S.No. : 03 LS1_EC_B_130519
Control Systems

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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

Control Systems

Date of Test : 13/05/2019

Answer Key

1.	(a)	7.	(d)	13.	(d)	19.	(c)	25.	(c)
2.	(d)	8.	(c)	14.	(b)	20.	(b)	26.	(c)
3.	(d)	9.	(c)	15.	(c)	21.	(c)	27.	(c)
4.	(c)	10.	(c)	16.	(a)	22.	(c)	28.	(c)
5.	(c)	11.	(a)	17.	(c)	23.	(c)	29.	(d)
6.	(c)	12.	(a)	18.	(b)	24.	(b)	30.	(c)

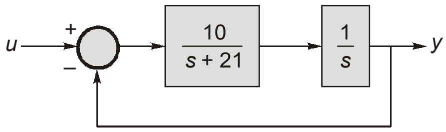
DETAILED EXPLANATIONS

1. (a)

From figure (a)

Eliminating first loop

$$\frac{y}{u} = \frac{\frac{10}{s(s+21)}}{1+\frac{10}{s(s+21)}} = \frac{10}{s^2+21s+10}$$



For the figure (b) $$\frac{y}{u} = \frac{\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}\times H} = \frac{10}{s^2+s+10H}$$

Comparing, $$s^2 + 21s + 10 = s^2 + s + 10H,$$
$$10H = 20s + 10,$$
$$H = 2s + 1$$

2. (d)

Centroid, $\sigma = -2$

Break away point $= -2$. In choice (c), break away point is between 1 and 2 and centroid = –2.

3. (d)

$\because$ $$G(j\omega) = \frac{(-\omega^2+4)(j\omega+2)}{(j\omega+9)(j\omega+8)(j\omega+6)}$$

$\therefore$ $G(j\omega) = 0$ at $\omega^2 = 4$

$\omega = 2$ rad/sec or $f = 0.318$ Hz

4. (c)

T_D the delay time, is not a desirable feature. It is the time taken by the system before starting to respond. Hence. T_S/T_D cannot be less than 1 as the system is unstable in this case. So the ratio T_S/T_D should be made as large as possible to make the sytem controllable.Therefore, the answer at (c) (greater than 10) is appropriate.

5. (c)

loop $L_1 = -2$

loop $L_2 = -4$

non-touching loop $L_1L_2 = 8$

forward path $P_1 = 4 \times 3 \times 2 = 24$

forward path $P_2 = 5$

$\therefore$ $$\frac{C}{R} = \frac{24+5}{1+2+4+8} = \frac{29}{15} = 1.933$$

6. (c)

$$t_r = \frac{\pi - \phi}{\omega_d}$$

where $$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

and $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

7. (d)

characteristic equation $= |(sI - A)|$

$$|(sI - A)| = \begin{vmatrix} s+1 & 0 \\ 0 & s+2 \end{vmatrix} = (s+1)(s+2) = s^2 + 3s + 2$$

comparing with second order characteristic equation

$$\omega_n = \sqrt{2}$$

$$2\xi\omega_n = 3$$

$$\xi = \frac{1.5}{\sqrt{2}} = 1.06$$

thus the system is over damped.

8. (c)

The transfer function has 3 poles since 1 pole introduces a decay of –20 dB/dec and 2 zeros because 1 zero adds a slope of 20 dB/dec.

9. (c)

$$\text{Gain margin} = \frac{1}{a}$$

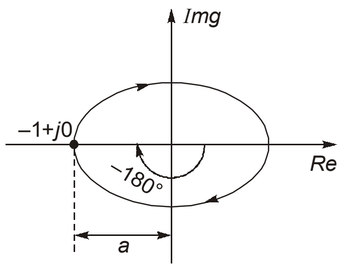
Where, $a = 1$

$$\text{G.M.} = \frac{1}{a} = 1$$

$$\text{G.M. in dB} = 20 \log 1 = 0$$

For phase margin, $\phi = -180°$

$$\text{P.M.} = 180 + \phi = 180 - 180 = 0$$



11. (a)

