{2} = 50 - 1 = 49\text{ krad/sec}$

now, $\omega_2 = \omega_0 + \frac{B}{2} = 50 + 1 = 51\text{ krad/sec}$

Hence, option (c) is incorrect.

29. (d)

Now, applying KCL at node A, we get,



$$I_1 = V_1 + (V_1 - V_1')$$

$$= 2V_1 - V_1'$$

$$I_1 = 2V_1 - \frac{1}{a}V_2$$

For I_2, we can write

$$I_2 = -\frac{1}{a}I_1' = -\frac{1}{a}\left[-V_1' + (V_1 - V_1')\right]$$

$$= -\frac{1}{a}V_1 + \frac{2}{a^2}V_2$$

$$\therefore \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{a} \\ -\frac{1}{a} & \frac{2}{a^2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the Z-parameter to not exist.

$$|Y| = 0$$

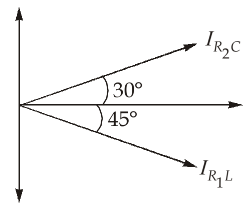
$$\therefore \quad |Y| = \frac{4}{a^2} - \frac{1}{a^2} = \frac{3}{a^2}$$

$$\because \quad |Y| \neq 0$$

Thus, no such value exist for which $|Y| = 0$.

30. (a)

From phasor, we can write



$$\tan 30° = \frac{X_C}{R_2}$$

$$\Rightarrow \quad R_2 = X_C\sqrt{3} = \frac{\sqrt{3}}{\omega C}$$

$$\tan 45° = \frac{X_L}{R_1}$$

$$\Rightarrow \quad R_1 = X_L = \omega L$$

$$R_1R_2 = \frac{\sqrt{3}}{\omega C} \times \omega L = \frac{L}{C}\sqrt{3}$$

$$R_1R_2 = \sqrt{3} = 1.732$$

we know

$$\frac{R_1 + R_2}{2} \geq \sqrt{R_1 R_2}$$

as arithmetic mean ≥ geometric mean ; (for non-negative real numbers)

$$R_1 + R_2 \geq 2\sqrt{\sqrt{3}}$$

$$R_1 + R_2 \geq 2(3)^{1/4}$$

Minimum value of $R_1 + R_2 = 2.63\ \Omega$

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