$$G(s) = \frac{1}{sT_1(1+sT_2)}$$

$$TF = \frac{\frac{1}{sT_1(1+sT_2)}}{1+\frac{1}{sT_1(1+sT_2)}} = \frac{1}{sT_1(1+sT_2)+1} = \frac{1}{s^2T_1T_2 + sT_1 + 1}$$

$$= \frac{1}{T_1T_2\left(s^2 + \frac{s}{T_2} + \frac{1}{T_1T_2}\right)}$$

$$\omega_n = \frac{1}{\sqrt{T_1T_2}};$$

$$\xi = \frac{1}{2}\sqrt{\frac{T_1}{T_2}}$$

for $\xi << 1$, $\Rightarrow$ $T_1 << T_2$

12. (a)

Zeros $s = -2$

Poles: $s = -1 + j2, -1 - j2$

The transfer function is

$$G(s) = \frac{K(s+2)}{(s+1-j2)(s+1+j2)}$$

where $K = \frac{\text{multiplication of vector lengths drawn from all poles}}{\text{multiplication of vector lengths drawn from all zeros}}$

$$= \frac{\sqrt{10}\times\sqrt{2}}{\sqrt{5}} = 2$$

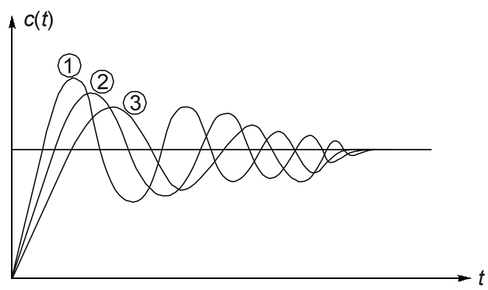
$$G(s=j1) = \frac{2(j1+2)}{(j1+1-j2)(j1+1+j2)} = \frac{2(2+j1)}{(1-j1)(1+j3)}$$

$$= \frac{2\times 2.236\angle 26.6^\circ}{1.4142\angle -45^\circ \times 3.162\angle 71.57^\circ} = 1\angle 0^\circ$$

Hence choice (a) is correct.

13. (d)

The poles of the three systems will have same real part and two poles for each system indicate a second order system. The envelope of the second order system with unit step input is governed by $e^{-\xi\omega_n t}$. Thus the second order system will have same envelope for the following systems.



14. (b)

ω_{pc} is given by

$$-\tan^{-1}(\omega_{pc}) - \tan^{-1}(2\omega_{pc}) - \tan^{-1}(3\omega_{pc}) = -180^\circ$$

$$\tan^{-1}(\omega_{pc}) + \tan^{-1}(2\omega_{pc}) = 180^\circ - \tan^{-1}(3\omega_{pc})$$

$$\frac{3\omega_{pc}}{1-2\omega_{pc}^2} = -3\omega_{pc}$$

$$1 = -1 + 2\omega_{pc}^2$$

$$\omega_{pc} = 1 \text{ rad/sec}$$

15. (c)

$$r(t) \longrightarrow \boxed{\frac{K}{s+a}} \longrightarrow c(t)$$

$\therefore$
$$C(s) = \frac{K}{s(s+a)}$$

$$C(s) = \frac{K/a}{s} - \frac{K}{a(s+a)}$$

$$c(t) = \frac{K}{a}(1-e^{-at})$$

$$c(\infty) = \frac{K}{a} = 1$$

$\therefore$
$$K = a$$

now analysing slope at $t = 0$, we get

$$\lim_{t\to 0}\frac{d}{dt}\left(\frac{K}{a}(1-e^{-at})\right) = \lim_{t\to 0} Ke^{-at} = \frac{1}{0.2} = 5$$

$\therefore$
$$K = 5$$
$$a = 5$$

16. (a)

At $\omega = 0$ $\quad \angle G(j\omega)H(j\omega) = -90°$

At $\omega = \infty$ $\quad \angle G(j\omega)H(j\omega) = -360°$

type 1 and order $= 4$

17. (c)

$$G(s) = \frac{K-2}{(s^2+2s+2)(s+1)+2k}$$

The characteristics equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K-2}{(s^2+2s+2)(s+1)+2k} = 0$$

or $\quad (s+1)(s^2+2s+2) + 2k + k - 2 = 0$

or $\quad s^3 + 2s^2 + 2s + s^2 + 2s + 2 + 3K - 2 = 0$

or $\quad s^3 + 3s^2 + 4s + 3K = 0$

Routh Array

s^3	1	4
s^2	3	$3K$
s^1	$\frac{12-3K}{3}$	0
s^0	$3K$	

For marginal stable, $12 - 3K = 0$

or $K = 4$

Auxilliary equation,

$$3s^2 + 3K = 0$$

for $s = j\omega$, $-3\,\omega^2 + 3 \times 4 = 0$

$$\omega^2 = 4$$

$$\omega = 2 \text{ rad/sec}$$

18. (b)

Characteristic equation,

$$1 + G(s)\,H(s) = 0$$

or, $s^2 + 15s + 200 = 0$

On comparison with standard second order characteristic equation we get,

$$2\xi\omega_n = 15 \quad \text{and} \quad \omega_n = \sqrt{200} \text{ rad/sec}$$

or $\xi = \dfrac{15}{2 \times \sqrt{200}} = 0.53$

Resonant peak, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}} = \dfrac{1}{2 \times 0.53\sqrt{1-0.53^2}}$

$$= 1.11$$

19. (c)

Characteristics equation $= 1 + G(s)\,H(s) = 0$

$$1 + \frac{Ke^{-2s}}{(s+1)(s+2)} = 0 \qquad \text{Where, } e^{-sT} = (1 - sT) \qquad \text{[We can approximate]}$$

$$1 + \frac{K(1-2s)}{(s+1)(s+2)} = 0$$

or $s^2 + 3s + 2 + K - 2Ks = 0$

$$s^2 + s(3 - 2K) + K + 2 = 0$$

Using Routh Array

s^2	1	$K+2$
s^1	$3-2K$	0
s^0	$K+2$	

For stability

$$3 - 2K > 0 \quad \text{...(i)}$$
$$K + 2 > 0 \quad \text{...(ii)}$$

From equation (i), $K < 1.5$

From equation (ii), $K > -2$

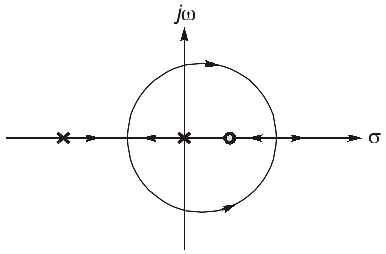
Therefore, $-2 < K < 1.5$

For absolute stability, $K = 1$

20. (b)

$$G(s) = \frac{K(s-1)}{s(s+2)}, \qquad K < 0$$

Since, $K < 0$ so, it have complementary root locus.



21. (c)

Closed loop transfer function,

$$T(s) = \frac{6}{s^2 + 5s + 6}$$

$$\text{Open loop T.F.} = \frac{6}{s^2 + 5s + 6 - 6} = \frac{6}{s(s+5)}$$

$$G(j\omega) = \frac{6}{j\omega(j\omega + 5)}$$

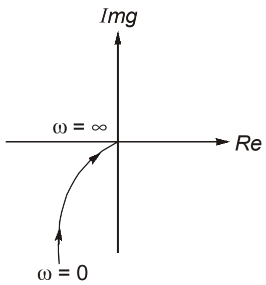
$$\text{Magnitude, } M = \frac{6}{\omega\sqrt{\omega^2 + 25}}$$

$$\text{Phase angle, } \phi = -90° - \tan^{-1}\left(\frac{\omega}{5}\right)$$

at, $\omega = 0$, $M = \infty$, $\phi = -90°$

at, $\omega = \infty$, $M = 0$, $\phi = -180°$

Polar plot



22. (c)

$$N = \frac{t_s}{T} = \frac{\text{settling time}}{\text{time period oscillation}} = \frac{4}{\xi\omega_n} \times \frac{\omega_d}{2\pi} = \frac{2\omega_n\sqrt{1-\xi^2}}{\pi\xi\omega_n}$$

$$N = \frac{2\sqrt{1-\xi^2}}{\pi\xi}$$

$$\Rightarrow \quad \frac{\pi\xi}{\sqrt{1-\xi^2}} = \frac{2}{N}$$

we know that, $$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\ln M_p = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$|\ln M_p| = \frac{\pi\xi}{\sqrt{1-\xi^2}} = \frac{2}{N}$$

$$|\ln M_p| \propto \frac{1}{N}$$

23. (c)

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = 30^\circ$$

ϕ_m = is positive, compensator is lead compensator

$$\frac{1-\alpha}{1+\alpha} = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = \frac{1}{3}$$

Also $$\omega_m = \frac{1}{T\sqrt{\alpha}} = \sqrt{3}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{T\sqrt{1}} = \sqrt{3}$$

$$\Rightarrow \quad T = 1$$

$$\text{zero is at } -\frac{1}{T} = -1$$

$$\text{pole is at } -\frac{1}{\alpha T} = -\frac{1}{\frac{1}{3}\cdot 1} = -3$$

$$\text{Transfer function} = \frac{s+1}{s+3}$$

24. (b)

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)+1}{1+(1+G(s))\cdot 1} = \frac{1+G(s)}{2+G(s)}$$

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial T}{T} \times \frac{G}{\partial G} = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

$$= \frac{2+G(s)-1-G(s)}{(2+G(s))^2} \times \frac{G(s)}{1+G(s)} \times 2+G(s) = \frac{G(s)}{(1+G(s))(2+G(s))}$$

25. (c)

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+a)(s+b)+s+K}$$

characteristic equation = $s^3 + (a + b)s^2 + (ab + 1)\ s + K$

Routh Table

$$\begin{array}{lll} s^3 & 1 & ab+1 \\ s^2 & a+b & K \\ s^1 & \dfrac{(a+b)(ab+1)-K}{a+b} & \\ s^0 & K & \end{array}$$

For oscillations s^1 Row must have zero element

$$(a + b)\ (ab + 1) = K$$

Also auxiliary equation

$A(s) = (a + b)s^2 + K$

at $s = j\omega$

$$(a+b)(-\omega^2)+K$$

$$\omega^2 = \frac{K}{a+b} = \frac{(a+b)(ab+1)}{(a+b)}$$

$$\omega = \left(\sqrt{ab+1}\right) \text{rad/sec}$$

26. (c)

The open loop transfer function

$$G(s)\ H(s) = \frac{K(s+1)(s+2)}{2s}$$

Calculation of 'K'

$$5\ \text{dB}\big|_{\omega = 0.1} = 20 \log K - 20 \log 0.1$$

$$K = 0.178$$

Calculation of ω_{gc}

$$G(s)\ H(s)\,\big|_{\omega = \omega_{gc}} = 1$$

$$\frac{0.178\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}}{2\omega} = 1$$

$$(\omega^4 + 5\omega^2 + 4) = 126.25\ \omega^2$$

$$\omega^4 - 121.25\ \omega^2 + 4 = 0$$

or $\omega = 0.18$ rad/sec and 11.22 rad/sec

27. (c)

Since the system is unity feedback $H(s) = 1$

Characteristic equation : $1 + G(s)\ H(s) = 0$

$$1+\frac{K(s^2+1)}{(s+1)(s+a)}\cdot(1) = 0$$

$$s^2 + (a+1)s + a + Ks^2 + K = 0$$
$$(1+K)s^2 + (a+1)s + (K+a) = 0$$

using Routh's tabular form

s^2	$(1+K)$	$(a+K)$
s^1	$(a+1)$	
s^0	$(a+K)$	

for stability of the system

$$(1+K) > 0 \Rightarrow K > -1$$
$$a+1 > 0 \Rightarrow a > (-1)$$
$$a+K > 0 \Rightarrow a > (-K) \Rightarrow a > 1 \quad \because K > (-1)$$

So, $a > 1$ is the correct choice.

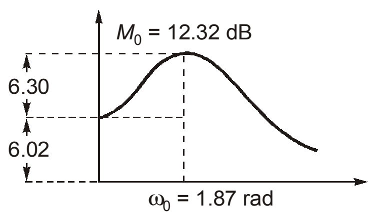
28. **(c)**

On observation

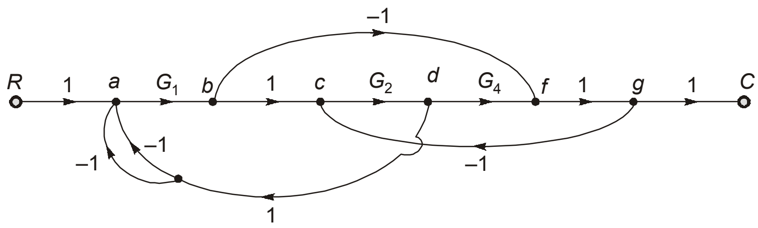
$$\omega_0 = \text{resonant frequency}$$
$$M_0 = \text{resonant peak}$$
$$\omega_0 = \omega_r = \omega_n\sqrt{1-2\xi^2}$$
$$M_0 = M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$
$$\text{transfer function} = \frac{C(s)}{R(s)} = \frac{2\times 4}{s^2+s+4} = 2\left(\frac{4}{s^2+s+4}\right)$$
$$2\xi\omega_n = 1$$
$$\omega_n = 2$$
$$\xi = 0.25$$
$$\omega_0 = 2\sqrt{1-2\times\frac{1}{16}}$$
$$\omega_0 = 2\sqrt{\frac{7}{8}} = 1.87 \text{ rad/sec}$$
$$M_0 = \frac{1}{2\times\frac{1}{4}\sqrt{1-\frac{1}{16}}} = \frac{1}{\frac{1}{2}\sqrt{\frac{15}{16}}} = \frac{1}{\frac{\sqrt{15}}{8}} = \frac{8}{\sqrt{15}}$$
$$M_0 = 20\log_{10}\left(\frac{8}{\sqrt{15}}\right) = 6.30 \text{ dB}$$

Since system has gain '2'.

at $\omega = 0$ gain is $10\log_{10}2 = 6.02$ dB



29. (d)



Applying Mason gain formula

$$L_1 = defgd = -G_2G_4$$
$$L_2 = bcdeb = L_3 = abcda = L_4 = \text{cfgdebc} = L_5 = cfgdeabc = -G_1G_2$$
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$
$$= 1 + G_2G_4 + 4G_1G_2$$
$$p_1 \text{ (forward path)} = G_1G_2G_4$$
$$p_2 \text{ (forward path)} = G_1(-1)$$

All loops are touching paths p_1 and p_2

$$\frac{C}{R} = \frac{G_1G_2G_4 - G_1}{1 + 4G_1G_2 + G_2G_4}$$

30. (c)

$$T(s) = \frac{8}{(s+10)^2}$$
$$s = j\omega$$
$$T(j\omega) = \frac{8}{(j\omega+10)^2}$$
$$|T(j\omega)| = \frac{8}{\left(\sqrt{\omega^2+10^2}\right)^2}$$
$$\text{output amplitude} = 2 \times \frac{8}{\left(\sqrt{\omega^2+10^2}\right)^2} \qquad \because \text{output amplitude} = \text{input amplitude} \times |T(\omega)|$$

where $\omega = 3$ rad/sec

$$\text{output amplitude} = \frac{2\times 8}{\left(\sqrt{9+100}\right)^2} = \frac{16}{109}$$
$$= 0.146 \approx 0.15$$

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